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Highlights

- Utilizing a real options approach, we develop an investment and financing model with a partial guarantee.
- We explicitly derive the pricing and timing of the option to invest for the cash flow with both diffusion and jump risk.
- If the funding gap rises, the option value decreases but the investment threshold first declines and then increases.
- The larger the guarantee level, the lower the option value and the later the investment.
- Raising guarantee levels reduce borrowers' risk-shifting incentives but do not change their incentives to replenish equity.

Investment and financing for SMEs with a partial guarantee and jump risk ☆

Pengfei Luo[a], Huamao Wang[b], Zhaojun Yang[a,*]

[a]*School of Finance and Statistics, Hunan University, Changsha, China.*
[b]*School of Mathematics, Statistics and Actuarial Science, University of Kent, UK.*

Abstract

We consider a small- and medium-sized enterprise (SME) with a funding gap intending to invest in a project, of which the cash flow follows a double exponential jump-diffusion process. In contrast to traditional corporate finance theory, we assume the SME is unable to get a loan directly from a bank and hence it enters into a partial guarantee agreement with an insurer and a lender. Utilizing a real options approach, we develop an investment and financing model with a partial guarantee. We explicitly derive the pricing and timing of the option to invest. We find that if the funding gap rises, the option value decreases but its investment threshold first declines and then increases. The larger the guarantee level, the lower the option value and the later the investment. The optimal coupon rate decreases with project risk and a growth of the guarantee level can effectively reduce agency conflicts.

Keywords: Finance, Investment analysis, Guarantee level, Real options,

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*Corresponding author. Tel: +86 731 8864 9918; Fax: +86 731 8868 4772.

Email addresses: `pengjuv1@163.com` (Pengfei Luo), `h.wang@kent.ac.uk` (Huamao Wang), `zjyang@hnu.edu.cn` (Zhaojun Yang)

Double exponential jump-diffusion process
JEL: G11, G13, G32

1. Introduction

Motivation. Small- and medium-sized enterprises (SMEs, henceforth) are the engine of the world economy. They provide a large number of job opportunities and create entrepreneurial spirit and technique innovation. Thus, they are crucial for fostering competitiveness and employment. Unfortunately, SMEs are severely limited by borrowing constraints when they have opportunities to invest in a project for business expansion. Particularly, they might not be able to borrow from banks at all, or they are offered with unfavourable lending conditions. As a result, they have to abandon potentially valuable investment opportunities. This situation becomes worse after the recent financial crisis. As reported by World Business Environment Survey, on average 43 (resp. 11) percent of businesses with 20 to 99 employees rate access to finance or cost of finance as a major constraint in developing (resp. developed) countries.[1] Indeed, SMEs incur more financing obstacles than large firms due to SMEs' low credibility and strong information asymmetry between lenders and borrowers, see, e.g., Andrikopoulos (2009). To alleviate this problem, Kang (2005) among others suggests that credit guarantees are effective in improving SMEs' credibility and information disclosure. In particular, Xiang and Yang (2015) develop a simple model and show that credit guarantees can completely eliminate the financing constraints in theory.

[1] Enterprise Surveys Database 2010; http://www.enterprisesurveys.org; "World Business Environment Survey" of more than 10,000 firms in 80 countries.

In reality, SMEs heavily rely upon credit guarantees for corporate financing in Asia, see the BIS Quarterly Review (Shim, 2006). In particular, Chinese entrepreneurs have invented a flexible and popular guarantee agreement, called a partial guarantee swap, which avoids incentive distortions usually caused by the existing government guarantee schemes of China. However, there are no papers to provide a quantitative research for such swap, let alone consider how to invest with it under a state-of-the-art jump-diffusion model.

Our work. In this paper, we consider an SME who intends to invest in an irreversible project with entry flexibility but has a funding gap. After the irreversible investment, the project generates the cash flow that follows a double exponential jump-diffusion process. In contrast to traditional corporate finance theory, we assume that the SME is unable to obtain a loan directly from a bank because of high project risk, low credibility, and strong information asymmetry. To overcome such financing constraint, the SME enters into a partial guarantee agreement with an insurer and a lender. According to the agreement, the lender lends cash to the SME and the insurer promises to undertake a fraction (guarantee level) of debt once the SME defaults. In return, the SME (borrower) allocates a fraction of equity and a fixed guarantee fee rate per unit time to the insurer.

We develop a real options model and discuss the SME's investment and financing strategies given the partial guarantee contract above. We derive the explicit formulas for the pricing and timing of the option to invest for the cash flow with both diffusion and jump risk. The two sources of project risk increase the option value and postpone investment. More importantly, we reveal that larger funding gaps or higher guarantee levels lead to the later investment and lower option values. Interestingly, raising the guarantee level can effectively reduce the borrower's moral hazard to increase equity values at the expense of the lender. Meanwhile, a growth in the guarantee level does not change the SME's incentive to replenish equity.

Literature review. Our work relates to the real options literature that takes into account the interaction between investment and financing decisions. Myers (1977) and Jensen and Meckling (1979) focus on the impact of stockholder-bondholder conflicts on a firm's financing and investment decisions. Boyle and Guthrie (2003) examine the effect of a financing constraint on investment timing and find that the financial constraint accelerates investment because the threat of future funding shortfalls lowers the value of waiting. Belhaj and Djembissi (2009) point out that debt financing costs reduce tax shields and consequently force entrepreneurs to postpone investment. Hirth and Uhrig-Homburg (2010) and Shibata and Nishihara (2015) illustrate that the investment threshold of a firm is a non-monotonic function of debt financing costs or debt issuance limits. Sundaresan, Wang and Yang (2015) develop a dynamic investment and financing model to investigate stockholder-bondholder conflicts.

The dynamics of cash flow generated by the project determines the pricing and timing of the option to invest. There are two kinds of well-known models to describe the dynamics: stochastic volatility models and the jump-diffusion processes. The latter seems more suitable to describe the dynamics of cash flow for a SME. A jump-diffusion model is first introduced by Merton (1976) to option pricing. Recently, a new jump-diffusion model, named the double exponential jump-diffusion process, is attracting more research interests due to two appealing properties of the double exponential distribution (Kou, 2002). First, its two-sided jumps and the leptokurtic feature of jump size lead to the peak and heavy tails of return distribution found in reality. Second, the double exponential distribution has a memoryless feature which facilitates the calculation of conditional means and variances. In such jump-diffusion framework, Kou and Wang (2003) study the first hitting time. Kou and Wang (2004) derive the solution for valuing an American option, and Chen and Kou (2009) investigate a variety of credit spreads. However, to the best of our knowledge, there are seldom papers studying real options with this well-behaved jump-diffusion model.

Our paper is also related with Yang and Zhang (2013), Yang and Zhang (2015a), Yang and Zhang (2015b), Xiang and Yang (2015), and Wang et al. (2015) which study an equity-for-guarantee swap or an option-for-guarantee swap. However, the flexible partial guarantee we discuss here is actually more popular with entrepreneurs due to its extra advantage. Specifically, SMEs can enter into a "personalized" guarantee contract by choosing appropriate combinations of a fixed guarantee fee and a fraction of equity as guarantee costs according to their investment projects. Moreover, to the best of our knowledge, all previous studies do not consider a real options problem based on a double exponential jump-diffusion process, let alone considering investment with a partial guarantee.

The structure of the article is as follows. Section 2 develops a model and discusses the pricing and timing of the option to invest. Section 3 presents numerical results and comparative static analysis. Section 4 concludes. The

Appendices present mathematical details and derivations.

2. The pricing and timing under a partial guarantee

The model. We assume an SME has a monopolistic and perpetual option to implement an irreversible investment project incurring a sunk cost I. The cash flow of the project before interest is observable and independent of the SME's capital structure. In contrast to a common continuous cash flow model, we assume the cash flow δ follows a double exponential jump-diffusion process of Chen and Kou (2009) under a given risk-neutral probability $\mathbb{Q}$, i.e.

$$\frac{d\delta_t}{\delta_{t-}} = \mu dt + \sigma dB_t + d\left(\sum_{i=1}^{N_t}(Z_i - 1)\right) - \lambda\xi dt, \tag{1}$$

where μ is a constant risk-adjusted growth rate, σ is a constant volatility, and the process $\{B_t, t \geq 0\}$ is a standard Brownian motion under $\mathbb{Q}$. In addition, for the jump part, ξ is the mean percentage jump size given by (A.4), $\{N_t, t \geq 0\}$ is a $\mathbb{Q}$-Poisson process with a constant intensity rate $\lambda > 0$, and the Z_i's are i.i.d nonnegative random variables. We assume all sources of randomness N, B, Z_i are independent under $\mathbb{Q}$.

We note that on the contrary to large companies, SMEs in fact can seldom issue bonds directly. To overcome such financing constraint, we introduce a new swap agreement among a borrower (SME), a lender (bank), and an insurer in contrast to the well-known corporate finance theory. Under the agreement, the bank lends at a given interest rate to the SME and if it defaults on the loan, the insurer must pay a certain part of the outstanding interest and principal to the bank. In return for the guarantee, the SME must allocate a fraction of its equity and a given guarantee fee rate to the insurer.

Specifically, after debt financing is provided, an SME pays a fixed coupon payment C per unit of time to the bank when the project is alive. After bankruptcy, the insurer pays a fixed payment kC $(0 \leq k \leq 1)$ per unit of time to the bank instead of the SME. The insurer gains a fraction ψ of the SME's equity together with a fixed cash guarantee fee rate g per unit of time only if default does not happen. The bankruptcy threshold is endogenously decided by the SME. When the firm goes bankrupt, the insurer takes over the remaining asset of the firm, suffering bankruptcy loss rate α $(0 < \alpha < 1)$. In addition, we assume the borrower incurs a proportional debt issuance cost sK $(0 \leq s < 1)$, where K is the amount of money borrowed.

The pricing of corporate securities. According to asset pricing theory, the value of a claim is given by the sum of its expected cash flow discounted at the risk-free interest rate r, where the expectation is taken with respect to the risk-neutral probability measure $\mathbb{Q}$. For this reason, the value $A(\delta_t)$ of the total cash flow δ is

$$A(\delta_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}\delta_s ds\right] = \frac{\delta_t}{r-\mu},\ t \geq 0. \tag{2}$$

There are four important parameters characterizing the double exponential jump process δ. They are four roots β_1, β_2, $-\beta_3$, and $-\beta_4$ of the equation $G(\beta) = r$, where $G(\beta)$ is given by (A.5).

According to Appendix A, the value $E(\delta_t)$ of equity is given by

$$\begin{aligned} E(\delta_t) &= (1-\chi_e)\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau_d} e^{-r(s-t)}(\delta_s - C - g)ds\right] \\ &= (1-\chi_e)\left\{\frac{\delta_t}{r-\mu} - \frac{C+g}{r}\left[1 - b_1\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3} - b_2\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]\right. \\ &\quad \left. - \frac{\delta_d}{r-\mu}\left[b_3\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3} + b_4\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]\right\}, \end{aligned} \tag{3}$$

where b_1, b_2, b_3 and b_4 are given in Appendix B, and the stopping time $\tau_d = \inf\{s \geq t : \delta_t \leq \delta_d\}$. We let $1-\chi_e = (1-\chi_d)(1-\chi_f)$, here χ_d is the tax rate of the effective dividends and χ_f is the tax rate of corporate profits.

The guarantee contract follows the widely-used assumption that the SME is able to make default decision endogenously to maximize the value of equity. Accordingly, the high-contact condition holds at the optimal bankruptcy boundary δ_d^*, i.e.

$$\left.\frac{\partial E(\delta_t)}{\partial \delta_t}\right|_{\delta_t=\delta_d^*} = 0. \tag{4}$$

Solving (4) leads to

$$\delta_d^* = \frac{(r-\mu)(C+g)}{r}\frac{\eta_2+1}{\eta_2}\frac{\beta_3\beta_4}{(\beta_3+1)(\beta_4+1)}. \tag{5}$$

In the same way, after bankruptcy the value of debt undertaken by the insurer, $P(\delta_t)$, is

$$\begin{aligned} P(\delta_t) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_{\tau_d}^{\infty} e^{-r(\tau_d-t)}kCds\right] \\ &= \frac{kC}{r}\left[b_1\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3} + b_2\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]. \end{aligned} \tag{6}$$

The value of debt $D(\delta_t)$ is given by

$$\begin{aligned} D(\delta_t) &= (1-\chi_p)\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau_d} e^{-r(s-t)}Cds\right] + (1-\chi_p)P(\delta_t) \\ &= (1-\chi_p)\frac{C}{r}\left\{1-(1-k)\left[b_1\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3} + b_2\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]\right\}, \end{aligned} \tag{7}$$

where χ_p is the personal tax rate of the interest payment. The value of cash guarantee fee $U(\delta_t)$ with the fixed rate g is

$$\begin{aligned} U(\delta_t) &= (1-\chi_p)\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau_d} e^{-r(s-t)} g ds\right] \\ &= (1-\chi_p)\frac{g}{r}\left\{1-\left[b_1\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3}+b_2\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]\right\}. \end{aligned} \tag{8}$$

The remaining value $R(\delta_t)$ of the firm net of bankruptcy costs is

$$\begin{aligned} R(\delta_t) &= (1-\chi_e)\mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_d-t)}(1-\alpha)A_{\tau_d}] \\ &= (1-\chi_e)(1-\alpha)\frac{\delta_d}{r-\mu}\left[b_3\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3}+b_4\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]. \end{aligned} \tag{9}$$

We assume that there are no arbitrage and other transaction costs except the debt financing cost. Hence, the value that the insurer receives should equal the value that s/he pays for a fair guarantee swap. In other words, the following equation must hold:

$$\psi E(\delta_t) + U(\delta_t) + R(\delta_t) = P(\delta_t), \tag{10}$$

where ψ is the fraction of the SME equity allocated to the insurer. Therefore, we have

$$\psi = \frac{P(\delta_t) - U(\delta_t) - R(\delta_t)}{E(\delta_t)} \tag{11}$$

and the total firm's value $V(\delta_t)$ is

$$V(\delta_t) = E(\delta_t) + D(\delta_t) - \psi E(\delta_t) - sD(\delta_t). \tag{12}$$

The pricing and timing of the option to invest. Now we solve the problem of pricing and timing of the option to invest under a given swap defined in the proceeding text. Let $F(\delta_t)$ be the value of the option to invest in a project that generates the cash flow following the time homogeneous Markov process (1). To compute the optimal investment time, it suffices to consider the stopping time $\tau_u = \inf\{\delta_t \geq \delta_u\}$ for an investment threshold δ_u. Therefore, we have

$$F(\delta_t) = \max_{\delta_u \geq \delta_t} \mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_u - t)}(V(\delta_{\tau_u}) - I)]. \tag{13}$$

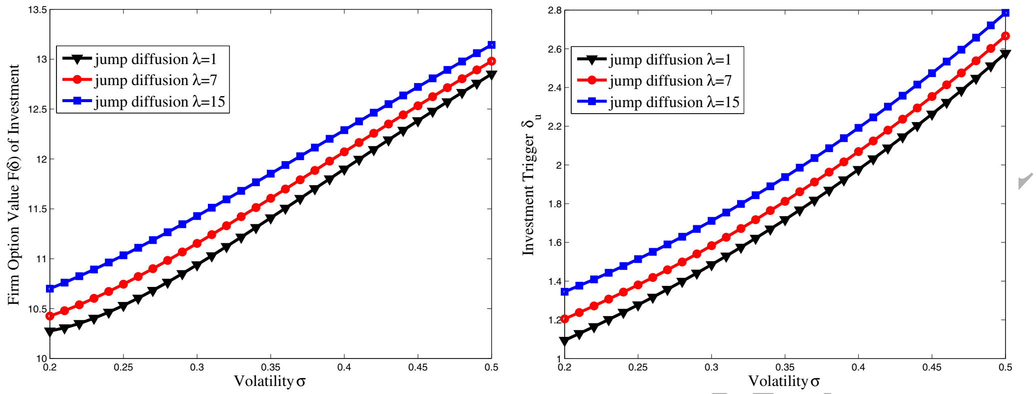
The optimal investment threshold δ_u^* satisfies the following high-contact condition:

$$\left.\frac{\partial F(\delta_t)}{\partial \delta_t}\right|_{\delta_t=\delta_u^*} = \left.\frac{\partial V(\delta_t)}{\partial \delta_t}\right|_{\delta_t=\delta_u^*}. \tag{14}$$

The optimal investment threshold δ_u^* and the value of the option to invest are explicitly presented in Appendix B.

3. Numerical results and analysis

Baseline parameter values. To make a reasonable comparison, the parameter values for the jump part are taken from Kou and Wang (2004), i.e. $p = 0.6$ ($q = 0.4$), $\eta_1 = 25$, $\eta_2 = 50$, and the jump intensity $\lambda = 7$. For the diffusion part we take the annualized risk-free interest rate $r = 0.05$, volatility $\sigma = 0.3$, bankruptcy loss rate $\alpha = 0.35$, and effective corporate tax rate $\chi_e = 0.2$ following Andrikopoulos (2009). The risk-adjusted growth rate $\mu = 0.01$ following Goldstein et al. (2001) since we assume the growth rate of an SME is not too high. The marginal cost of debt financing $s = 0.05$, which falls in the ballpark with the estimates (6.09%) in Eckbo et al. (2007). The interest payments are taxed at a personal rate $\chi_p = 0.1$ falling in the tax ranges in

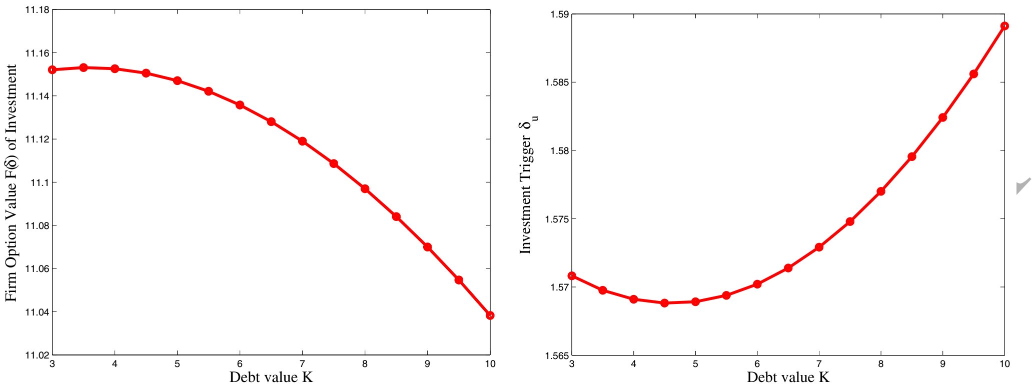


(a) option value versus project risk (b) investment trigger versus project risk

Figure 1: The figure displays (a) the value of the option and (b) the investment trigger for different values of project risk .

many countries. We assume that the current cash flow rate $\delta_0 = 1$ and the sunk cost $I = 10$ following Chen et al. (2010). We let the fixed guarantee fee rate $g = 0.01$ and the guarantee level $k = 0.6$. These values exclude some obviously uninteresting cases, such as an immediate default ($\delta_0 < \delta_b$), exercising the option immediately ($\delta_u < \delta_0$), and a negative fraction ($\psi < 0$) of equity contributing to guarantee costs. The coupon rate is optimal unless otherwise stated, i.e. it is determined endogenously by maximaizing the value of the SME.

The effects of project uncertainty on the pricing and timing of the option. Figure 1(a) illustrates that as we expect, the value of the option to invest increases with project risk. A similar conclusion is also pointed out by Kou et al. (2005). Consistent with this result, Figure 1(b) states that the investment trigger is an increasing function of the project risk, which accords with Kou et al. (2005). Actually, the two figures further document the well known



(a) option value versus funding gap (b) investment trigger versus funding gap

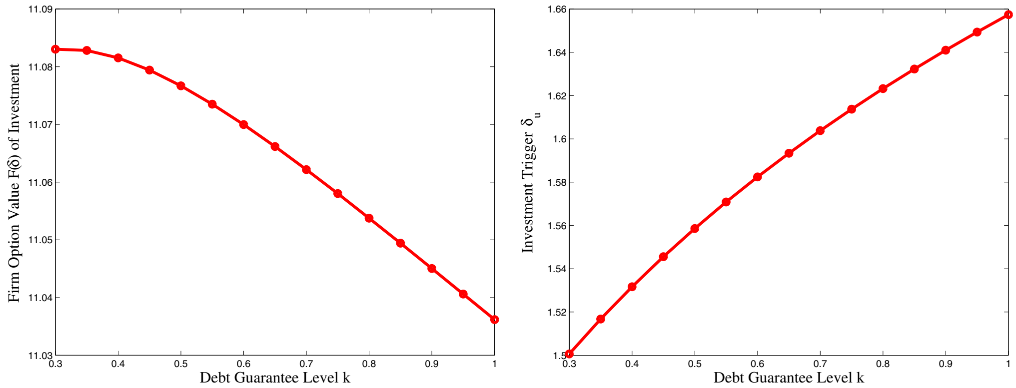
Figure 2: The figure plots (a) the value of the option and (b) the investment trigger for different funding gaps (debt values), where the coupon rate varies accordingly with the funding gap.

conclusion in real options theory, i.e. the larger the project risk, the higher the value of the option to invest and the later the investment time.

The effects of a funding gap on the pricing and timing of the option. According to pecking order theory in corporate finance, a borrower (SME) should only borrow the minimum money for starting a project. We acknowledge this theory and examine the case where only the funding gap is financed through borrowing. Accordingly, we utilize Figures 2(a) and 2(b) to describe how the funding gap, i.e. the debt value K, impacts on the pricing and timing of the option while the coupon rate varies accordingly with the gap. It turns out that as the gap rises, the option value decreases but the investment threshold first descends and then increases. The former happens because there are a financing cost and an extra tax on the payment of the insurer to the lender due to the guarantee. As a matter of fact, the insurer and the lender have a zero net gain or loss from the guarantee agreement, i.e. the tax shields or loss and bankruptcy costs are totally harvested or incurred at last by the SME. Therefore, a larger gap requires the SME to pay more financing costs and more extra amount in tax, which induces a deduction in the option value.

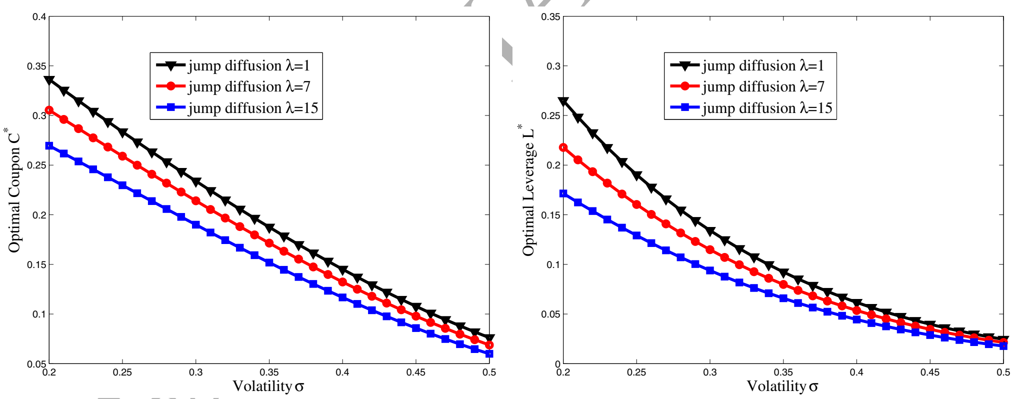
The effects of guarantee level on investment option. To explore the effects of guarantee level k on the pricing and timing of the option to invest, we take a fixed funding gap $K = 9$, while the coupon rate C varies accordingly. Figures 3(a) and 3(b) indicate that the option value deceases but the investment threshold increases with the guarantee level. Because of the aforementioned arguments in the last paragraph, these observations follow from the fact that a growth of guarantee level not only leads to a larger extra amount in tax paid by the insurer, ceteris paribus, but also decreases the coupon rate. Indeed, a smaller coupon rate implies a lower tax shield, though it reduces the bankruptcy costs by decreasing the default threshold from (5).

The effects of project risk on optimal capital structure. Xiang and Yang (2015) show that an SME can totally eliminate financing constraints thanks to an equity-for-guarantee swap. For this reason, the coupon rate could be endogenously determined under a guarantee swap. In other words, the entrepreneur could take an optimal capital structure to maximize the SME value, though s/he has to pay the corresponding guarantee costs. Figure 4(a) plots that the optimal coupon rate decreases with project risk. This phenomenon follows from two opposite forces. On one hand, the higher the project risk, the larger the investment threshold as shown in Figure 1(b) and the less the bankruptcy probability after exercising the option. Accordingly, issuing more debt can obtain more tax shields while bankruptcy costs only



(a) option value versus guarantee level (b) investment trigger versus guarantee level

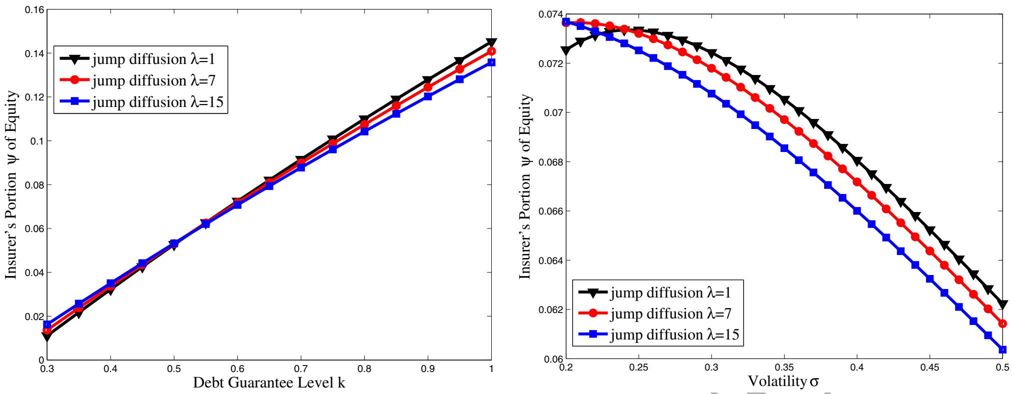
Figure 3: The figure shows (a) the value of the option and (b) the investment trigger for different guarantee levels. The given debt value is K=9 and the coupon rate changes accordingly with the guarantee level.



(a) optimal coupon versus project risk (b) optimal leverage versus project risk

Figure 4: The figure depicts (a) optimal coupon rate and (b) optimal leverage for different values of project risk .

increase a little. As a result, the optimal coupon rate should increase. On the other hand, a larger project risk leads to a higher default probability

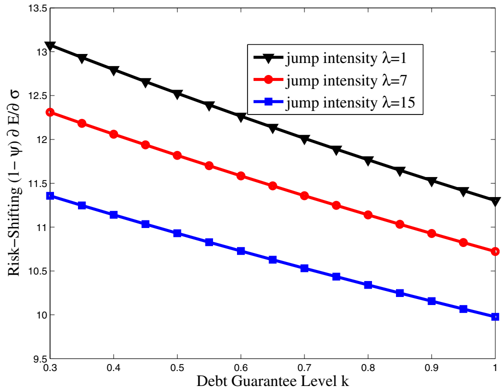


(a) insurer's fraction versus guarantee level (b) insurer's fraction versus project risk

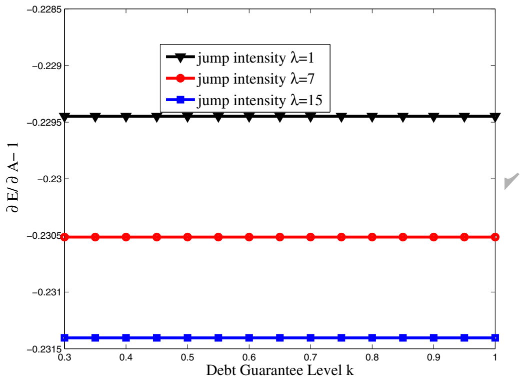
Figure 5: The figure illustrates the fraction of equity allocated to the insurer for different values of (a) guarantee level and (b) project risk.

and hence the SME should issue less debt, i.e. the coupon rate should be reduced, to decrease bankruptcy costs. In addition, Figure 4(b) depicts that the optimal leverage decreases with project risk, which means that the value of the SME increases faster than the optimal debt level with project risk. This phenomenon is not very obvious but is consistent with the conclusions in Mauer and Sarkar (2005). In particular, under the assumption of our baseline parameter values, the optimal leverage ratio is consistent with empirical averages. For example, Hall et al. (2004) and Shim (2006) show that the average debt leverage ratios range from 5 percent to 60 percent.

Guarantee costs versus guarantee level and project risk. Figure 5(a) plots that at a given fixed guarantee fee g and coupon rate $C = 0.8$, the fraction of equity allocated to the insurer increases with the debt guarantee level. This is in agreement with intuition. Furthermore, it shows that at a low guarantee level, the higher the jump risk, the larger the fraction but if the guarantee



(a) risk-shifting versus guarantee level



(b) Debt overhang versus guarantee level

Figure 6: The figure demonstrates (a) the risk-shifting incentive and (b) the debt overhang for different values of debt guarantee level. The given coupon rate $C = 0.8$ and the profit flow level $\delta = 1.8$ after investment.

level is high enough, the opposite holds true. Figure 5(b) reveals further that if the diffusive volatility is low, the fraction increases with project risk but when the diffusive volatility is high, the fraction falls. This phenomenon is caused by two opposite factors. One increases the fraction due to the higher default probability generated from a larger project risk but the other decreases the fraction since the value of equity increases with project risk and therefore a smaller fraction of equity is enough in return for the guarantee.

Asset substitution and debt overhang. To compare two candidate capital structures, we generally check the inefficiencies arising from asset substitution and debt overhang. For the first inefficiency, we compute risk-shifting incentives, which are measured by the rate of change of the borrower's equity value with respect to the diffusive volatility of the project, i.e. $\frac{(1-\psi)\partial E}{\partial \sigma}$. The larger the rate, the stronger the risk-shifting incentive of the borrower.

Consequently, Figure 6(a) demonstrates clearly that the risk-shifting incentive decrease globally with the debt guarantee level. This conclusion further explains that such guarantee not only totally eliminate the financing constraints as argued by Xiang and Yang (2015), but also dramatically decreases the inefficiencies arising from asset substitution. To describe the second inefficiency, which arises from debt overhang, we compute the rate of change of total equity value with respect to the total cash flow value A minus 1, i.e. $\frac{\partial E}{\partial A} - 1$. It represents the net value received by shareholders after they invest one unit of value in the firm. Figure 6(b) implies that this inefficiency is invariant with the guarantee level.

4. Conclusions

There are a large number of SMEs all over the world undergoing financing constraints, which have been inducing a huge loss of social welfare for a long time. This problem has been attracting much attention from researchers and practitioners.

In this paper, we solve an SME's problem of pricing and timing of the option to invest in a project, which generates the cash flow following a double exponential jump-diffusion process. The SME has a funding gap to start the project and the gap is financed by entering into a partial guarantee agreement. We provide an explicit solution of the option value and the investment threshold. We show that the option value and investment threshold increase with project risk. If the funding gap rises, the option value decreases but the investment threshold first declines and then increases. The larger the guarantee level, the lower the option value and the later the investment. The optimal coupon rate and optimal leverage decrease with project risk. At a given fixed guarantee fee rate, the guarantee level and project risk have an ambiguous effect on the fraction of equity allocated to the insurer if coupon rate varies accordingly to keep capital structure optimal. While debt overhang is independent of guarantee levels, the inefficiency arising from asset substitution decreases as guarantee levels rise.

In essence, it is the most important role played by a partial guarantee that the guarantee succeeds in exchanging a partial future cash flow of an SME for cash available at investment time to finance the SME's funding gap. In this way, financing constraints are in fact completely eliminated. From this perspective, the partial guarantee we discuss here is similar to a mortgage loan agreement, which has greatly improved our welfare level.

Appendices

Appendix A The double exponential jump-diffusion process

Clearly, Equation (1) has the following unique solution:

$$\delta_t = \delta_0 e^{(\mu - \frac{\sigma^2}{2} - \lambda\xi)t + \sigma B_t} \prod_{i=1}^{N_t} Z_i. \tag{A.1}$$

Introducing the variables $Y_i := \ln(Z_i)$, one has

$$\delta_t = \delta_0 e^{X_t}, \text{ where } X_t = \left(\mu - \frac{\sigma^2}{2} - \lambda\xi\right) t + \sigma B_t + \sum_{i=1}^{N_t} Y_i. \tag{A.2}$$

The random variables Y_i follow an asymmetric double exponential distribution with density

$$f(y) = p\eta_1 e^{-\eta_1 y}\mathbf{1}_{\{y\geq 0\}} + q\eta_2 e^{\eta_2 y}\mathbf{1}_{\{y<0\}}, \quad \eta_1 > 1,\ \eta_2 > 0, \tag{A.3}$$

where $p, q \geq 0$ with $p + q = 1$ represent the probabilities of upward and downward jumps. The means of the two exponential distributions are $1/\eta_1$ and $1/\eta_2$, respectively. The mean percentage jump size ξ has the solution

$$\xi = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1. \tag{A.4}$$

Introduce the Laplace exponent $G(\cdot)$ of X such that the moment-generating function $\mathbb{E}_t^{\mathbb{Q}}[e^{\beta X_t}] = \exp[G(\beta)t]$, where $G(\beta)$ is defined as (Chen and Kou, 2009)

$$\begin{aligned} G(\beta) :=& \frac{1}{2}\sigma^2\beta^2 + \left[\mu - \frac{\sigma^2}{2} - \lambda\left(\frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1\right)\right]\beta \\ &+ \lambda\left(\frac{p\eta_1}{\eta_1 - \beta} + \frac{q\eta_2}{\eta_2 + \beta} - 1\right). \end{aligned} \tag{A.5}$$

The equation $G(\beta) = r$ has four roots: $\beta_1, \beta_2, -\beta_3, -\beta_4$, where $-\infty < -\beta_4 < -\eta_2 < -\beta_3 < 0 < \beta_1 < \eta_1 < \beta_2 < \infty$.

Let $\tau_u(\delta)$ be the first passage time of the "upward barrier" for the process δ, $\tau_u(\delta) = \inf\{t \geq 0 : \delta_t \geq \delta_u\}$. Let $\tau_d(\delta)$ be the first passage time of the "downward barrier" for the process δ, $\tau_d(\delta) = \inf\{t \geq 0 : \delta_t \leq \delta_d\}$. Thanks to Kou and Wang (2003), we obtain the following equations

$$\mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_u - t)}] = \frac{\eta_1 - \beta_1}{\beta_2 - \beta_1}\frac{\beta_2}{\eta_1}\left(\frac{\delta_u}{\delta_t}\right)^{-\beta_1} + \frac{\beta_2 - \eta_1}{\beta_2 - \beta_1}\frac{\beta_1}{\eta_1}\left(\frac{\delta_u}{\delta_t}\right)^{-\beta_2}, \tag{A.6}$$

$$\mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_u - t)}\delta_{\tau_u}^{\zeta}] = \delta_u^{\zeta}\left[\frac{\eta_1 - \beta_1}{\beta_2 - \beta_1}\frac{\beta_2 - \zeta}{\eta_1 - \zeta}\left(\frac{\delta_u}{\delta_t}\right)^{-\beta_1} + \frac{\beta_2 - \eta_1}{\beta_2 - \beta_1}\frac{\beta_1 - \zeta}{\eta_1 - \zeta}\left(\frac{\delta_u}{\delta_t}\right)^{-\beta_2}\right].$$

$$ \tag{A.7} $$

$$\mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_d-t)}] = \frac{\eta_2-\beta_3}{\eta_2}\frac{\beta_4}{\beta_4-\beta_3}\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3} + \frac{\beta_4-\eta_2}{\eta_2}\frac{\beta_3}{\beta_4-\beta_3}\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}, \tag{A.8}$$

$$\mathbb{E}_t^{\mathbb{Q}}[e^{-r(\tau_d-t)}\delta_{\tau_d}^{\zeta}] = \delta_d^{\zeta}\left[\frac{\eta_2-\beta_3}{\beta_4-\beta_3}\frac{\beta_4+\zeta}{\eta_2+\zeta}\left(\frac{\delta_d}{\delta_t}\right)^{\beta_3} + \frac{\beta_4-\eta_2}{\beta_4-\beta_3}\frac{\beta_3+\zeta}{\eta_2+\zeta}\left(\frac{\delta_d}{\delta_t}\right)^{\beta_4}\right]. \tag{A.9}$$

Appendix B Solutions to option value and investment threshold

For the convenience of expressions, we introduce the following coefficients: $b_1 = \frac{\eta_2-\beta_3}{\eta_2}\frac{\beta_4}{\beta_4-\beta_3}$, $b_2 = \frac{\beta_4-\eta_2}{\eta_2}\frac{\beta_3}{\beta_4-\beta_3}$, $b_3 = \frac{\eta_2-\beta_3}{\beta_4-\beta_3}\frac{\beta_4+1}{\eta_2+1}$, $b_4 = \frac{\beta_4-\eta_2}{\beta_4-\beta_3}\frac{\beta_3+1}{\eta_2+1}$,
$b_5 = \frac{\eta_1-\beta_1}{\beta_2-\beta_1}\frac{\beta_2-1}{\eta_1-1}$, $b_6 = \frac{\beta_2-\eta_1}{\beta_2-\beta_1}\frac{\beta_1-1}{\eta_1-1}$, $b_7 = \frac{\eta_1-\beta_1}{\beta_2-\beta_1}\frac{\beta_2}{\eta_1}$, $b_8 = \frac{\beta_2-\eta_1}{\beta_2-\beta_1}\frac{\beta_1}{\eta_1}$,
$b_9 = \frac{\eta_1-\beta_1}{\beta_2-\beta_1}\frac{\beta_2+\beta_3}{\eta_1+\beta_3}$, $b_{10} = \frac{\beta_2-\eta_1}{\beta_2-\beta_1}\frac{\beta_1+\beta_3}{\eta_1+\beta_3}$, $b_{11} = \frac{\eta_1-\beta_1}{\beta_2-\beta_1}\frac{\beta_2+\beta_4}{\eta_1+\beta_4}$, $b_{12} = \frac{\beta_2-\eta_1}{\beta_2-\beta_1}\frac{\beta_1+\beta_4}{\eta_1+\beta_4}$.

From equation (14), the firm-maximizing investment threshold δ_u^* satisfies the equation

$$d_0 + d_1\left(\frac{\delta_d}{\delta_u}\right)^{(-1)} + d_2\left(\frac{\delta_d}{\delta_u}\right)^{\beta_3} + d_3\left(\frac{\delta_d}{\delta_u}\right)^{\beta_4} = 0, \tag{B.1}$$

where we denote:

$d_0 = \{[(1-\chi_p)-(1-\chi_e)]\frac{C+g}{r} - s(1-\chi_p)\frac{C}{r} - I\}(b_7\beta_1 + b_8\beta_2)$,

$d_1 = (1-\chi_e)(b_5\beta_1 + b_6\beta_2 - 1)\frac{\delta_d}{r-\mu}$,

$d_2 = (h_2 - \chi_p\frac{kC}{r})b_1a_1 - (1-\chi_e)\alpha\frac{\delta_d}{r-\mu}\, b_3a_1$,

$d_3 = (h_2 - \chi_p\frac{kC}{r})b_2a_2 - (1-\chi_e)\alpha\frac{\delta_d}{r-\mu}\, b_4a_2$,

$a_1 = b_9\beta_1 + b_{10}\beta_2 + \beta_3$,

$a_2 = b_{11}\beta_1 + b_{12}\beta_2 + \beta_4$,

$h_2 = -[(1-\chi_p)-(1-\chi_e)]\frac{C+g}{r} + s(1-k)\frac{(1-\chi_p)C}{r}$.

According to (B.1), (A.6), (A.7), and (13), the option value is given by

$$\begin{aligned} F =&(1-\chi_e)\frac{\delta_u}{r-\mu}X_1 - \left(-[(1-\chi_p)-(1-\chi_e)]\frac{C+g}{r} + s\frac{(1-\chi_p)C}{r} + I\right)X_2 \\ &+ \left(h_2 - \chi_p\frac{kC}{r}\right)(b_1X_3 + b_2X_4) - (1-\chi_e)\alpha\frac{\delta_d}{r-\mu}(b_3X_3 + b_4X_4), \end{aligned} \tag{B.2}$$

where we denote:

$X_1 = b_5(\frac{\delta_u^e}{\delta_t})^{-\beta_1} + b_6(\frac{\delta_u^e}{\delta_t})^{-\beta_2}$,

$X_2 = b_7(\frac{\delta_u^e}{\delta_t})^{-\beta_1} + b_8(\frac{\delta_u^e}{\delta_t})^{-\beta_2}$,

$X_3 = [b_9(\frac{\delta_u^e}{\delta_t})^{-\beta_1} + b_{10}(\frac{\delta_u^e}{\delta_t})^{-\beta_2}](\frac{\delta_d}{\delta_u^e})^{\beta_3}$,

$X_4 = [b_{11}(\frac{\delta_u^e}{\delta_t})^{-\beta_1} + b_{12}(\frac{\delta_u^e}{\delta_t})^{-\beta_2}](\frac{\delta_d}{\delta_u^e})^{\beta_4}$.

References

Andrikopoulos, A., 2009. Irreversible investment, managerial discretion and optimal capital structure. Journal of Banking & Finance 33 (4), 709–718.

Belhaj, M., Djembissi, B., 2009. Optimal investment under credit constraints. Annals of Economics and Statistics 93-94, 259–277.

Boyle, G. W., Guthrie, G. A., 2003. Investment, uncertainty, and liquidity. Journal of Finance 58 (5), 2143–2166.

Chen, H., Miao, J., Wang, N., 2010. Entrepreneurial finance and nondiversifiable risk. Review of Financial Studies 23(12), 4348–4388.

Chen, N., Kou, S., 2009. Credit spreads, optimal capital structure, and implied volatility with endogenous default and jump risk. Mathematical Finance 19 (3), 343–378.

Eckbo, B. E., Masulis, R. W., Norli, Ø., 2007. Security offerings. In: Eckbo, B. E. (Ed.), Handbook of Corporate Finance: Empirical Corporate Finance, vol.1., Elsevier, North-Holland, pp. 233–373.

Goldstein, R., Ju, N., Leland, H., 2001. An EBIT-based model of dynamic capital structure. Journal of Business 74 (4), 483–512.

Hall, G. C., Hutchinson, P. J., Michaelas, N., 2004. Determinants of the capital structures of european smes. Journal of Business Finance & Accounting 31 (5-6), 711–728.

Hirth, S., Uhrig-Homburg, M., 2010. Investment timing when external financing is costly. Journal of Business Finance & Accounting 37 (7-8), 929–949.

Jensen, M. C., Meckling, W. H., 1979. Theory of the firm: Managerial behavior, agency costs, and ownership structure. Springer.

Kang, D., 2005. Corporate distress and restructuring policy of Korean small and medium sized enterprises: Role of credit guarantee scheme. In: 2005 KDI-KAEA conference, July 15, 2005, Korea Development Institute, Seoul.

Kou, S., Petrella, G., Wang, H., 2005. Pricing path-dependent options with jump risk via Laplace transforms. Kyoto Economic Review 74 (1), 1–23.

Kou, S. G., 2002. A jump-diffusion model for option pricing. Management Science 48 (8), 1086–1101.

Kou, S. G., Wang, H., 2003. First passage times of a jump diffusion process. Advances in applied probability 35 (2), 504–531.

Kou, S. G., Wang, H., 2004. Option pricing under a double exponential jump diffusion model. Management Science 50 (9), 1178–1192.

Mauer, D. C., Sarkar, S., 2005. Real options, agency conflicts, and optimal capital structure. Journal of Banking & Finance 29 (6), 1405–1428.

Merton, R. C., 1976. Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics 3 (1), 125–144.

Myers, S. C., 1977. Determinants of corporate borrowing. Journal of Financial Economics 5 (2), 147–175.

Shim, I., 2006. Corporate credit guarantees in Asian. BIS Quarterly Review, December 2006.

Shibata, T., Nishihara, M., 2015. Investment timing, debt structure, and financing constraints. European Journal of Operational Research 241 (2), 513–526.

Sundaresan, S., Wang, N., Yang, J., 2015. Dynamic investment, capital structure and debt overhang. Review of Corporate Finance Studies 4, 1–42.

Wang, H., Yang, Z., Zhang, H., 2015. Entrepreneurial finance with equity-for-guarantee swap and idiosyncratic risk. European Journal of Operational Research 241 (3), 863–871.

Xiang, H., Yang, Z., 2015. Investment timing and capital structure with loan guarantees. Finance Research Letters 13, 179–187.

Yang, Z., Zhang, H., 2013. Optimal capital structure with an equity-for-guarantee swap. Economics Letters 118 (2), 355–359.

Yang, Z., Zhang, C., 2015a. Two new equity default swaps with idiosyncratic risk. International Review of Economics and Finance 37, 254–273.

Yang, Z., Zhang, C., 2015b. The pricing of two newly invented swaps in a jump-diffusion model. Annals of Economics and Finance, forthcoming.