Accounting for Japanese Business Cycles: A Quest for Labor Wedges

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The Japanese business cycle from 1980 to 2007 portrays a less contemporaneous correlation of labor with output than in the United States, and in addition labor tends to lead output by one quarter. A canonical real business cycle model cannot account for these facts. This paper uses the business cycle accounting method following Chari, Kehoe, and McGrattan (2007) and shows that efficiency and labor market distortions are important in accounting for the quarterly business cycle fluctuation patterns in Japan. Fiscal and monetary variables such as labor income tax, money growth, and interest rates cannot fully account for the distortions in the Japanese labor market.

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I. Introduction

The Japanese business cycle from 1980 to 2007 portrays a correlation of labor with output that is less contemporaneous than in the United States, and in addition labor tends to lead output by one quarter. A canonical real business cycle model cannot account for these facts. Using the business cycle accounting method of Chari, Kehoe, and McGrattan (2007), this paper shows that efficiency and labor market distortions are important in accounting for the patterns of quarterly business cycle fluctuation in Japan. I assess the impacts of fiscal and monetary shocks as possible candidates for the labor wedges.

The business cycle accounting model consists of the representative household, firm, and government. The government affects markets by exogenously purchasing goods while charging distortionary labor and investment taxes. The firm faces exogenous shocks to the production process. These exogenous variables are called resource wedges, labor wedges, investment wedges, and efficiency wedges. The wedges are computed as residuals in the equilibrium conditions using the data for output, consumption, investment, and labor supply. Finally, the model is simulated using the computed wedges.

Several studies use the business cycle accounting method to analyze the medium-term business cycle fluctuations during recession periods. Chari, Kehoe, and McGrattan (2007) find that labor and efficiency wedges are important in accounting for the Great Depression and 1981 recession in the United States. Kersting (2008) finds that the labor wedges are important in accounting for the early 1980s in the U.K. recession. Cociuba and Ueberfeldt (2008) find that labor and efficiency wedges are important in accounting for Canadian business cycles over the 1961–2005 period. Lama (2011) uses a small open-economy version of business cycle accounting and finds that labor and efficiency wedges are important in accounting for output drops in Argentina, Brazil, Chile, Colombia, Mexico, and Peru. Otsu (2010) also uses a small open-economy version and finds that efficiency wedges are important in accounting for the 1998 crises in Hong Kong, South Korea, and Thailand.

Several studies also analyze the Japanese economy using the business cycle accounting method. Kobayashi and Inaba (2006) use a deterministic version of the business cycle accounting model to show that efficiency and labor wedges are important in accounting for the lost decade. Chakraborty (2009) shows that efficiency and investment wedges are important in accounting for the boom in the 1980s, while labor wedges are important in accounting for the recession during the 1990s. Otsu and Pyo (2009) show that the positive contemporaneous correlation between efficiency and investment wedges may have amplified the effect of financial frictions, especially during the boom in the 1980s. Saijo (2008) finds that efficiency wedges are important in accounting for the output drop, while labor and capital wedges are important in accounting for the slow recovery of output during the Great Depression in Japan. Unlike these studies, I focus on the HP-filtered high-frequency fluctuation of the Japanese economy to show that labor wedges are important in accounting for the correlation between labor and output.

Related studies investigate Japanese labor market issues using dynamic general equilibrium models. Braun *et al.* (2006) assess the cyclical features of Japan and the U.S. labor market variables over the 1960–2000 period. They find that the adjustment of labor supply is made mostly at the intensive margin in Japan, whereas it is done at the extensive margin in the United States, and that fluctuations in hours worked per worker lead the business cycles, while those of employment lag the business cycle in Japan. They show that the first fact can be accounted for by the differences in the elasticities of hours across gender and economies. They show that it is difficult to account for the second fact using a variation of the real business cycle model, and conjecture that there was some distortion in the labor market independent of productivity shocks. Otsu (2011) shows that shocks to the intensity of labor can account for the lead in hours and the lag of employment. In this paper, I focus on total hours worked following the business cycle accounting method, and show that labor wedges are important in accounting for the fluctuations in the aggregate labor supply.

The business cycle accounting model does not specify the underlying source of the labor wedge. In fact, many different shocks can manifest themselves as labor wedges. Labor income tax is the usual suspect; however, its fluctuation is not observed on a quarterly basis. I also show that interest rate shocks in a working capital model, money growth shocks in a cash-in-advance constraint model, and money growth shocks in a labor union sticky-wage model can all be mapped into a prototype business cycle accounting model with labor wedges. I simulate these monetary models and show that monetary shocks cannot fully account for the labor market distortions.

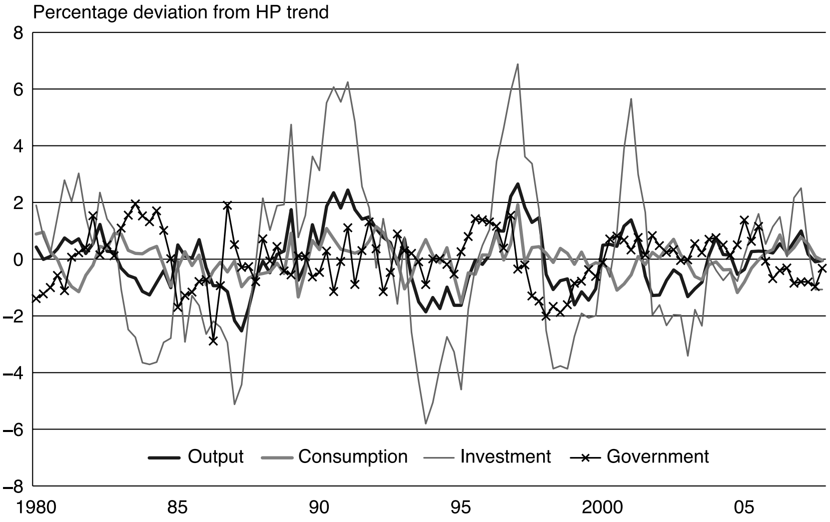
The remaining of the paper is organized as follows. In Section II, the Japanese business cycle data are presented in comparison with the U.S. data. In Section III, the business cycle accounting model is described. In Section IV, the quantitative results are shown. In Section V, possible sources of the labor wedges are assessed. Section VI concludes the paper.

II. Data

This section presents the quarterly fluctuations of key macroeconomic variables in Japan over the 1980–2007 period.[1] Output is defined as GDP plus the imputed flow of service from consumer durables.[2] Consumption is defined as the consumption expenditure on nondurables and services and the imputed flow of services from consumer durables.[3] Investment is defined as the gross capital formation and the expenditure on household durables. Government expenditures include only government consumption, while government investment is included in gross capital formation. Capital stock is defined as the sum of residential capital stock, nonresidential capital stock, government capital stock, and the stock of consumer durables. The quarterly stock data for each asset are estimated by interpolating the annual data for the beginning of 1980 and 2000 using the quarterly investment data for each asset.[4] Labor supply is defined as the

1. I choose this time period to remove the large effects of the oil shocks in the 1970s and the recent crisis in 2008.
2. The trade balance is not reported in this section for simplicity, as we will be constructing a closed-economy model. In the model section, the sum of government purchases and the trade balance is defined as a wedge in the resource constraint follwing Chari, Kehoe, and McGrattan (2007).
3. Imputed rent is already included in the service consumption data.
4. The stock data for residential, nonresidential, and government capital are from Hayashi and Prescott (2002), while the stock data for durable goods are from the Economic and Social Research Institute website. Since the

Figure 1 Japanese Business Cycles (Expenditure)



number of workers times the number of hours worked per workers, that is, total hours worked. The source of the Japanese data is the System of National Accounts (SNA) data offered by the Cabinet Office Economic and Social Research Institute website for national accounts and the Labor Force Survey for labor statistics.

Figure 1 plots the fluctuations of HP-filtered per capita output, consumption investment, and government expenditures. The output series shows the recessions in the early 1990s after the bursting of the "bubble" in the mid-1990s after the Asian crisis, and in the early 2000s after the bursting of the IT bubble.[5] Consumption and investment are both procyclical. Investment is much more volatile than output, while consumption is less volatile than output. The fluctuation of government expenditure does not seem to have a clear relationship with output. Figure 2 plots the fluctuations of HP-filtered per capita output, capital stock, and labor. Labor is procyclical for most of the period, except for the early 1980s. The fluctuation of capital clearly lags that of output.

Table 1 presents the statistical features of the data presented in Figures 1 and 2. The first column reports the standard deviation of each variable, while the second column reports the ratio of the standard deviation of each variable relative to that of output. The remainder of the table reports the cross-correlation of each variable with output. For comparison, the U.S. data are presented in the lower panel. The source of the U.S. data is the Bureau of Economic Affairs website for national accounts, and the Bureau of Labor Statistics website for labor statistics.

durable stock data are not available for 1980, I assume that the ratio of durable stock to other stock in 1980 is the same as that in 2000.

5. The 1990s are known as the lost decade where the average growth rate (0.89 percent) was substatially lower than that of the previous decade (2.27 percent). This medium-term fluctuation disappears once the data are HP filtered. The main objective of this paper is to analyze the high-frequency business cycle fluctuation. The nonfiltered data and results are presented in Appendix 1.

Figure 2 Japanese Business Cycle (Production)

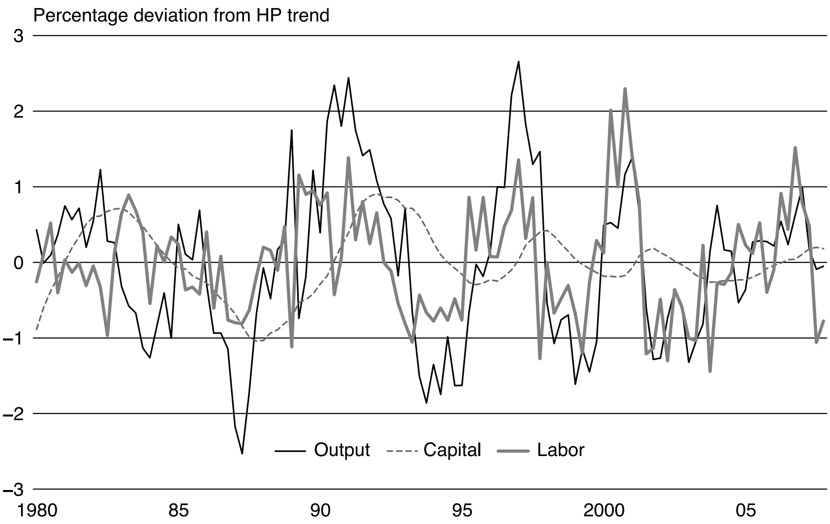


Table 1 Business Cycle Features

[1] Japanese Economy (1980–2007)

	St. dev. of v		Correlation of output with						
v	Percent	Rel. y	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.06	1.00	0.46	0.61	0.79	1.00	0.79	0.61	0.46
Consumption	0.60	0.56	0.15	0.11	0.18	0.38	0.28	0.29	0.36
Investment	2.87	2.70	0.58	0.67	0.80	0.89	0.72	0.54	0.35
Government	0.94	0.88	0.14	0.10	0.06	0.00	−0.07	−0.12	−0.19
Capital	0.45	0.42	−0.25	−0.11	0.03	0.18	0.36	0.50	0.60
Labor	0.74	0.69	0.36	0.47	0.55	0.45	0.40	0.24	0.12

[2] U.S. Economy (1980–2007)

	St. dev. of v		Correlation of output with						
v	Percent	Rel. y	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.22	1.00	0.50	0.68	0.86	1.00	0.86	0.68	0.50
Consumption	0.74	0.60	0.56	0.68	0.78	0.84	0.73	0.61	0.47
Investment	4.37	3.57	0.54	0.69	0.84	0.94	0.76	0.54	0.33
Government	0.94	0.77	−0.07	−0.12	−0.12	−0.03	−0.02	0.01	0.06
Capital	0.33	0.27	−0.04	0.07	0.21	0.43	0.55	0.64	0.71
Labor	1.31	1.07	0.36	0.55	0.75	0.89	0.87	0.78	0.66

The business cycle features of Japan are similar to those of the U.S. economy in several aspects. First, consumption, government purchases, and capital stock are less volatile than output, while investment is far more volatile than output. Second, all variables are procyclical except for government purchases, which are acyclical in both countries. Finally, the fluctuation of capital lags that of output.

The key difference between Japan and the United States is the fluctuation of the labor supply. First, the labor supply in Japan is much less volatile relative to output than in the United States. The standard deviation of labor relative to output is 0.69 in Japan, while it is 1.07 in the United States. Second, the contemporaneous correlation of consumption and labor supply to output is much lower in Japan than in the United States. The correlation coefficients are 0.38 and 0.45 in Japan, while they are 0.84 and 0.89 in the United States, respectively. Finally, the fluctuation of labor supply leads output in Japan, while in the United States the fluctuations are coincident. The second and third points are important, as it turns out that they cannot be replicated by a standard real business cycle model.[6]

III. Business Cycle Accounting Model

The business cycle accounting model of Chari, Kehoe, and McGrattan (2007) is based on a neoclassical closed-economy model that consists of a representative firm, household, and the government. The firm produces a final good from capital and labor using a constant-returns-to-scale production technology that faces exogenous disturbances in production efficiency. The infinitely lived representative household gains utility from consumption and leisure. The household owns capital stock and labor endowment, and decides how much to consume, invest, and work. The government imposes distortionary labor and investment taxes on the household, spends on government purchases, and rebates the remainder to the household via lump-sum transfers. Labor and investment taxes, government purchases, and production efficiency are computed as “wedges” in equilibrium conditions and are taken as exogenous. The detailed description is as follows.

A. Firm

The firm produces a final good with a Cobb-Douglas production function,

$$Y_t = z_t K_t^{\theta} (\Gamma_t l_t)^{1-\theta},$$

where Y_t is output, z_t is production efficiency—that is, total factor productivity (TFP)—K_t is capital stock, l_t is labor, θ is the income share of capital, and Γ_t is the labor-augmented technical progress. We assume that the labor-augmenting technical progress grows at a constant rate γ such that $\Gamma_t = (1+\gamma)\Gamma_{t-1}$. Labor is defined as

$$l_t = h_t * e_t,$$

where e_t is the number of workers employed per the number of the adult population, and h_t is the average weekly hours worked per worker divided by the maximum possible hours available.[7] In a standard neoclassical growth model, real per capita

6. The standard deviation of investment relative to that of investment in Japan is lower (2.7) than that in the United States (3.57). This is also an interesting difference between the two economies, which is left as a subject for future research.

7. I assume that the maximum hours available for work are 14 × 7 hours, taking into account sleeping and other activities necessary to maintain minimum standards of living.

macroeconomic variables such as output, consumption, investment, government expenditure, and capital stock grow at the rate of the labor-augmenting technical progress γ along the balanced growth path. Therefore, I detrend these growing variables with the trend Γ_t so that all variables are stationary. The detrended variables are denoted by lowercase letters.

The firm maximizes its profit defined by the value of production net of costs of hiring labor and renting capital stock from the household. That is,

$$\max \pi_t = y_t - w_t l_t - r_t k_t, \tag{1}$$

where w_t is the real wage and r_t is the real capital rental rate, and the detrended production function is

$$y_t = z_t k_t^{\theta} l_t^{1-\theta}. \tag{2}$$

Profits will always equal zero due to perfect competition.

B. Household

The lifetime utility for the representative household depends on consumption c_t and labor l_t:

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{3}$$

where β $(0 < \beta < 1)$ is the subjective discount rate. For the periodical preference function, $u(\cdot)$, I assume Cobb-Douglas preferences:

$$u(c_t, l_t) = \Psi \log(c_t) + (1 - \Psi) \log(1 - l_t),$$

which are commonly used in the macroeconomic literature.[8]

The representative household maximizes the lifetime utility (3) subject to the budget constraint

$$(1 - \tau_t^l) w_t l_t + r_t k_t + \pi_t + \tau_t = c_t + (1 + \tau_t^x) x_t, \tag{4}$$

and the capital law of motion

$$\eta k_{t+1} = x_t + (1 - \delta) k_t, \tag{5}$$

where η is the constant growth trend of the labor-augmented technical progress and population, x_t is investment, τ_t^l and τ_t^x are labor income and investment tax rates, τ_t is the lump-sum government transfer, and δ is the depreciation rate of capital stock.

8. This is a special case of a general form

$$u = \frac{(c_t^{\Psi}(1 - l_t)^{1-\Psi})^{1-\sigma}}{1 - \sigma},$$

with $\sigma = 1$.

C. Government

The government collects distortionary taxes, spends on exogenous government purchases g_t and rebates the remaining to the household using lump-sum transfer. Thus, the government budget constraint is

$$\tau_t + g_t = \tau_t^l w_t l_t + \tau_t^x x_t. \tag{6}$$

Note that the transfer can be negative, in which case the government collects lump-sum taxes from the household.

Combining the household budget constraint (4), the firms profits (1), and the government budget constraints (6), we get the resource constraint

$$y_t = c_t + x_t + g_t. \tag{7}$$

D. Wedges

There are four exogenous variables in this model: resource wedges g_t, labor wedges τ_t^l, investment wedges τ_t^x, and efficiency wedges z_t. Although the wedges are defined as government purchases and taxes in the model, we do not use the data for these. Instead, the wedges are measured using the following equilibrium conditions.

Resource wedges are measured using the resource constraint (7). Notice that they include all GDP expenditure components net of consumption and investment, which comprises government purchases and the trade balance. Labor wedges are measured using the labor first-order condition:

$$\frac{1-\Psi}{\Psi}\frac{c_t}{1-l_t} = (1-\tau_t^l)(1-\theta)\frac{y_t}{l_t}. \tag{8}$$

Notice that labor wedges need not be labor taxes. Any other distortion in the labor market creates a wedge between the marginal rate of substitution of labor to consumption and the marginal product of labor. Investment wedges are measured using the capital Euler equation:

$$\eta\frac{\Psi}{c_t}(1+\tau_t^x) = \beta E_t\left[\frac{\Psi}{c_{t+1}}\left(\theta\frac{y_{t+1}}{k_{t+1}} + (1-\delta)(1+\tau_{t+1}^x)\right)\right]. \tag{9}$$

Finally, efficiency wedges are measured using the production function (2).

The economic interpretation is as follows. An increase in resource wedges is considered a loss of resources for the economy. Through a pure negative income effect, consumption and investment decrease while labor and output increase. An increase in labor wedges is considered a decrease in the real wage that the household receives. Investment decreases through a negative income effect, while consumption, labor, and output decrease predominantly through a substitution effect.[9] An increase in investment wedges reduces the return on investment the household receives. Consumption increases, while investment decreases due to the substitution effect. The increase in consumption causes

9. Consumption and labor both decrease, because consumption becomes expensive relative to leisure. This effect dominates the negative income effect, which tends to increase labor.

an income effect, which increases leisure. As a result, labor and output decrease. An increase in efficiency wedges directly increases output and the labor demand.[10] The increase in income leads to an increase in consumption and investment.

The wedges are assumed to follow the following stochastic process:

$$\tilde{s}_t = P_{4\times4}\tilde{s}_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0_{4\times1}, Q_{4\times4}), \tag{10}$$

where $\tilde{s}_t = (\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t)'$, while variables with "˜" indicate the log deviation from their steady-state values, and $\varepsilon_t = (\varepsilon_{gt}, \varepsilon_{lt}, \varepsilon_{xt}, \varepsilon_{zt})'$.

E. Competitive Equilibrium

The competitive equilibrium is, $\{c_t, l_t, k_{t+1}, y_t, x_t, \tau_t, w_t, r_t, g_t, \tau_t^l, \tau_t^x, z_t\}_{t=0}^{\infty}$ such that

(1) Households optimize given $\{w_t, r_t, \tau_t, \tau_t^l, \tau_t^x\}$ and k_0.
(2) Firms optimize given $\{w_t, r_t, z_t\}$.
(3) Markets clear and the government budget constraint (6) holds.
(4) The resource constraint (7) holds.
(5) Shocks follow the process (10).

IV. Quantitative Analysis

The quantitative analysis is carried out in four steps. First, we obtain the parameter values by calibration and estimation. Next, we solve for linear decision rules of the endogenous variables. Then we back out the shocks using data and these decision rules. Finally, we plug the shocks one by one into the decision rules and compare their impacts on the economy.

A. Parameters

This subsection describes the procedure by which we obtained the values of the parameters. The calibrated parameter values are listed in Table 2.

The income share of capital θ is computed directly from data using the definition

$$\theta = \frac{\text{capital income} + \text{flow income from consumer durables}}{\text{GNP} + \text{flow income from consumer durables}}.$$

Table 2 Parameter Values

θ	Capital share	0.388
δ	Depreciation	0.022
β	Discount factor	0.987
Ψ	Preference weight	0.228
γ	Technology growth	0.0035
n	Population growth	0.0019

10. The increase in labor demand drives up the real wage. The substitution effect tends to increase the labor supply. On the other hand, the increase in income tends to decrease income. In an infinite-horizon model, it is usually the case that the substitution effect dominates the income effect.

Population growth rate n is computed directly from data for the population of people older than 15 years.

The growth rate of labor-augmenting technical progress is the trend growth rate of Solow residuals estimated with ordinary least squares. The log of Solow residuals is defined as

$$\ln SR_t = \ln \Gamma_t^{1-\theta} + \ln z_t = \ln \Gamma_0 + (1-\theta)t \ln(1+\gamma) + \ln z_t, \tag{11}$$

from (2) and is directly computable using data for output, capital, and labor. Thus, we can estimate γ from a regression of Solow residuals on a linear trend t and a constant:

$$\ln SR_t = a + bt + u_t. \tag{12}$$

That is, from (11) and (12), $\gamma \approx \ln(1+\gamma) = b/(1-\theta)$. The growth trend η is computed as

$$\eta = (1+\gamma)(1+n),$$

where the constant population growth rate n is computed directly from data.

The depreciation rate δ is computed directly from (5) using capital and investment data. Then from (9), the discount factor β is computed as

$$\beta = \frac{\eta}{\theta \frac{y}{k} + 1 - \delta},$$

where the capital output ratio k/y is computed directly from data and I assume that investment taxes are zero in the steady state. Also, from (8), the utility parameter Ψ is computed as

$$\frac{1-\Psi}{\Psi} = (1-\theta)\frac{y}{l}\frac{1-l}{c},$$

where the consumption share of output c/y is computed directly from data and I assume that labor taxes are zero in the steady state.

Finally, the parameters in the stochastic process (10) are estimated with Bayesian estimation using the data for output, consumption, labor, and investment. I rely on structural estimation, because investment wedges are not directly observable. The estimated values are

$$P = \begin{vmatrix} 0.81 & 0.19 & 0.31 & 0.02 \\ 0.14 & 0.88 & -0.33 & -0.10 \\ 0.02 & -0.01 & 0.95 & -0.04 \\ 0.07 & -0.04 & -0.08 & 0.92 \end{vmatrix},$$

$$Q = \begin{vmatrix} 0.42 & 0.16 & -0.00 & 0.10 \\ 0.16 & 0.35 & 0.02 & 0.17 \\ -0.00 & 0.02 & 0.01 & 0.02 \\ 0.10 & 0.17 & 0.02 & 0.12 \end{vmatrix} * 1.00E-03.$$

B. Computing Wedges

Given all parameters values, the model can be solved quantitatively. I use the solution method following Uhlig (1999) to solve for linear decision rules. Having obtained the decision rules, I compute the entire series of exogenous variables $\{\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t\}$ using data for $\{\tilde{y}_t, \tilde{c}_t, \tilde{l}_t, \tilde{x}_t\}$. Specifically, the procedure of computing wedges is as follows:

(1) Compute linear decision rules

$$\tilde{k}_{t+1} = A_{1\times 1}\tilde{k}_t + B_{1\times 4}(\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t)',$$

$$(\tilde{y}_t, \tilde{c}_t, \tilde{l}_t, \tilde{x}_t)' = C_{4\times 1}\tilde{k}_t + D_{4\times 4}(\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t)',$$

while A, B, C, and D are matrixes containing the corresponding linear decision rule coefficients.

(2) Assume $\tilde{k}_0 = 0$.

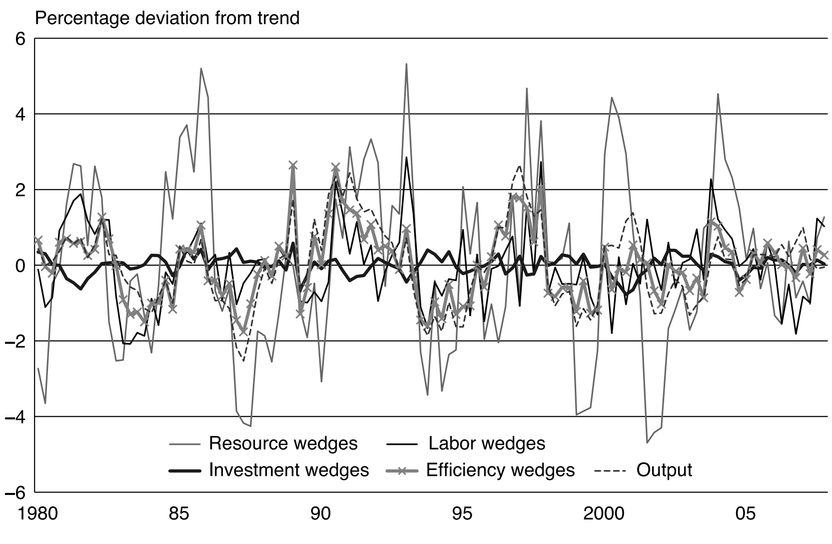
(3) Given $\tilde{k}_0$, $(\tilde{g}_0, \tau_0^l, \tau_0^x, \tilde{z}_0)' = D^{-1}(\tilde{y}_0, \tilde{c}_0, \tilde{l}_0, \tilde{x}_0)'$.

(4) Given $\tilde{k}_0$ and $(\tilde{g}_0, \tau_0^l, \tau_0^x, \tilde{z}_0)'$, $\tilde{k}_1 = B(\tilde{g}_0, \tau_0^l, \tau_0^x, \tilde{z}_0)'$.

(5) Given $\tilde{k}_1$, $(\tilde{g}_1, \tau_1^l, \tau_1^x, \tilde{z}_1)' = D^{-1}(\tilde{y}_t, \tilde{c}_t, \tilde{l}_t, \tilde{x}_t)' - D^{-1}C\tilde{k}_1$ and so on.

Figure 3 plots the estimated HP-filtered wedges.[11] Resource, labor, and efficiency wedges move along together with output, whereas investment wedges move in the opposite direction. The volatility of resource wedges is much larger than that of output, while the volatility of investment wedges is much smaller than that of output.

Figure 3 Wedges



11. The linearly detrended (non-HP-filtered) wedges are presented in Appendix 1.

Table 3 Business Cycle Features of Estimated Wedges

	St. dev. of v		Correlation of output with						
v	Percent	Rel. y	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.06	1.00	0.46	0.61	0.79	1.00	0.79	0.61	0.46
Resource wedge	2.40	2.25	0.01	0.22	0.36	0.54	0.43	0.31	0.24
Labor wedge	1.05	0.99	0.05	0.13	0.19	0.37	0.27	0.23	0.16
Investment wedge	0.25	0.24	−0.14	−0.30	−0.38	−0.24	−0.37	−0.32	−0.17
Efficiency wedge	0.92	0.87	0.40	0.50	0.64	0.90	0.65	0.50	0.38

The cyclical properties of the wedges are summarized in Table 3. Resource, labor, and efficiency wedges are procyclical and their fluctuations are coincident with output fluctuation, while investment wedges are countercyclical and coincident. The correlation of efficiency wedges with output is much higher than that of other wedges.

C. Simulation

The model is simulated by plugging in the computed wedges into the linear decision rules of output, consumption, investment, and the labor supply. In order to focus on the impact of each wedge on high-frequency cyclical comovements of these variables in Japan, I present the HP-filtered results. Non-HP-filtered results are presented in Appendix 1.

Figure 4 shows the cyclical features of the HP-filtered simulation results using each wedge one by one. The results show that efficiency wedges are important in accounting for the output fluctuation, investment wedges are important in accounting for fluctuation in investment, and labor wedges are important in accounting for the fluctuation in labor, while both labor and efficiency wedges are important in accounting for the fluctuation in consumption.

Table 4 reports the results of the simulation with only efficiency wedges.[12] There are several important discrepancies between the results and the data. First, the persistence of output and investment is too low. Second, the contemporaneous correlations of consumption and labor with output are too high. Third, the volatility of labor is too low. Finally, the model cannot account for the lead in labor.

Table 5 reports the results with both efficiency and labor wedges. The results show that all three discrepancies mentioned above are improved. The persistence of output and investment increases, the contemporaneous correlations of consumption and labor with output are closer to the data, the volatility of labor is equivalent to the data, and the model reproduces the lead in labor. Therefore, we conclude that labor wedges are important in accounting for the output and labor correlation pattern in Japan. The remaining question is, what is the fundamental source of the labor wedges.

12. This is almost equivalent to the real business cycle model. The only difference is that in this simulation efficiency wedges affect the expectation of the future values of other wedges. Although the realization of other wedges is always zero, expectations that they will be non-zero have some effect on the outcome.

Figure 4 Simulated Output

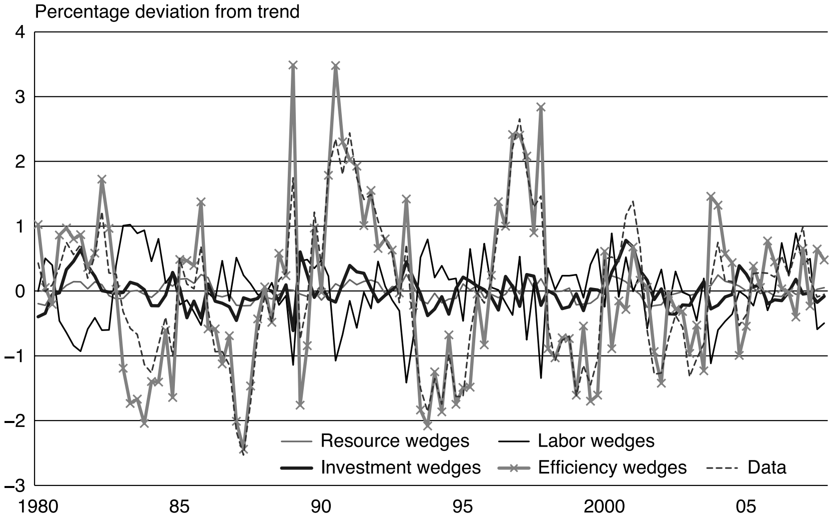


Table 4 Business Cycle Accounting Results with Efficiency Wedges

	St. dev. of v		Correlation of output with						
v	Percent	Rel. y	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.25	1.00	0.34	0.47	0.57	1.00	0.57	0.47	0.34
Consumption	0.63	0.50	0.22	0.36	0.50	0.92	0.64	0.59	0.51
Investment	3.57	2.86	0.37	0.49	0.58	0.99	0.53	0.41	0.27
Labor	0.53	0.42	0.41	0.50	0.57	0.94	0.44	0.30	0.15

Table 5 Business Cycle Accounting Results with Efficiency and Labor Wedges

	St. dev. of v		Correlation of output with						
v	Percent	Rel. y	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	0.98	1.00	0.43	0.51	0.67	1.00	0.67	0.51	0.43
Consumption	0.49	0.50	0.25	0.35	0.54	0.81	0.67	0.59	0.58
Investment	2.95	3.01	0.46	0.52	0.66	0.98	0.62	0.44	0.35
Labor	0.64	0.65	0.30	0.29	0.33	0.28	0.13	−0.01	−0.04

V. Sources of Labor Wedges in Japan

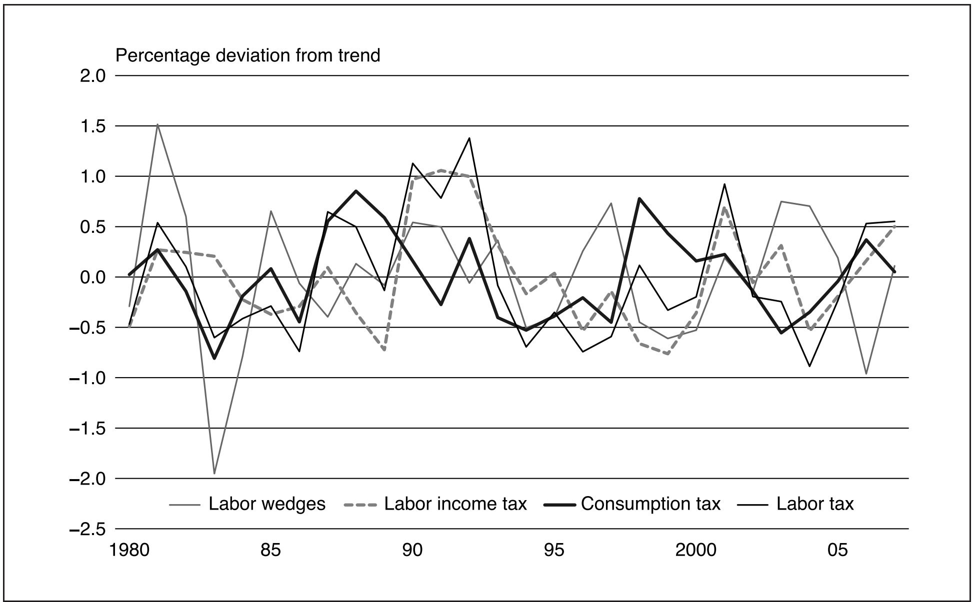
As shown in Chari, Kehoe, and McGrattan (2007), there are several fundamental sources of labor wedges. In this section, I show that labor income taxes, money growth shocks, and real interest rate shocks can be mapped into the prototype model with labor wedges. I assess whether these monetary and fiscal variables can account for the fluctuation pattern of the labor wedges in Japan.

A. Labor Tax

Since labor wedges are defined as taxes on labor income in the model, I will investigate the data for taxes. Not only labor income taxes but also consumption taxes must be considered, as they affect the relative price of labor to consumption, that is, the effective real wage.[13] Therefore, the effective tax rate on labor is the sum of labor income and consumption tax rates. The effect of labor taxes on the long-run declining trend on Japanese labor input is discussed in Otsu (2009). In this subsection, I focus on their impacts on the labor market in the business cycle frequency.[14]

Unfortunately, the tax data are only available in the annual frequency. Therefore, I compare them with the annual average of the labor wedges. Figure 5 plots the HP-filtered annual tax data from McDaniel (2007) along with the labor wedges. The figure shows that the fluctuation of the annual labor wedges is highly correlated with the fluctuation of labor taxes in the early 1980s, late 1980s, and early 2000s. Most of the movement in the effective labor tax during the early 1980s comes from the consumption tax, while that in the late 1980s and early 2000s comes from the labor income tax. However, since we do not observe tax changes every quarter, we cannot directly infer their contributions to the quarterly fluctuations of labor wedges in Japan.

Figure 5 Labor Wedges and Labor Tax



13. Time-varying consumption taxes also affect intertemporal choices of consumption, and thus can also be considered to contribute to investment wedges.

14. Braun (1994) and McGrattan (1994) show that fluctuations in distortionary taxes are important in accounting for the postwar U.S. business cycle fluctuations. The former focuses on the annual impacts of distortionary taxes, while the latter estimates the quarterly stochastic process of taxes from annual tax data with maximum likelihood estimation.

B. Monetary Shocks

Monetary shocks such as money growth or nominal interest rate shocks can create distortions in the labor market.[15] In this subsection, I show that a working capital model with nominal interest rate shocks, a cash-in-advance model, and a sticky wage model with money growth shocks can be mapped into the business cycle accounting model with labor wedges. I simulate each model with the observed monetary shocks and discuss their impacts on the Japanese business cycle over the 1980–2007 period.

1. Working capital model

Labor working capital models such as Christiano and Eichenbaum (1992) assume that the firm must borrow to pay its wage bill. I assume a simple setting in which firms borrow cash from the government in the beginning of the period and pay it back at the end of the period. The government exogenously sets the interest rate on the loans to the firm Q_t based on its monetary policy stance. Since the borrowing cost is included in the labor cost, shocks to the interest rate can be interpreted as labor wedges.

The firm's problem will change accordingly as follows.

$$\max \pi_t = y_t - Q_t w_t l_t - r_t k_t,$$

where Q_t is the interest payment the firm must make. The firm's first-order condition for labor will be

$$Q_t w_t = (1-\theta)\frac{y_t}{l_t}.$$

If $1/Q_t = (1-\tau_{lt})$, the labor working capital model is observationally equivalent to the prototype model with labor wedges. Therefore, shocks to nominal interest rates operate as labor wedges. That is, a rise in the interest rates will increase the cost of labor and reduce the labor supply.

2. Inflation tax model

Cash-in-advance models such as Cooley and Hansen (1989) give rise to labor wedges from money growth shocks. With a cash-in-advance constraint, the consumer must hold cash to purchase consumption goods. Since labor income cannot be used to buy goods in the current period, money growth shocks affect the effective price of labor relative to consumption. Therefore, money growth shocks create labor wedges in the form of inflation tax.

The consumer's budget constraint with money without wedges is

$$w_t l_t + r_t k_t + \frac{M_{t-1}}{P_t} + \pi_t + \tau_t = c_t + x_t + \frac{M_t}{P_t}. \quad (13)$$

Assume that the consumer holds money due to a standard cash-in-advance constraint:

$$\frac{M_{t-1}}{P_t} + \tau_t \geq c_t, \quad (14)$$

15. In a Keynesian view, optimal monetary policy can reduce the inefficiency in the labor market through this distortionary effect.

where M_t is the money supply and P_t is the price of consumption goods relative to money. The amount of cash held to purchase goods in the current period is predetermined during the previous period. The labor first-order condition will be

$$\frac{1-\Psi}{\Psi}\frac{c_t}{1-l_t} = \frac{\lambda_t}{\lambda_t + \mu_t} w_t,$$

where λ and μ are the Lagrange multipliers on the constraints (13) and (14), respectively. If $\lambda_t/(\lambda_t + \mu_t) = (1 - \tau_{lt})$ the cash-in-advance model is observationally equivalent to the prototype model with labor wedges. An increase in money growth $M_t/M_{t-1} = \varphi_t$ that raises the expected inflation reduces the expected return on labor income and thus should reduce the labor supply.

3. Sticky wage model

Chari, Kehoe, and McGrattan (2007) show that a model with sticky wages due to labor unions following Cole and Ohanian (2002) can be mapped into a prototype model with labor wedges. Given the predetermined wage contract, an unexpected change in the inflation rate will create a discrepancy between the marginal product of labor and the marginal rate of substitution of leisure to consumption, which can be interpreted as an increase in labor wedges.

In this model, there is a continuum of identical labor unions j that have monopolistic power over the differentiated labor supply. The aggregate labor is defined as

$$l_t = \left[\int (l_t^j)^\nu \, dj\right]^{\frac{1}{\nu}},$$

where ν is the markup of the labor union. The final good firm hires differentiated labor from each union j according to a labor demand function

$$l_t^j = \left[\frac{w_{t-1}}{w_{t-1}^j}\right]^{\frac{1}{1-\nu}} l_t,$$

where w_{t-1} is the aggregate wage for labor at period t that is predetermined in period $t-1$. The labor first-order condition is

$$(1-\theta)\frac{y_t}{l_t} = \frac{w_{t-1}}{i_t}.$$

An unexpected increase in the money growth rate causes inflation, which will lower the labor cost and raise labor demand, whereas an expected increase in inflation will be factored in w_{t-1} by the union and will not affect the real cost of labor.[16]

16. The wage rate is set according to the optimality condition:

$$w_{t-1} = \frac{1}{\nu}\frac{1-\Psi}{\Psi}\frac{E_{t-1}\left[\frac{l_t}{1-l_t}\right]}{E_{t-1}\left[\frac{l_t}{c_t}\frac{1}{i_t}\right]}.$$

This shows that only expected inflation will be incorporated in the predetermined wage rate. See Appendix 2 for details on the union problem.

4. Quantitative analysis

Table 6 summarizes the behavior of monetary variables over the 1980–2007 period. The call rate is positively correlated to output and lags it in terms of the timing of fluctuation. Because Japan adopted the so-called zero interest rate policy from 1999, the moments over the 1980–98 period are reported separately in the fourth row. I find that the fluctuation and the correlation with output are higher than the sample including the zero interest rate period. According to the working capital model, the procyclical interest rates should reduce the procyclicality of the labor supply. The level of M1 is positively correlated to output and leads it in terms of fluctuation, while the growth rate of M1 is negatively correlated to output and lags it in terms of fluctuation. According to the inflation tax model, the countercyclicality of the money growth should increase the correlation between output and labor. On the other hand, according to the sticky wage model, the countercyclicality of the money growth should decrease the correlation of labor with output. Therefore, interest rate shocks with working capital and money growth shocks with staggered price settings might be able to account for the labor wedges, while inflation tax does not seem to be a plausible candidate for this.

To investigate the quantitative impacts of monetary shocks on the Japanese economy, I simulate each model with monetary shocks and efficiency wedges, that is, TFP. The deep parameters in each model are calibrated in standard fashion. To conduct a stochastic simulation, I assume the following stochastic processes:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \varepsilon_{zt}, \quad \varepsilon_{zt} \sim N(0, \sigma_z^2),$$

$$\tilde{Q}_t = \rho_Q \tilde{Q}_{t-1} + \varepsilon_{Qt}, \quad \varepsilon_{Qt} \sim N(0, \sigma_Q^2),$$

$$\tilde{\varphi}_t = \rho_\varphi \tilde{\varphi}_{t-1} + \varepsilon_{\varphi t}, \quad \varepsilon_{\varphi t} \sim N(0, \sigma_\varphi^2).$$

The estimated values are $(\rho_z, \rho_Q, \rho_\varphi) = (0.98, 0.97, -0.02)$. TFP is included in all exercises, because we have already seen that they are important in accounting for the fluctuation in output. By adding monetary shocks, we can investigate their contribution to the labor wedges.

Table 7 reports the simulation results of each model. The first and second rows report the moments of labor computed from the data and the business cycle

Table 6 Business Cycle Features of Monetary Variables

	St. dev. of v		Correlation of output with						
v	Percent	Rel. y	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.06	1.00	0.46	0.61	0.79	1.00	0.79	0.61	0.46
Labor wedges	1.05	0.99	0.05	0.13	0.19	0.37	0.27	0.23	0.16
Call rate	0.21	0.20	0.22	0.30	0.34	0.38	0.40	0.39	0.38
Call rate (−98)	0.26	0.22	0.26	0.34	0.38	0.41	0.44	0.45	0.46
M1	4.10	3.86	0.21	0.17	0.12	0.04	−0.06	−0.10	−0.11
M1 growth	2.82	2.65	0.07	−0.04	−0.06	−0.10	−0.13	−0.04	0.01

Table 7 Simulation Results of Monetary Models

	St. dev. of l		Correlation of output with labor						
l	Percent	Rel. y	$l(-3)$	$l(-2)$	$l(-1)$	$l(0)$	$l(1)$	$l(2)$	$l(3)$
Data	0.74	0.69	0.36	0.47	0.55	0.45	0.40	0.24	0.12
Efficiency wedge	0.53	0.42	0.41	0.50	0.57	0.94	0.44	0.30	0.15
Working capital	0.41	0.36	0.30	0.37	0.44	0.81	0.29	0.13	−0.01
Inflation tax	1.08	0.67	0.33	0.51	0.59	0.99	0.53	0.41	0.27
Sticky wage	6.48	1.59	−0.09	0.02	−0.17	0.97	−0.23	−0.01	−0.16

accounting model with only efficiency wedges. The main discrepancies are the low contemporaneous correlation of labor and output and the lead in labor from output.

The third row reports the results from the working capital model. The contemporaneous correlation of labor with output falls from 0.94 to 0.81 by adding the interest rate shock. This is due to the depressing effect and procyclicality of the interest rates. Therefore, interest rate shocks with working capital do contribute to the labor wedge in terms of contemporaneous correlation between labor and output.[17] However, the model cannot replicate the lead in labor.

The fourth row reports the results of the inflation tax model. The lead in labor cannot be replicated with this model. Furthermore, the contemporaneous correlation of labor with output is higher than the model without monetary shocks. This is due to the countercyclicality of the money growth shock. However, the inflation tax does improve the model in terms of the volatility of labor relative to that of output.

The fifth row reports the results of the sticky wage model. The volatility of labor and the contemporaneous correlation of labor with output are too high. Moreover, there is no persistence in labor. In fact, the impact of money growth shocks dominates that of efficiency wedges and output loses persistence in the sticky wage setting. Chari, Kehoe, and McGrattan (2000) show that monetary shocks with staggered price setting cannot replicate the persistence in aggregate variables. This is why monetary models introduce habit preferences and investment adjustment costs to replicate the persistence of output, consumption, the price level, and so on.

C. Other Candidates

Misspecification of the preference function can result in labor wedges in the business cycle accounting model. For instance, preferences with habit persistence:

$$u(c_t, c_{t-1}, l_t) = \Psi \log(c_t - bc_{t-1}) + (1 - \Psi)\log(1 - l_t),$$

and preferences without income effects on labor as in Greenwood, Hercowitz, and Huffman (1988):

$$u(c_t, l_t) = \log(c_t - \chi l_t^{\nu}),$$

17. If we disregard the post-zero interest rate policy period after 1999, the volatility of labor increases from 0.41 percent to 0.46 percent, while other correlation patterns are not affected.

imply different marginal rates of substitution

$$\frac{1-\Psi}{\Psi}\frac{1}{1-l_t}\left(\frac{1}{c_t-bc_{t-1}}-E_t\left[\frac{\beta b}{c_{t+1}-c_t}\right]\right)^{-1}, \tag{15}$$

and

$$\frac{\chi l_t^{\nu-1}}{c_t-\chi l_t^{\nu}},$$

respectively.[18] If either of these preferences were the true preference function, labor wedges would arise simply because the marginal rate of substitution in the business cycle accounting model was false. The obvious challenge in assessing this issue is to claim these alternative preferences as "true" and the log preferences as "false." A similar discussion can be made for the production function.

Variable preference weights Ψ, that is, preference shocks, are observationally equivalent to labor wedges. By definition, anything that increases the household's subjective value of consumption relative to leisure can be considered a preference shock. Nonetheless, preference weights are not directly observable unless the model endogenously links them to observable shocks. In Braun, Ikeda, and Joines (2009), the preference weights on consumption and leisure depend on the family size. Their quantitative analysis shows that the drop in the birthrate led to a long-run decline in labor supply through this endogenous shift in preference weights.[19] Obviously, however, demographics do not fluctuate in the short run. It is not clear whether other underlying shocks are causing preference weights to fluctuate quarterly.

Fluctuations in the markup to monopolistically competitive firms, often defined as cost-push shocks in New Keynesian models, are also observationally equivalent to labor wedges. By definition, the markup is the ratio of the optimal price to the marginal cost, where the marginal cost is the ratio of nominal wages to the marginal product of labor. Therefore, in equilibrium the markup is equal to the ratio of the marginal product of labor to the real wage. Thus, stochastic markup shocks are observationally equivalent to labor wedges. There are several sources of stochastic markup fluctuations. For instance, changes in the bargaining power of labor unions can cause a fluctuation in the markup. Steinsson (2003) shows that stochastic shocks to the household elasticity of substitution among the differentiated goods manifest themselves as markup shocks. However, it is not clear how to measure the union bargaining power or the curvature on the preference function from the data.

18. Note that non-separable preferences,

$$u(c_t, l_t) = \frac{(c_t^{\Psi}(1-l_t)^{1-\Psi})^{1-\sigma}}{1-\sigma},$$

have the same marginal rate of substitution as the log utility. Therefore, the computed labor wedges will be the same as the benchmark case. Nonetheless, their impacts will be different, as the Euler equation will differ due to the non-separability of consumption and leisure in the marginal utility of consumption.

19. The head of the household maximizes the utility of the family. Every family member consumes, while only the head of the household goes to work. A decrease in family members increases the preferece weight of leisure and hence leads to a decrease in labor.

Finally, expectational shocks can also generate labor wedges. Consider the habit preferences discussed above that gives the marginal rate of substitution as (15). Clearly, a shock to expected future consumption can affect the marginal rate of substitution, which is observationally equivalent to labor wedges in the benchmark model. Fujiwara, Hirose, and Shintani (2011) estimate multi-period expectational shocks to future TFP in a dynamic stochastic general equilibrium (DSGE) model using Bayesian estimation. This will enable us to investigate whether expectational shocks can account for the correlation pattern of labor with output. However, the expectational shocks cannot be directly observed.

VI. Conclusion

This paper applies the business cycle accounting method to the Japanese economy and shows that the fluctuations in labor wedges are important in order to replicate the low contemporaneous correlation between labor and output and the lead of labor to output. Fiscal and monetary variables such as labor income tax, money growth, and interest rates cannot fully account for the distortions in the Japanese labor market. Further understanding of the quarterly Japanese labor market fluctuations requires a deeper assessment of additional sources of labor wedges such as preference shocks, markup shocks, and expectational shocks, as well as the functional form of aggregate preferences.

APPENDIX 1: NONFILTERED RESULTS

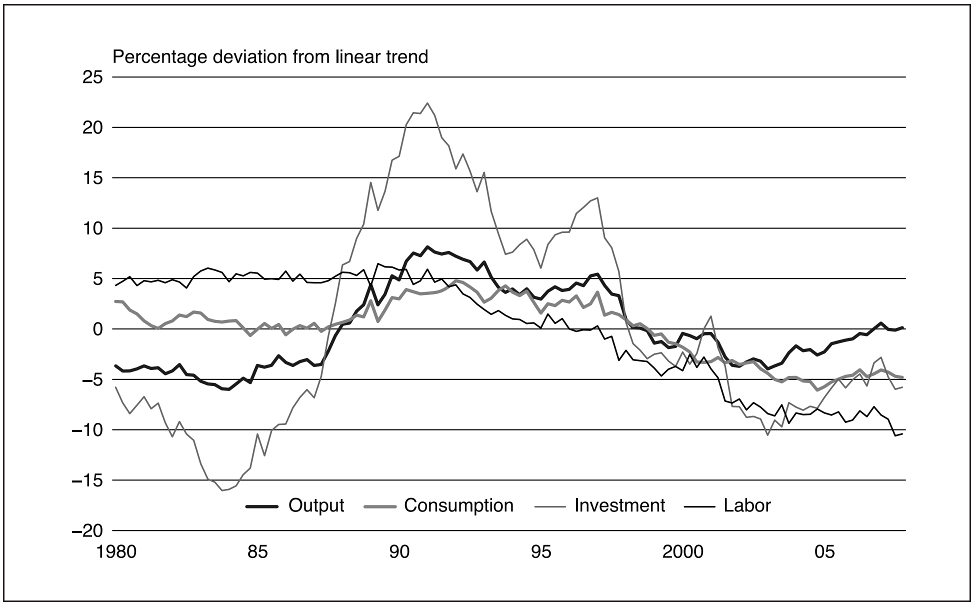
Appendix Figure 1 presents the data for the observable variables used for the estimation and simulation. Output, consumption, and investment are detrended by the growth trend η. The non-HP-filtered data clearly show the abrupt growth in the late 1980s and the lost decade in the 1990s. Consumption has been declining relative to output, implying a decline in the consumption share of output.[20] Labor is declining throughout the entire period.

Appendix Figure 2 plots the nonfiltered wedges. Resource, labor, and efficiency wedges have growing trends. Government expenditure is growing relative to output in Japan over the entire period, which explains the growth in resource wedges. The growth in labor wedges reflects the declining trend in labor. The growth in efficiency wedges reflects the fact that labor is declining in the data, whereas it is assumed to be stationary in the model. Although this discrepancy is unpleasant, it does not have a major impact on the estimation or simulation results in the paper.

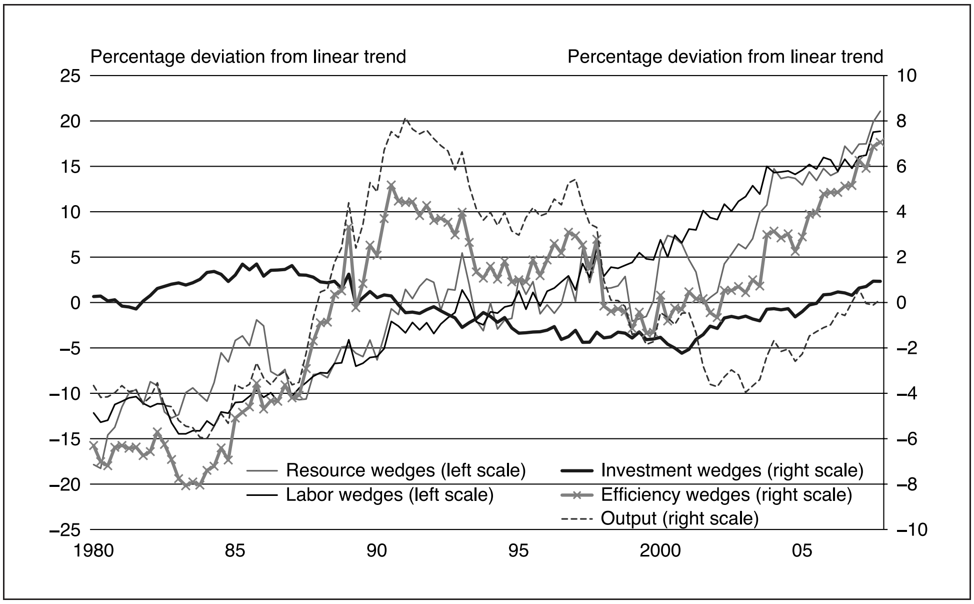
Appendix Figure 3 plots the nonfiltered simulated output. Resource and investment wedges do not have much impact on output fluctuation. Efficiency wedges account for most of the ups and downs of output; however, they overstate the growth throughout the entire period. Labor wedges cause output to decline throughout the entire period. The combination of labor and efficiency wedges reproduces output almost perfectly. This result is consistent with the finding of Kobayashi and Inaba (2006).

20. This is mainly due to the increase in government expenditures over the entire period.

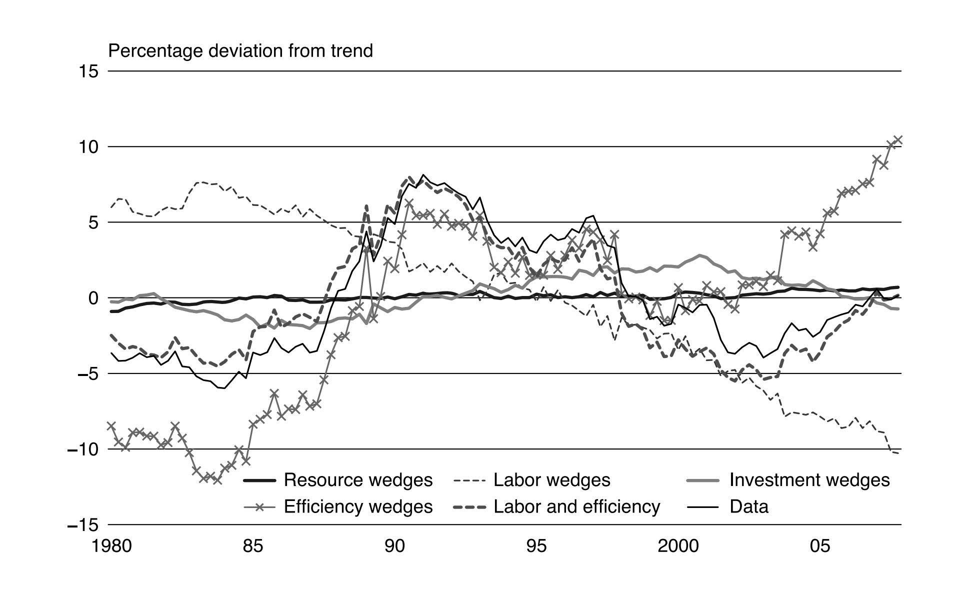
Appendix Figure 1 Japanese Business Cycles (Nonfiltered)



Appendix Figure 2 Wedges (Nonfiltered)



Appendix Figure 3 Simulated Output (Nonfiltered)



APPENDIX 2: MONETARY MODELS

The following describes the detail of the monetary models introduced in Section V.

A. Working Capital Model

1. Firm's problem

$$\max \pi_t = y_t - Q_t w_t l_t - r_t k_t,$$

subject to

$$y_t = z_t k_t^{\theta} l_t^{1-\theta}.$$

2. Household problem

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t [\Psi \log(c_t) + (1-\Psi)\log(1-l_t)],$$

subject to

$$w_t l_t + r_t k_t + \pi_t + \tau_t = c_t + x_t,$$

$$\eta k_{t+1} = x_t + (1-\delta)k_t.$$

3. Government budget constraint

$$(Q_t - 1) w_t l_t = \tau_t + g.$$

4. Equilibrium conditions

We have five equations, five endogenous variables $\{y, c, x, l, k\}$ and two exogenous variables $\{z_t, Q_t\}$.

$$y_t = z_t k_t^{\theta} l_t^{1-\theta},$$

$$y_t = c_t + x_t + g,$$

$$\eta k_{t+1} = x_t + (1-\delta)k_t,$$

$$\eta \frac{\Psi}{c_t} = \beta E_t \left[\frac{\Psi}{c_{t+1}} \left(\theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right],$$

$$(1-\theta)\frac{y_t}{l_t} = Q_t \frac{1-\Psi}{\Psi} \frac{c_t}{1-l_t}.$$

B. Inflation Tax Model

1. Firm's problem

$$\max P_t \pi_t = P_t y_t - W_t l_t - R_t k_t,$$

subject to

$$y_t = z_t k_t^{\theta} l_t^{1-\theta}.$$

2. Household problem

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t [\Psi \log(c_t) + (1-\Psi)\log(1-l_t)],$$

subject to

$$W_t l_t + R_t k_t + M_{t-1} + P_t \pi_t + P_t \tau_t = P_t c_t + P_t x_t + M_t,$$

$$\eta k_{t+1} = x_t + (1-\delta)k_t,$$

$$\frac{M_{t-1}}{P_t} \geq c_t.$$

3. Government budget constraint

$$M_t - M_{t-1} = P_t \tau_t + P_t g,$$

$$M_t = \varphi_t M_{t-1}.$$

4. Change of variables

$$m_t = \frac{M_t}{P_t}, \quad w_t = \frac{W_t}{P_t}, \quad r_t = \frac{R_t}{P_t}, \quad i_t = \frac{P_t}{P_{t-1}}.$$

5. Equilibrium conditions

We have nine equations, nine endogenous variables $\{y, c, x, l, k, m, i, w, \lambda\}$, where λ is the Lagrange multiplier on the household budget constraint, and two exogenous variables $\{z_t, \varphi_t\}$.

$$y_t = z_t k_t^{\theta} l_t^{1-\theta},$$

$$y_t = c_t + x_t + g,$$

$$\eta k_{t+1} = x_t + (1-\delta)k_t,$$

$$\eta\lambda_t = \beta E_t\left[\lambda_{t+1}\left(\theta\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right)\right],$$

$$\lambda_t = \frac{1}{w_t}\frac{1-\Psi}{1-l_t},$$

$$\lambda_t = \beta E_t\left[\frac{1}{i_{t+1}}\frac{\Psi}{c_{t+1}}\right],$$

$$w_t = (1-\theta)\frac{y_t}{l_t},$$

$$m_t = c_t + g,$$

$$i_t = \frac{P_t}{P_{t-1}} = \frac{m_{t-1}}{m_t}\frac{M_t}{M_{t-1}} = \frac{m_{t-1}}{m_t}\varphi_t.$$

C. Sticky Wage Model

1. Firm's problem

a. Profit maximization

$$\max P_t\pi_t = P_t y_t - W_{t-1}l_t - R_t k_t,$$

subject to

$$y_t = z_t k_t^{\theta} l_t^{1-\theta}.$$

b. Labor cost minimization

$$\min \int W_{t-1}^{j} l_t^{j}\, dj,$$

subject to

$$l_t = \left[\int (l_t^{j})^{\nu}\, dj\right]^{\frac{1}{\nu}}.$$

2. Union's problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\Psi \log(c_t^j) + (1-\Psi)\log(1-l_t^j) + \chi \log\left(\frac{M_t}{P_t}\right)\right],$$

subject to

$$W_{t-1}^j l_t^j + R_t k_t^j + M_{t-1}^j + P_t \pi_t^j + P_t \tau_t^j = P_t c_t^j + P_t x_t^j + M_t^j,$$

$$\eta k_{t+1}^j = x_t^j + (1-\delta) k_t^j,$$

$$l_t^j = \left[\frac{W_{t-1}}{W_{t-1}^j}\right]^{\frac{1}{1-\nu}} l_t.$$

3. Government budget constraint

$$M_t - M_{t-1} = P_t \tau_t + P_t g,$$

$$M_t = \varphi_t M_{t-1}.$$

4. Change of variables

$$m_t = \frac{M_t}{P_t}, \quad w_t = \frac{W_t}{P_t}, \quad r_t = \frac{R_t}{P_t}, \quad i_t = \frac{P_t}{P_{t-1}}.$$

5. Equilibrium conditions

All unions are identical, so we can drop the super-script j. We have eight equations, eight endogenous variables $\{y, c, x, l, k, m, w, i\}$ and two exogenous variables $\{z_t, \varphi_t\}$.

$$y_t = z_t k_t^{\theta} l_t^{1-\theta},$$

$$\eta k_{t+1} = x_t + (1-\delta) k_t,$$

$$y_t = c_t + x_t + g,$$

$$\eta \frac{\Psi}{c_t} = \beta E_t \left[\frac{\Psi}{c_{t+1}}\left(\theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right)\right],$$

$$w_t = \frac{1}{\nu}\frac{1-\Psi}{\Psi}\frac{E_t\left[\frac{l_{t+1}}{1-l_{t+1}}\right]}{E_t\left[\frac{l_{t+1}}{c_{t+1}}\frac{1}{i_{t+1}}\right]},$$

$$i_t(1-\theta)\frac{y_t}{l_t} = w_{t-1},$$

$$\frac{\chi}{m_t} = \frac{\Psi}{c_t} - \beta E_t\left[\frac{\Psi}{c_{t+1}}\frac{1}{i_{t+1}}\right],$$

$$i_t = \frac{m_{t-1}}{m_t}\varphi_t.$$

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