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RESEARCH ARTICLE

Delay independent decentralised output feedback control for large scale systems with nonlinear interconnections†

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In this paper, a stabilisation problem for a class of large scale systems with nonlinear interconnections is considered. All the uncertainties are nonlinear and are subject to the effects of time delay. A decentralised static output feedback variable structure control is synthesised and the stability of the corresponding closed loop system is analysed based on the Lyapunov Razumikhin approach. A set of conditions is developed to guarantee that the large scale interconnected system is stabilised uniformly asymptotically. Further study shows that the conservatism can be reduced by employing additive controllers if the known interconnections are separated into matched and mismatched parts. It is not required that the subsystems are square. The designed controller is independent of time delay and thus it does not require memory. Simulation results show the effectiveness of the proposed approach.

Keywords: Decentralised control, interconnected systems, Lyapunov Razumikhin approach, static output feedback, time varying delay.

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1 Introduction

Large scale interconnected systems are often modelled as dynamical equations composed of interconnections between a collection of lower-dimensional subsystems. It is well-known that for

† The original ideas informing this paper were presented at the 12th International Workshop on Variable Structure Systems (VSS), Mumbai, India, January, 2012. The current version is a modification and extension which contains detailed proofs omitted from the conference version. Simulation results, some remarks and additional information have been added in this paper.
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a large-scale interconnected system, a perturbation of one subsystem can both affect the other subsystems as well as the overall performance of the network. Such a class of systems are often characterised by geographical separation. Issues such as the economic cost and reliability of communication links have to be considered. Specifically, when the information transfer channels between subsystems are blocked, only local information is available and in this case only decentralised schemes are possible. This has provided impetus to explore decentralised control strategies (see Bakule (2008) and the references therein).

Decentralised output feedback control has received much attention and many interesting results have been obtained. Adaptive control techniques have been employed by Jain and Khorrami (1997), but only parametric uncertainty is dealt with in this approach. Yan et al. (1998) proposed a control scheme to deal with structural uncertainties but it is required that each subsystem is square i.e. the dimension of the input is equal to the dimension of the output. It should be noted that time delay is another important factor which makes the study of large scale interconnected systems complex (Richard 2003). Mahmoud and Bingulac (1998) considered a class of interconnected systems where delay does not appear in the interconnections. However, the interconnections between two or more physical systems are often accompanied by phenomena such as material transfer, energy transfer and information transfer, which, from a mathematical point of view, can be represented by delay elements (Michiels and Niculescu 2007). This has motivated the study of large scale time delay interconnected systems, and many results have been achieved (Bakule 2008, Hua and Ding 2011, Ye et al. 2012). However most of the existing results consider situations where all the system states are available. The associated decentralized output feedback results for time delay large-scale interconnected systems are very few (Hua et al. 2008, Zhou 2008). An output feedback decentralised control scheme is given in Mahmoud and Qureshi (2012) for the case of discrete interconnected systems. A class of nonlinear interconnected systems with triangular structure is considered in Hua et al. (2008), and a large scale system composed of a set of single input single output subsystems with dead zone input is considered in Zhou (2008). In both Hua et al. (2008) and Zhou (2008), the control schemes are based on dynamical output feedback which increases the computation greatly due to the associated closed-loop system possessing possibly twice the order of the actual plant.

In many of the existing control schemes, controllers are explicitly dependent on time delay (Bekiaris-Liberis and Krstic 2013, Yan et al. 2010) and/or limitations on the rate of change of the time delay must be imposed (Fridman and Dambrine 2009). In addition, as pointed out in Hua et al. (2008), most of the existing variable structure controllers for nonlinear systems use knowledge of the delay explicitly and hence require memory, which is difficult to implement in practice especially for the case of time-varying delay. Although a memoryless control for a class of linear systems was proposed based on a back-stepping approach in Hua et al. (2008), the nonlinear uncertainty is required to be matched and it is assumed that all the system states are available. A memoryless sliding mode control scheme is given in Yan (2003), but all the nonlinear terms are assumed to be matched and are without time delay. This renders the associated sliding mode dynamics to be delay free and thus there is no delay involved in the stability analysis of the sliding mode dynamics. More recently, a class of nonlinear time delay interconnected systems is considered by Yan et al. (2013). However, it is required that the time delay is precisely known and each subsystem is square. Further, the results given in Yan et al. (2013) do not render themselves suitable for extension to the non-square case.

In this paper, a variable structure control is synthesised to stabilise a class of large scale time delay systems with nonlinear interconnections. The bounds on the uncertainties are nonlinear and involve time delay states. A decentralised variable structure control scheme using only output information is proposed which is independent of time delay. Based on the Lyapunov Razumikhin approach, sufficient conditions are derived such that the closed-loop system formed by the designed control and the large scale interconnected systems is uniformly asymptotically stable. Limitation on the rate of change of the time delay is unnecessary. A compensator, which increases the required computation levels for large-scale interconnected systems, is not required either. Further study shows that the effects of the known interconnections can be largely rejected if they are separated into matched and mismatched parts and dealt with separately. Unlike the work in Yan et al. (2013), it is not required that the subsystem is square, and it is not required that the time delay is known. Thus the controller does not require memory. Simulation results show that the approach developed in this paper is effective.

Notation: In this paper, $\mathbb{R}^+$ denotes the nonnegative set of real numbers $\{t \mid t \geq 0\}$. The symbol $\mathcal{C}_{[a,b]}$ represents the set of $\mathbb{R}^n$-valued continuous function on $[a, b]$. The symbol I_n denotes the $n \times n$ unit matrix. The expression $A > 0$ ($A < 0$) means that A is symmetric positive (negative) definite and $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) represents its maximum (minimum) eigenvalue. For vectors $x = (x_1, x_2, \ldots, x_{n_1})^T \in \mathbb{R}^{n_1}$ and $y = (y_1, y_2, \ldots, y_{n_2})^T \in \mathbb{R}^{n_2}$, the expression $f(x, y)$ denotes a function $f(x_1, \ldots, x_{n_1}, y_1, \ldots, y_{n_2})$ defined on $\mathbb{R}^{n_1+n_2}$. Finally, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

2 System description and basic assumptions

Consider a time-varying delay interconnected system composed of n n_i-th order subsystems described by

$$\dot{x}_i = A_i x_i + B_i \big(u_i + \xi_i(t, x_i, x_{id_i})\big) + F_i(x) + \psi_i(t, x, x_d) \tag{1}$$

$$y_i = C_i x_i, \qquad i = 1, 2, \ldots, n, \tag{2}$$

where $x := \mathrm{col}(x_1, \ldots, x_n)$, $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{p_i}$ are the state variables, inputs and outputs of the i-th subsystem respectively. The triples (A_i, B_i, C_i) represent constant matrices of appropriate dimensions with B_i and C_i of full rank. The functions $\xi_i(\cdot)$ are matched uncertainties in the i-th subsystem. The function vectors $F_i(x) \in \mathbb{R}^{n_i}$ are known and analytic interconnections with $F_i(0) = 0$. The terms $\psi_i(t, x, x_d)$ are uncertain interconnections of the i-th subsystem. The symbols $x_{id_i} := x_i(t - d_i)$ and $x_d := \mathrm{col}(x_{1d_1}, x_{2d_2}, \cdots, x_{nd_n})$ are the delayed states, and the symbols $d_i := d_i(t)$ denote the time-varying delays which are assumed to be known, nonnegative and bounded in $\mathbb{R}^+$, that is

$$\overline{d}_i := \sup_{t \in \mathbb{R}^+} \{d_i(t)\} < \infty, \qquad i = 1, 2, \ldots, n$$

The initial conditions associated with the time delays are given by

$$x_i(t) = \phi_i(t), \qquad t \in [-\overline{d}_i, 0]$$

where $\phi_i(\cdot)$ are continuous in $[-\overline{d}_i, 0]$ for $i = 1, 2, \ldots, n$. It is assumed that all the nonlinear functions are smooth enough such that the unforced interconnected system has a unique continuous solution.

In this paper, the local case will be considered. In order to simplify the description, the considered domain will not be stated unless it is necessary. However, each variable's dimension will be clearly identified. Note, if all the relevant conditions hold globally, the developed results can be extended to the global case.

Since the function vectors $F_i(x)$ are known and analytic with $F_i(0) = 0$, there exist analytic function matrices $\Phi_i(\cdot) \in \mathbb{R}^{n_i \times \sum_{i=1}^{n} n_i}$ such that (see Banks and Al-jurani (1994))

$$F_i(x) = \Phi_i(x) x, \qquad i = 1, 2, \ldots, n$$

Partition the matrices $\Phi_i(\cdot)$ as

$$\Phi_i(x) := \left[\, \Phi_{i1}(x)\ \Phi_{i2}(x) \cdots\ \Phi_{in}(x) \,\right]$$

where $\Phi_{ij}(\cdot) \in \mathbb{R}^{n_i \times n_j}$ for $i, j = 1, 2, \ldots, n$. It follows from $x = \mathrm{col}(x_1, x_2, \cdots, x_n)$ that the interconnection terms $F_i(x)$ can be expressed by

$$F_i(x) = \sum_{j=1}^{n} \Phi_{ij}(x) x_j \tag{3}$$

It should be noted that the matrices $\Phi_{ij}(\cdot)$ which satisfy equation (3) are not unique. One way to find the matrices $\Phi_i(\cdot)$ and thus $\Phi_{ij}(\cdot)$ is presented in Yan and Dai (1999).

Definition 1. For the large-scale interconnected system (1)–(2), the systems

$$\dot{x}_i = A_i x_i + B_i(u_i + \xi_i\big(t, x_i, x_{id_i})\big) \tag{4}$$

$$y_i = C_i x_i, \qquad\qquad i = 1, 2, \ldots, n, \tag{5}$$

are called the i-th isolated subsystems and the systems

$$\dot{x}_i = A_i x_i + B_i u_i \tag{6}$$

$$y_i = C_i x_i, \qquad\qquad i = 1, 2, \ldots, n, \tag{7}$$

are said to be the i-th nominal isolated subsystems.

System (1)–(2) can be considered as being generated by interconnecting the isolated subsystems (4)–(5). The following basic assumptions are required.

Assumption 1. There exist known continuous functions $\rho_i(\cdot)$, $\varpi_i(\cdot)$, $\alpha_{ij}(\cdot)$ and $\beta_{ij}(\cdot)$ such that for $i, j = 1, 2, \ldots, n$

$$\|\xi_i(t, x_i, x_{id_i})\| \le \rho_i(t, y_i) + \varpi_i(t, y_i)\|x_{id_i}\| \tag{8}$$

$$\|\psi_i(t, x, x_d)\| \le \sum_{j=1}^{n} \alpha_{ij}(t, x)\|x_j\| + \sum_{j=1}^{n} \beta_{ij}(t, x)\|x_{jd_j}\| \tag{9}$$

Remark 1. Assumption 1 describes the limitations on the uncertainties that can be tolerated by the system. It is not required that the interconnections are described or bounded by functions of the system outputs as in Rodellar et al. (1993), Yan et al. (1998). Furthermore, unlike Rodellar et al. (1993), Yan et al. (1998), Mahmoud and Bingulac (1998), the bounds on the uncertain interconnections are nonlinear and involve the time delay state variables.

Assumption 2. The triples (A_i, B_i, C_i) are output feedback stabilisable for $i = 1, 2, \ldots, n$.

Assumption 2 is fundamental and implies that there exist matrices K_i such that for any $Q_i > 0$, the equations

$$-Q_i = (A_i - B_i K_i C_i)^T P_i + P_i(A_i - B_i K_i C_i) < 0, \qquad i = 1, 2, \ldots, n \tag{10}$$

have unique solutions $P_i > 0$.

Assumption 3. There exist matrices E_i such that

$$B_i^T P_i = E_i C_i \tag{11}$$

where the matrices P_i satisfy (10) for $i = 1, 2, \ldots, n$.

Remark 2. Assumption 2 together with Assumption 3 describes a structural property associated with the nominal isolated subsystems (A_i, B_i, C_i) in (6)–(7), which is the standard Constrained Lyapunov Problem (CLP) (see e.g., Galimidi and Barmish 1986). A similar limitation has been imposed by many authors (Galimidi and Barmish 1986, Choi 2008, Cheng 1998). Necessary and sufficient conditions for solving the CLP can be found in Galimidi and Barmish (1986) and Edwards et al. (2007).

The objective of this paper is, under the assumption that all the isolated subsystems (6)–(7) are output feedback stabilisable, to design a variable structure control law of the form

$$u_i = u_i(t, y_i), \qquad i = 1, 2, \ldots, n \tag{12}$$

such that the closed-loop system formed by applying the control law in (12) to the large scale interconnected system (1)–(2), is uniformly asymptotically stable even in the presence of the uncertainties and time delays. Since the control u_i in (12) are only dependent on the time t and the i-th subsystem's output y_i, and are independent of time delay, they constitute a delay independent decentralised static output feedback control.

3 Decentralised static output feedback control design

In this section, a decentralised output feedback controller which is independent of the time delay will be proposed for the large scale interconnected system (1)–(2).

Consider the control law

$$u_i = -K_i y_i - \frac{1}{2\varepsilon_i^a} E_i y_i \varpi_i^2(t, y_i) + u_i^a(t, y_i), \qquad i = 1, 2, \ldots, n \tag{13}$$

where $K_i \in \mathcal{R}^{m_i \times p_i}$ are design parameters satisfying (10), $\varpi_i(\cdot)$ are given in (8), $\varepsilon_i^a > 0$ are constant and the terms $u_i^a(\cdot)$ are defined by

$$u_i^a(\cdot) := \begin{cases} -\frac{E_i y_i}{\|E_i y_i\|}\rho_i(t, y_i), & E_i y_i \neq 0 \\ 0, & E_i y_i = 0 \end{cases} \tag{14}$$

where E_i satisfy (11). Since the structure of the control u_i in (13) are variable due to the terms $u_i^a(\cdot)$ in (14), they are called a variable structure control. Clearly each element u_i is decentralised because it is only dependent on the time t and the local output y_i. Thus the u_i in (13) are called decentralised output feedback variable structure controllers throughout the paper.

The following result is now ready to be presented:

Theorem 1. Under Assumptions 1–3, the closed-loop system formed by applying the control (13)–(14) to system (1)–(2) is uniformly asymptotically stable if

$$\gamma := \inf_x \left\{ \lambda_{\min}\big(W^T(\cdot) + W(\cdot)\big) \right\} > 0$$

where $W(\cdot) = [w_{ij}(\cdot)]_{2n\times 2n}$ is a function matrix defined by

$$w_{ij}(\cdot) = \begin{cases} \lambda_{\min}(Q_i) - q\lambda_{\max}(P_i) - 2\|P_i\Phi_{ii}(x)\| - 2\alpha_{ii}(t,x_i)\|P_i\|, & 1 \leq i = j \leq n \\ \lambda_{\min}(P_i) - \varepsilon_i^a, & n+1 \leq i = j \leq 2n \\ -2\|P_i\Phi_{ij}(x)\| - 2\alpha_{ij}(t,x)\|P_i\|, & i \neq j \quad \text{and} \quad 1 \leq i,j \leq n \\ -2\beta_{i(j-n)}(t,x_{j-n})\|P_i\|, & 1 \leq i \leq n, \text{ and } j > n \\ -2\beta_{(i-n)j}(t,x_j)\|P_{i-n}\|, & i > n, \text{ and } 1 \leq j \leq n \\ 0, & \text{otherwise} \end{cases}$$

for constants $q > 1$ and $\varepsilon_i^a > 0$, where $\alpha_{ij}(\cdot)$ and $\beta_{ij}(\cdot)$ satisfy (9) for $i,j = 1,2,\ldots,n$.

Proof: Applying the control (13)–(14) into system (1)–(2) and considering the equation (3), the corresponding closed-loop system can be described by

$$\begin{aligned} \dot{x}_i = A_i x_i + B_i\Big(-K_iC_ix_i - \frac{1}{2\varepsilon_i^a}E_iy_i\varpi_i^2(t,y_i) + u_i^a(t,y_i) + \xi_i\big(t,x_i,x_{id_i})\Big) \\ + \sum_{j=1}^n \Phi_{ij}(x)x_j + \psi_i(t,x,x_d) \end{aligned} \tag{15}$$

where $u_i^a(\cdot)$ are given by (14) and $\Phi_{ij}(\cdot)$ satisfy equations (3) for $i,j = 1,2,\ldots,n$. For system (15), consider the Lyapunov function candidate

$$V(x(t)) = V(x_1(t), x_2(t), \cdots, x_n(t)) = \sum_{i=1}^n x_i^T(t)P_ix_i(t) \tag{16}$$

where $P_i > 0$ satisfy equation (10) for $i = 1,2,\ldots,n$. Then, the time derivative of $V(\cdot)$ along the trajectories of system (15) is given by

$$\begin{aligned} \dot{V} = -\sum_{i=1}^n x_i^TQ_ix_i + 2\sum_{i=1}^n x_i^TP_iB_i\Big(-\frac{1}{2\varepsilon_i^a}E_iy_i\varpi_i^2(t,y_i) + u_i^a(t,y_i)\Big) + 2\sum_{i=1}^n x_i^TP_iB_i\xi_i(t,x_i,x_{id_i}) \\ +2\sum_{i=1}^n\sum_{j=1}^n x_i^TP_i\Phi_{ij}(x)x_j + 2\sum_{i=1}^n x_i^TP_i\psi_i(t,x,x_d) \end{aligned} \tag{17}$$

From (8), (11) and Young's inequality, it follows that for any $\varepsilon_i^a > 0$

$$\begin{aligned} x_i^TP_iB_i\xi_i(t,x_i,x_{id_i}) &= (E_iy_i)^T\xi_i(t,x_i,x_{id_i}) \\ &\leq \|E_iy_i\|\rho_i(t,y_i) + \|E_iy_i\|\varpi_i(t,y_i)\|x_{id_i}\| \\ &\leq \|E_iy_i\|\rho_i(t,y_i) + \frac{1}{2\varepsilon_i^a}\|E_iy_i\|^2\varpi_i^2(t,y_i) + \frac{\varepsilon_i^a}{2}\|x_{id_i}\|^2 \end{aligned} \tag{18}$$

From (11) and the definition of $u_i^a(\cdot)$ in (14), it follows that

i) if $E_iy_i = 0$, then $u_i^a(\cdot) = 0$, and thus

$$x_i^TP_iB_iu_i^a(t,y_i) + \|E_iy_i\|\rho_i(t,y_i) = 0$$

ii) if $E_i y_i \neq 0$, from the definition of $u_i^a(\cdot)$ in (14),

$$\begin{aligned}
&x_i^T P_i B_i u_i^a(t, y_i) + \|E_i y_i\| \rho_i(t, y_i) \\
&\leq -(E_i y_i)^T \frac{E_i y_i}{\|E_i y_i\|} \rho_i(t, y_i) + \|E_i y_i\| \rho_i(t, y_i) \\
&= 0
\end{aligned}$$

Therefore, from i) and ii) above,

$$x_i^T P_i B_i u_i^a(t, y_i) + \|E_i y_i\| \rho_i(t, y_i) \leq 0, \qquad i = 1, 2, \cdots, n \tag{19}$$

Further, from (11),

$$\begin{aligned}
&-\frac{1}{2\varepsilon_i^a} x_i^T P_i B_i E_i y_i \varpi_i^2(t, y_i) + \frac{1}{2\varepsilon_i^a} \|E_i y_i\|^2 \varpi_i^2(t, y_i) \\
&= -\frac{1}{2\varepsilon_i^a} x_i^T C_i^T E_i^T E_i y_i \varpi_i^2(t, y_i) + \frac{1}{2\varepsilon_i^a} \|E_i y_i\|^2 \varpi_i^2(t, y_i) \\
&= -\frac{1}{2\varepsilon_i^a} (E_i y_i)^T E_i y_i \varpi_i^2(t, y_i) + \frac{1}{2\varepsilon_i^a} \|E_i y_i\|^2 \varpi_i^2(t, y_i) = 0
\end{aligned} \tag{20}$$

Therefore, from (18), (19) and (20)

$$\begin{aligned}
&\sum_{i=1}^n x_i^T P_i B_i \Big(-\frac{1}{2\varepsilon_i^a} E_i y_i \varpi_i^2(t, y_i) + u_i^a(t, y_i) \Big) + \sum_{i=1}^n x_i^T P_i B_i \xi_i(t, x_i, x_{id_i}) \\
&\leq -\sum_{i=1}^n \frac{1}{2\varepsilon_i^a} x_i^T P_i B_i E_i y_i \varpi_i^2(t, y_i) + \sum_{i=1}^n x_i^T P_i B_i u_i^a(t, y_i) + \sum_{i=1}^n \|E_i y_i\| \rho_i(t, y_i) \\
&\quad + \sum_{i=1}^n \frac{1}{2\varepsilon_i^a} \|E_i y_i\|^2 \varpi_i^2(t, y_i) + \sum_{i=1}^n \frac{\varepsilon_i^a}{2} \|x_{id_i}\|^2 \\
&\leq \frac{1}{2} \sum_{i=1}^n \varepsilon_i^a \|x_{id_i}\|^2
\end{aligned} \tag{21}$$

From (9),

$$\begin{aligned}
x_i^T P_i \psi_i(t, x, x_d) &\leq \|x_i\| \, \|P_i\| \sum_{j=1}^n \big(\alpha_{ij}(t, x) \|x_j\| + \beta_{ij}(t, x) \|x_{jd_j}\| \big) \\
&= \sum_{i=1}^n \big(\alpha_{ij}(t, x) \|P_i\| \, \|x_i\| \, \|x_j\| + \beta_{ij}(t, x) \|P_i\| \, \|x_i\| \|x_{jd_j}\| \big)
\end{aligned} \tag{22}$$

Applying (21) and (22) to equation (17) yields

$$\begin{aligned}
\dot{V} \leq &-\sum_{i=1}^n x_i^T Q_i x_i + \sum_{i=1}^n \varepsilon_i^a \|x_{id_i}\|^2 + 2 \sum_{i=1}^n \sum_{j=1}^n x_i^T P_i \Phi_{ij}(x) x_j \\
&+2 \sum_{i=1}^n \sum_{j=1}^n \big(\alpha_{ij}(t, x) \|P_i\| \, \|x_i\| \, \|x_j\| + \beta_{ij}(t, x) \|P_i\| \, \|x_i\| \|x_{jd_j}\| \big)
\end{aligned} \tag{23}$$

From the definition of $V(\cdot)$ in (16), it is clear that

$$V(x_{1d_1}, x_{2d_2}, \dots, x_{nd_n}) \le qV(x_1, x_2, \dots, x_n), \qquad (q > 1)$$

implies that

$$q\sum_{i=1}^{n}\lambda_{\max}(P_i)\|x_i\|^2 - \sum_{i=1}^{n}\lambda_{\min}(P_i)\|x_{id_i}\|^2 \ge q\sum_{i=1}^{n}x_i^T P_i x_i - \sum_{i=1}^{n}x_{id_i}^T P_i x_{id_i} \ge 0 \tag{24}$$

Therefore, from (24) and (23), it follows that when $V(x_{1d_1}, \dots, x_{nd_n}) \le qV(x_1, \dots, x_n)$,

$$\begin{aligned}
\dot{V} \le & -\sum_{i=1}^{n}\lambda_{\min}(Q_i)\|x_i\|^2 + \sum_{i=1}^{n}\varepsilon_i^a\|x_{id_i}\|^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n}\|P_i\Phi_{ij}(x)\|\,\|x_i\|\,\|x_j\| \\
& +2\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\alpha_{ij}(t,x)\|P_i\|\,\|x_i\|\,\|x_j\| + \beta_{ij}(t,x)\|P_i\|\,\|x_i\|\|x_{jd_j}\|\right) \\
& +q\sum_{i=1}^{n}\lambda_{\max}(P_i)\|x_i\|^2 - \sum_{i=1}^{n}\lambda_{\min}(P_i)\|x_{id_i}\|^2 \\
\le & -\sum_{i=1}^{n}\Big(\lambda_{\min}(Q_i) - q\lambda_{\max}(P_i)\Big)\|x_i\|^2 - \sum_{i=1}^{n}\Big(\lambda_{\min}(P_i) - \varepsilon_i^a\Big)\|x_{id_i}\|^2 \\
& +2\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\|P_i\Phi_{ij}(x)\| + \alpha_{ij}(t,x)\|P_i\|\Big)\|x_i\|\,\|x_j\| \\
& +2\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}(t,x)\|P_i\|\,\|x_i\|\,\|x_{jd_j}\| \\
= & -\frac{1}{2}Y\left(W^T(\cdot) + W(\cdot)\right)Y^T \\
\le & -\frac{1}{2}\lambda_{\min}\left(W^T(\cdot) + W(\cdot)\right)\left(\|x\|^2 + \|x_d\|^2\right) \\
\le & -\frac{1}{2}\gamma\|x\|^2
\end{aligned}$$

where $Y := \left[\,\|x_1\| \cdots \|x_n\|\ \|x_{1d_1}\| \cdots \|x_{nd_n}\|\,\right]$. Hence, by applying Lemma 1 in the Appendix, the conclusion follows from $\gamma > 0$. ∇

Remark 3. From inequalities (9) in Assumption 1, the bounds on the uncertain interconnections are dependent on the system states, and thus they cannot be employed in the control design since static output feedback is used in this paper. The effects of such interconnections have been reflected through $\alpha_{ij}(t,x)$ and $\beta_{ij}(t,x)$ in the matrix W in Theorem 1. From Lemma 1 in the Appendix, it is straightforward to see that the result in Theorem 1 can be extended to the global case if $\gamma := \inf_x \left\{\lambda_{\min}\big(W^T(\cdot) + W(\cdot)\big)\right\} > 0$ holds globally.

It is well known that one of the main challenges for large scale interconnected systems is to deal with interconnections. It is assumed that the function matrices $\Phi_{ij}(\cdot)$ in the decomposition (3) are only dependant on the i-th system's outputs y_i, that is

$$\Phi_{ij}(x) = \Phi_{ij}(y_i), \qquad i, j = 1, 2, \dots, n$$

In this case, the known interconnections $\Phi_i(x)$ in system (1) are described by

$$\Phi_i(x) = \sum_{i=1}^{n} \Phi_{ij}(y_i)x_j, \qquad i = 1, 2, \ldots, n \tag{25}$$

where $\Phi_{ij}(\cdot) \in \mathbb{R}^{n_i \times n_j}$. It is clear to see that the expressions (25) include linear interconnections as a special case in which the matrices $\Phi_{ij}(\cdot)$ are constant.

In order to reduce the effects of the interconnections, the objective now is to separate the interconnections into matched and mismatched contributions, and then try to reject the effects of the accessible parts by appropriate additive control elements. Denote the l-th column vector of the matrix $\Phi_{ij}(y_i)$ by $\Phi_{ij}^{(l)}(y_i)$ for $l = 1, 2, \ldots, n_j$. For the given input matricies B_i, it is assumed that $\mathrm{Im}(B_i)$ represents the image of the matrix B_i, and $(\mathrm{Im}(B_i))^{\perp}$ denotes the orthogonal complimentary space of $\mathrm{Im}(B_i)$. Using basic matrix theory, decompose the vector $\Phi_{ij}^{(l)}(y_i)$ as

$$\Phi_{ij}^{(l)}(y_i) = (\Phi_{ij}^{(l)}(y_i))^a + (\Phi_{ij}^{(l)}(y_i))^b$$

such that $(\Phi_{ij}^{(l)}(y_i))^a \in \mathrm{Im}(B_i)$ and $(\Phi_{ij}^{(l)}(y_i))^b \in (\mathrm{Im}(B_i))^{\perp}$ for $l = 1, 2, \ldots, n_j$. Let

$$\Phi_{ij}^a(y_i) := \left[(\Phi_{ij}^{(1)}(y_i))^a \;(\Phi_{ij}^{(2)}(y_i))^a \;\cdots\; (\Phi_{ij}^{(n_j)}(y_i))^a\right]$$

$$\Phi_{ij}^b(y_i) := \left[(\Phi_{ij}^{(1)}(y_i))^b \;(\Phi_{ij}^{(2)}(y_i))^b \;\cdots\; (\Phi_{ij}^{(n_j)}(y_i))^b\right]$$

It is straightfoward to see that $\Phi_{ij}(y_i)$ has the following decomposition

$$\Phi_{ij}(y_i) = \Phi_{ij}^a(y_i) + \Phi_{ij}^b(y_i), \qquad i, j = 1, 2, \ldots, n \tag{26}$$

where

$$\Phi_{ij}^a(y_i) = B_i \widetilde{\Phi}_{ij}(y_i), \qquad i, j = 1, 2, \ldots, n \tag{27}$$

for some $\widetilde{\Phi}_{ij}(y_i) \in \mathbb{R}^{m_i \times n_j}$.

Then, consider the following control law

$$u_i = -K_i y_i - \frac{1}{2\varepsilon_i^a} E_i y_i \varpi_i^2(t, y_i) + u_i^a(t, y_i) + u_i^b(t, y_i), \qquad i = 1, 2, \ldots, n \tag{28}$$

where K_i and $u_i^a(\cdot)$ are given in (13), and the additive control element $u_i^b(\cdot)$ is defined by

$$u_i^b(\cdot) = \begin{cases} -\frac{E_i y_i}{\|E_i y_i\|^2} \sum_{j=1}^{n} \left(\frac{1}{2\varepsilon_i^b} \|(E_i y_i)^T \widetilde{\Phi}_{ij}(y_i)\|^2\right), & E_i y_i \neq 0 \\ 0, & E_i y_i = 0 \end{cases} \tag{29}$$

where $\widetilde{\Phi}_{ij}(y_i)$ satisfy (27). It should be noted that the control (28) is generated by adding the term (29) to the control (13).

Corollary 1. Assume that the interconnections of system (1)–(2) can be expressed in (25). Then, under Assumptions 1–3, the closed-loop system formed by applying the control (28) to the system (1)–(2) is uniformly asymptotically stable if $\inf_x \left\{\lambda_{\min}\left(\Gamma^T(\cdot)+\Gamma(\cdot)\right)\right\} > 0$ where the matrix $\Gamma(\cdot) = [\Gamma_{ij}(\cdot)]_{2n\times 2n}$ is defined by

$$\Gamma_{ij}(\cdot) = \begin{cases} \lambda_{\min}(Q_i) - q\lambda_{\max}(P_i) - 2\alpha_{ii}(t,x_i)\|P_i\| - \sum_{j=1}^{n}\varepsilon_j^b, & 1 \le i = j \le n \\ -2\|P_i\Phi_{ij}^b(y_i)\| - 2\alpha_{ij}(t,x)\|P_i\|, & i \ne j \quad \text{and} \quad 1 \le i,j \le n \\ w_{ij}(\cdot), & \text{otherwise} \end{cases}$$

for some constants $\varepsilon_j^b > 0$ and $q > 1$ where the functions $w_{ij}(\cdot)$ are defined in Theorem 1 and the matrices $\Phi_{ij}^b(\cdot)$ are defined in (26).

Proof: From (25), (26) and (27),

$$\begin{aligned} &\sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t,y_i) + \sum_{i=1}^{n} x_i^T P_i \Phi_i(x) \\ &= \sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t,y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i B_i \widetilde{\Phi}_{ij}(y_i)x_j + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i \Phi_{ij}^b(y_i)x_j \end{aligned} \tag{30}$$

Based on the structure of the control in (29), consider the following two cases:

i) if $E_iy_i = 0$, then from (11) and (29),

$$\begin{aligned} &\sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t,y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i B_i \widetilde{\Phi}_{ij}(y_i)x_j \\ &= \sum_{i=1}^{n}(E_iy_i)^T u_i^b(t,y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n}(E_iy_i)^T\widetilde{\Phi}_{ij}(y_i)x_j = 0 \end{aligned}$$

ii) if $E_iy_i \ne 0$, then from (11), the definition of $u_i^b(\cdot)$ in (29) and Young's inequality,

$$\begin{aligned} &\sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t,y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i B_i \widetilde{\Phi}_{ij}(y_i)x_j \\ &= \sum_{i=1}^{n}(E_iy_i)^T u_i^b(t,y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n}(E_iy_i)^T\widetilde{\Phi}_{ij}(y_i)x_j \\ &\le \sum_{i=1}^{n}(E_iy_i)^T u_i^b(t,y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{1}{2\varepsilon_i^b}\|(E_iy_i)^T\widetilde{\Phi}_{ij}(y_i)\|^2 + \frac{\varepsilon_i^b}{2}\|x_j\|^2\right) \\ &= -\sum_{i=1}^{n}(E_iy_i)^T\frac{E_iy_i}{\|E_iy_i\|^2}\left(\sum_{j=1}^{n}\frac{1}{2\varepsilon_i^b}\|(E_iy_i)^T\widetilde{\Phi}_{ij}(y_i)\|^2\right) + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2\varepsilon_i^b}\|(E_iy_i)^T\widetilde{\Phi}_{ij}(y_i)\|^2 \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\varepsilon_i^b}{2}\|x_j\|^2 \\ &= \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{\varepsilon_j^b}{2}\right)\|x_i\|^2 \end{aligned}$$

From the analysis in i) and ii) above, it follows that

$$\sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t, y_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i B_i \widetilde{\Phi}_{ij}(y_i) x_j \le \sum_{i=1}^{n} \Big(\sum_{j=1}^{n} \frac{\varepsilon_j^b}{2} \Big) \|x_i\|^2 \tag{31}$$

By applying (31) to (30),

$$2\sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t, y_i) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i \Phi_i(x) \le 2\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i \Phi_{ij}^b(y_i) x_j + \sum_{i=1}^{n} \Big(\sum_{j=1}^{n} \varepsilon_j^b \Big) \|x_i\|^2 \tag{32}$$

Hence, the conclusion follows by following the proof of Theorem 1. ∇

Remark 4. The proof of Corollary 1 shows that using the decomposition (26), the term $\sum_{i=1}^{n}\sum_{j=1}^{n} x_i^T P_i B_i \widetilde{\Phi}_{ij}(y_i) x_j$ which results from the matched interconnections can be largely rejected by the designed control (29) by choosing the positive parameters ε_j^b small enough, although this approach may result in high gain control. The numerical example in Section 4 will show that the conservatism can be reduced by employing the additive term (29).

4 Illustrative example

In order to illustrate the results obtained, consider an interconnected system described by

$$\dot{x}_1 = \underbrace{\begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 2 & 2 \end{bmatrix}}_{A_1} x_1 + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B_1} (u_1 + \xi_1(t, x_1, x_{1d_1})) + \underbrace{\begin{bmatrix} 0.1x_{12}x_{22} \\ -0.1x_{21} \\ (5x_{21} - 5x_{22})x_{12} \end{bmatrix}}_{F_1(x)} + \psi_1(t, x, x_d) \tag{33}$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} 10 & 15 \\ -30 & 1 \end{bmatrix}}_{A_2} x_2 + \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{B_2} \Big(u_2 + \xi_2(t, x_2, x_{2d_2})\Big) + \underbrace{\begin{bmatrix} 0.1x_{11} + 2x_{12} - 6x_{13} \\ (x_{22} - x_{21})(-2x_{12} + 6x_{13}) \end{bmatrix}}_{F_2(x)}$$
$$+\psi_2(t, x, x_d) \tag{34}$$

$$y_1 = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{C_1} x_1, \qquad y_2 = \underbrace{\begin{bmatrix} -1 & 1 \end{bmatrix}}_{C_2} x_2, \tag{35}$$

where $x_1 := \mathrm{col}(x_{11}, x_{12}, x_{13}) \in \mathbb{R}^3$ and $x_2 := \mathrm{col}(x_{21}, x_{22}) \in \mathbb{R}^2$ are states, $y_1 = \mathrm{col}(y_{11}, y_{12}) \in \mathbb{R}^2$ and $y_2 \in \mathbb{R}^1$ are outputs, and $u_1, u_2 \in \mathbb{R}^1$ are inputs. The uncertainties $\xi_i(\cdot)$ and the uncertain interconnections $\psi_i(\cdot)$ for $i = 1, 2$ satisfy

$$\|\xi_1(t, x_1, x_{1d_1})\| \leq \underbrace{(2 + y_{11})^2 \sin^4(y_{12}t)}_{\rho_1(t,y_1)} + \underbrace{|y_{12}y_{11} \sin t|}_{\varpi_1(t,y_1)} \|x_{1d_1}\|$$

$$\|\xi_2(t, x_2, x_{2d_2})\| \leq \underbrace{3|y_2| \exp\{-t\}}_{\rho_2(t,y_2)} + \underbrace{y_2^2 |\sin t|}_{\varpi_2(t,y_2)} \|x_{1d_2}\|$$

$$\|\psi_1(t, x, x_d)\| \leq \underbrace{\frac{1}{3}|x_{11} \cos x_{22}|}_{\alpha_{12}(t,x)} \|x_2\| + \underbrace{\frac{1}{4}|x_{12}| \sin^2 t}_{\beta_{12}(t,x)} \|x_{2d_2}\|, \qquad \psi_2(t, x, x_d) = 0$$

where the bounds on $\psi_1(\cdot)$ imply that $\alpha_{11}(\cdot) = \beta_{11}(\cdot) = 0$ and the fact that $\psi_2(\cdot) = 0$ shows that $\alpha_{21}(\cdot) = \alpha_{22}(\cdot) = \beta_{21}(\cdot) = \beta_{22}(\cdot) = 0$. The interconnections $F_1(\cdot)$ and $F_2(\cdot)$ can be expressed in (25) as follows

$$F_1(\cdot) = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\Phi_{11}(y_1)} x_1 + \underbrace{\begin{bmatrix} 0 & 0.1 \\ -0.1 & 0 \\ 5y_{11} & -5y_{11} \end{bmatrix}}_{\Phi_{12}(y_1)} x_2$$

$$F_2(\cdot) = \underbrace{\begin{bmatrix} 0.1 & 2 & -6 \\ 0 & -2y_2 & 6y_2 \end{bmatrix}}_{\Phi_{21}(y_2)} x_1 + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Phi_{22}(y_2)} x_2$$

It is straightforward to see that the decompositions (26) and (27) hold with

$$\tilde{\Phi}_{11}(y_1) = 0, \quad \Phi_{11}^b(y_1) = 0, \quad \Phi_{12}(y_1) = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B_1} \underbrace{\begin{bmatrix} 2.5y_{11} & -2.5y_{11} \end{bmatrix}}_{\tilde{\Phi}_{12}(y_1)} + \underbrace{\begin{bmatrix} 0 & 0.1 \\ -0.1 & 0 \\ 0 & 0 \end{bmatrix}}_{\Phi_{12}^b(y_1)}$$

$$\Phi_{21}(y_2) = \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{B_2} \underbrace{\begin{bmatrix} 0 & 2y_2 & -6y_2 \end{bmatrix}}_{\tilde{\Phi}_{21}(y_2)} + \underbrace{\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Phi_{21}^b(y_2)}, \quad \tilde{\Phi}_{22}(y_2) = 0, \quad \Phi_{22}^b(y_2) = 0$$

Let $K_1 = [1 \quad 3]$, $K_2 = -8$, $Q_1 = 8I_3$ and $Q_2 = I_2$. Then the solutions to the equations (10) and (11) are

$$P_1 = I_3, \qquad P_2 = \begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1.25 \end{bmatrix}, \qquad E_1 = [0 \quad 2], \qquad E_2 = -1$$

Let $\varepsilon_i = \varepsilon_i^a = \varepsilon_i^b = 0.1$ for $i = 1, 2$ and $q = 1.01$. Based on the parameters above, the control (29) is well defined. By direct computation,

$$\Gamma = \begin{bmatrix} 6.7900 & -0.2 - \frac{2}{3}|x_{11} \cos x_{22}| & 0 & -0.6667 - \frac{1}{2}|x_{12}| \sin^2 t \\ -0.2550 & 4.2850 & 0 & 0 \\ 0 & -0.6667 - \frac{1}{2}|x_{12}| \sin^2 t & 0.9000 & 0 \\ 0 & 0 & 0 & 0.9000 \end{bmatrix}$$

which is positive definite in the domain

$$\Omega = \{(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}) \mid |x_{11}| \leq 10.5, \, |x_{12}| \leq 3.1, \, x_{13}, x_{21}, x_{22} \in \mathbb{R}\}$$

Hence from Corollary 1, the system (33)–(35) is stabilised by the control (13). Simulation results presented in Figures 1 and 2 show the results obtained are effective.

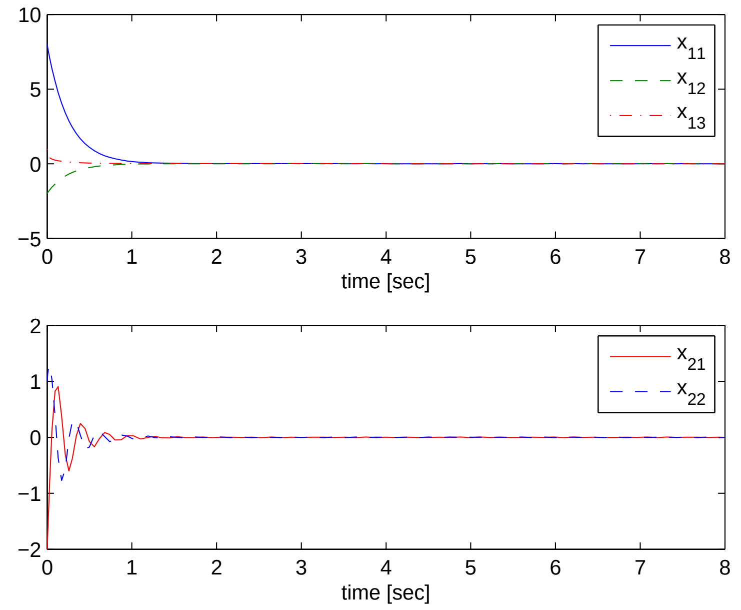

Figure 1. The time response of the state variables of system (33)–(35)

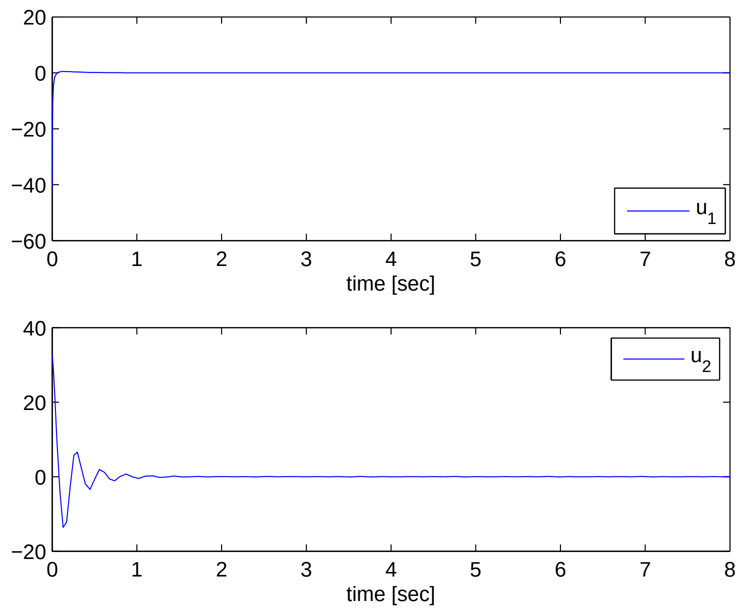

Figure 2. The time response of the control signals

Remark 5. Consider comparison of the matrix W in Theorem 1 and the matrix Γ in Corollary 1.

By direct computation it follows that

$$W = \begin{bmatrix} 6.9900 & -14.1435 - \frac{2}{3}|x_{11}\cos x_{22}| & 0 & -0.6667 - \frac{1}{2}|x_{12}|\sin^2 t \\ -17.8891 & 4.4850 & 0 & 0 \\ 0 & -0.6667 - \frac{1}{2}|x_{12}|\sin^2 t & 0.9000 & 0 \\ 0 & 0 & 0 & 0.9000 \end{bmatrix}$$

It is straightforward to check that $W + W^T$ is not positive definite even if $x_{11} = 0$ and $x_{12} = 0$, and thus Theorem 1 cannot be applied to the system (33)–(35). This confirms the result stated in Remark 4.

5 Conclusions

This paper has presented control strategies for a class of interconnected systems with time varying delays. It is not required that the subsystems are square. The proposed controllers are decentralised, independent of the time delay and based only on output information, which is convenient for real implementation. The limitation on the rate of change of the time varying delay is not required, as is required using the Lyapunov-Krasovskii approach. This paper has presented an approach to deal with cases where there are nonlinear time delay bounds on the mismatched interconnections when memoryless control is required.

6 Appendix

Consider the time-delay system

$$\dot{x}(t) = f(t, x(t - d(t))) \tag{36}$$

with initial condition

$$x(t) = \phi(t), \qquad t \in [-\overline{d}, 0]$$

where $f : \mathbb{R}^+ \times \mathcal{C}_{[-\overline{d},0]} \mapsto \mathbb{R}^n$ takes $\mathbb{R}\times$ (bounded sets of $\mathcal{C}_{[-\overline{d},0]}$) into bounded sets in $\mathbb{R}^n$; $d(t)$ is the time-varying delay and $\overline{d} := \sup_{t\in\mathbb{R}^+}\{d(t)\} < \infty$.

Lemma 1. Consider the system (36). If there exists a function $V_0(x) = x^T P x$ with $P > 0$ such that for $d \in [-\overline{d}, 0]$, the time derivative of V_0 along the solution of system (36) satisfies

$$\dot{V}_0(x) \le -q_1\|x\|^2 \quad \text{if} \quad V_0(x(t+d)) \le q_2 V_0(x(t)) \tag{37}$$

for some $q_1 > 0$ and $q_2 > 1$, then system (36) is uniformly asymptotically stable. Further, if (37) holds in $\mathbb{R}^n$, then system (36) is globally uniformly asymptotically stable.

Proof: By using the well-known Razumikhin Theorem (see, eg. Gu et al. (2003)), it is straightforward to see that the result follows by directly extending Lemma 5 of Appendix A in Yan et al. (2010) to the global case. ∇

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