Warranty return policies for products with unknown claim causes and their optimisation

Shaomin Wu[1]

Kent Business School, University of Kent, Canterbury CT2 7PE, UK

Abstract

In practical warranty services management, faults may not always be found in claimed items by warranty service agents, which is the well-known no-fault-found phenomenon (for example, caused by a loose connection between parts, or simply human error). This phenomenon can contribute more than 40% of reported service faults in electronic products and it can be due to faults of manufacturers or product users. Little research, however, considers this phenomenon in warranty management since faults are normally assumed to be found in the claimed items. On the basis of different levels of testing, this paper proposes three warranty return policies, which decide whether new items should be sent to warranty claimants or not. It then derives and compares the expected costs of the policies, and obtains the optimal warranty periods under supply chain environments. The paper illustrates the results with artificially generated data.

Keywords: supply chain, optimisation, game theory, cost benefit analysis, warranty management

1. Introduction

Product warranty is a contractual obligation incurred by a manufacturer (or retailer) in connection with the sale of a product. It has become increasingly more important in consumer and commercial transactions and is widely used to serve many different purposes (Karim and Suzuki, 2005; Wu, 2012; Wu, 2013). The US Congress has enacted several warranty acts (UCC, Magnusson Moss Warranty Act, Tread Act, etc.) over the last

[1] Email: s.m.wu@kent.ac.uk. Tel: +44(0) 1227 827940.
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100 years. The European Union (EU) passed legislation requiring a two-year warranty for all products sold in Europe (Murthy and Djamaludin, 2002).

Warranty expense is one of the operating expenses for manufacturers. A product might be sold with a warranty agreement and the manufacturer needs to cover labour and parts needed for repairs or replacement within the warranty period. As a consequence, warranty incurs tremendous cost in the manufacturing industries. For example, the automotive industry spends roughly $10–$13 billion per year in the U.S. on warranty claims and up to $40 billion globally (MSX International Inc, 2010).

Although warranty only covers items that have failed, it has been noted that faults may not always be found in claimed items, which is also referred to as *no-fault-found* (NFF) (Prakash et al., 2009; Wu, 2011; Huang et al., 2011). Brombacher (1999) showed that the observed categories of reliability problems were distributed as: components 21%; customers 17%, apparatus 24% and no fault found 38%. On these statistics, the author further interpreted that *the reliability failures in products were split into problems on a component level, problems on "internal product level" (e.g. interaction problems) and problems on a customer/application level. This analysis showed the largest single group where the cause of the failure remained unknown.* The *no-fault–found* (NFF) phenomenon is a big problem when dealing with multipart products. For example, the NFF contributes on average to 45% of reported service faults in electronic products (Jones and Hayes, 2001), and the problem of NFFs in aircraft electronic equipment has long plagued operators (Ramsey, 2005). The problem is not new, but many believe it is getting worse, in part because today's highly complex products are equipped with more and more electronic sensors, computers, control functions and wires (Ramsey, 2005).

Our literature review shows, however, that the following assumption has been imposed with no explanation in most of the existing research on warranty management:

Fault can always be found in claimed items by warranty service agents. That is, all claimed items are failed ones.

Following the above assumption, research in the literature normally takes one of the following two assumptions: (1) for repairable products, claimed items are returned to the claimants after repair; or (2) for non-repairable products, new items are returned to the claimants. Such assumptions may simplify the calculation process. However, as mentioned above, in practice, fault might not always be found in claimed items, for which two methods can therefore be used to handle warranty claims. (1) A new item is returned to a claimant if fault is found in her claimed item, and (2) the original claimed item (without any maintenance conducted on it) is returned to the claimant if no fault is found in her claimed item. This will of course raise another question, which is the ability to diagnose the real fault in the claimed items.

A couple of authors have conducted cost-benefit analysis for product returns with the NFF phenomenon (see, Prakash et al., 2009; Wu, 2011; Huang et al., 2011, for example). Prakash et al. (2009) presented a manufacturing process adjustment to eliminate warranty related NFF product failures in the field when all key product characteristics measured are within design tolerances. Huang et al. (2011) suggested using a coordination mechanism to resolve the profit conflict in a reverse supply chain in the presence of false failure returns. Wu (2011) derived the expected warranty costs for repairable products when the NFF phenomenon is considered and found that the expected claim cost per individual product incurred by NFF is sensitive to the total number of products sold.

It's widely accepted that reducing NFF has the potential for dramatic cost savings across the industry, particularly in terms of additional spares, logistics, workshop time, test equipment and training (Burchell, 2007).

NFF is also referred to as intermittent failures, which is the loss of some functions or performance characteristics of a product for a limited period of time until subsequent recovery of the function. Users may experience a failure and restart the item (for example, computers) and it runs OK. When the item is taken to a service agent, the repairman might not experience this failure when the item is being inspected. As a consequence, the warranty service agent may develop different product return policies: they may either return the claimed item to the claimant, or may send a new item to her. Different return policies can apparently incur different cost. For example, misdiagnosing a failed item to be non-failed and then returning it to the claimant can cause losses directly relating to the manufacturer. Such losses can be: cost of repairing or replacing, cost of customer dissatisfaction, loss of customer good will, and loss of market share, for example. However, misdiagnosing a non-failed item to be failed and sending a new item to the claimant may only incur the cost of the new product. Analysing such return policies is therefore crucially important for service suppliers. This motivates the authors to write this paper, which analyses and further derives the expected costs of three return policies.

Under different return policies, the following interesting questions can emerge:

(a) What is the expected cost of each return policy?

(b) Which return policy should be adopted under a given cost setting?

(c) What are the optimal warranty periods under a supply chain environment?

This paper answers the above three questions. It proposes three product return polices, derives their expected cost, and optimises warranty periods under two supply chain environments. As little research on those issues exists in the literature, the paper develops novelty.

The rest of this paper is structured as follows. Section 2 includes assumptions and notation. Section 3 derives the expected costs of three return policies. Section 4 compares the costs derived from Section 3 and derives optimal warranty periods for base warranty and extended warranty, considering supply chain environments. Section 5 offers discussion on estimation of the parameters assumed in the paper. Section 6 gives numerical examples, and Section 7 concludes the paper.

2. Settings and notation

Suppose that the following general assumptions hold.

- **Causes of claims**. A claim can be reported to the warranty provider (manufacturer/retailer) due to one of the following three causes: known faults, unknown faults, and human error. To avoid ambiguity in writing, we refer to the claims due to known faults, unknown faults, and human error as claim causes 1, 2 and 3, respectively. That is, claim cause 1 is due to known faults, with which an item is not repaired and a new item should be sent to the claimant. Claim cause 2 is due to unknown faults that are caused by the manufacturing side, but it may not be detected. Human error, ie., human error, can also cause a claim and it can be an intended or an unintended human error, and it is caused by the product users. Either claim cause 2 or claim cause 3 might be diagnosed correctly or incorrectly: the real cause is revealed if diagnosed correctly, and they are classified as NFF if diagnosed incorrectly. That is, NFF can be due to claim cause 2 or claim cause 3.
- **Testing techniques**. There are two types of testing techniques available.
  (a) Type I testing T_1: it is an initial testing and aims to identify claim cause 1. This type can only identify known faults, or claim cause 1, and it cannot detect claim causes 2 or 3.
  (b) Type II testing T_2: which is a more sophisticated testing than Type I testing and it aims to take a further diagnosis on those items in which no fault has been found

with Type I testing. The probability that claim causes 2 and 3 can be detected and confirmed with Type II testing is ρ ($0 \leq \rho \leq 1$).

- **Return policies**. Once a claimed item is received, one of the following three return policies is applied.
  (a) Return Policy 1. Once a claimed item is received, a new and identical item will be sent to the claimant.
  (b) Return Policy 2. Once a claimed item is received, it will be tested with Type I testing.
    - if claim cause 1 is confirmed in the claimed item, a new item will be sent to the claimant,
    - if no fault is confirmed in the claimed item, the original claimed item will be returned to the claimant.
  (c) Return Policy 3. Once a claimed item is received, it will be tested with Type I testing. Then
    - if claim cause 1 is confirmed in the claimed item, a new item will be sent to the claimant;
    - if no fault can be confirmed in the claimed item, the claimed item will be tested with Type II testing. If claim cause 2 can be confirmed with Type II testing, then a new and identical item is be sent to the claimant. Otherwise, the claimed item is returned to the claimant.
- **Independence**. The occurrences of the three claim causes are statistically independent. Each failure mechanism leading to a particular type of failure (i.e., failure cause) proceeds independently of every other one, at least until a failure occurs.

- **Maintenance.** No maintenance, neither corrective maintenance nor preventive maintenance, is conducted on the product. If no fault is found in Return Policy 2 or Return Policy 3, the claimed item is returned to the claimant and the hazard rate function of the item is not altered.
- **Warranty policy**. Only non-renewing warranty policy is considered, that is, under this policy, the manufacturer/retailer offers a satisfactory service only within the original warranty period, and an item with a confirmed failure is replaced by the manufacturer at no cost to the buyer or at a pre-specified cost to the buyer within the original warranty period, and the original warranty is not renewable.
- **Warranty processing time**. Assume that time on processing a claimed item is negligible.

In this paper, we use the following notation.

Notation

$F_i(t)$	Cumulative distribution function (cdf) of time to failure due to claim cause *i*, where *i*=1,2,3.
$f_i(t)$	$f_i(t) = dF_i(t)/dt$ with *i*=1,2,3.
$\lambda_i(u)$	Failure intensity function corresponding to $F_i(t)$, *i*=2,3.
$\Lambda_i(t)$	$\Lambda_i(t) = \int_0^t \lambda_i(u)du$, *i*=2,3.
$m_i(t)$	Renewal function corresponding to the cdf $F_i(t)$, where *i*=1,2,3.
c_{32}	Expected cost of diagnosing claim cause 3 to claim cause 2
c_{23}	Expected cost of diagnosing claim cause 2 to claim cause 3
c_a	Expected administration cost per claim
c_n	Cost of returning a new item
c_{t1}	Expected cost of Type I testing per item
c_{t2}	Expected cost of Type II testing per item
ρ	Probability of correctly diagnosing claim causes 2 and 3
$C_k(t)$	Expected cost of return policy k per an item, within time interval (0,*t*), where *k*=1,2,3
w	Length of a warranty period

3. Expected costs of return policies

All of the three Return Policies can correctly detect claim cause 1, which results in returning new items.

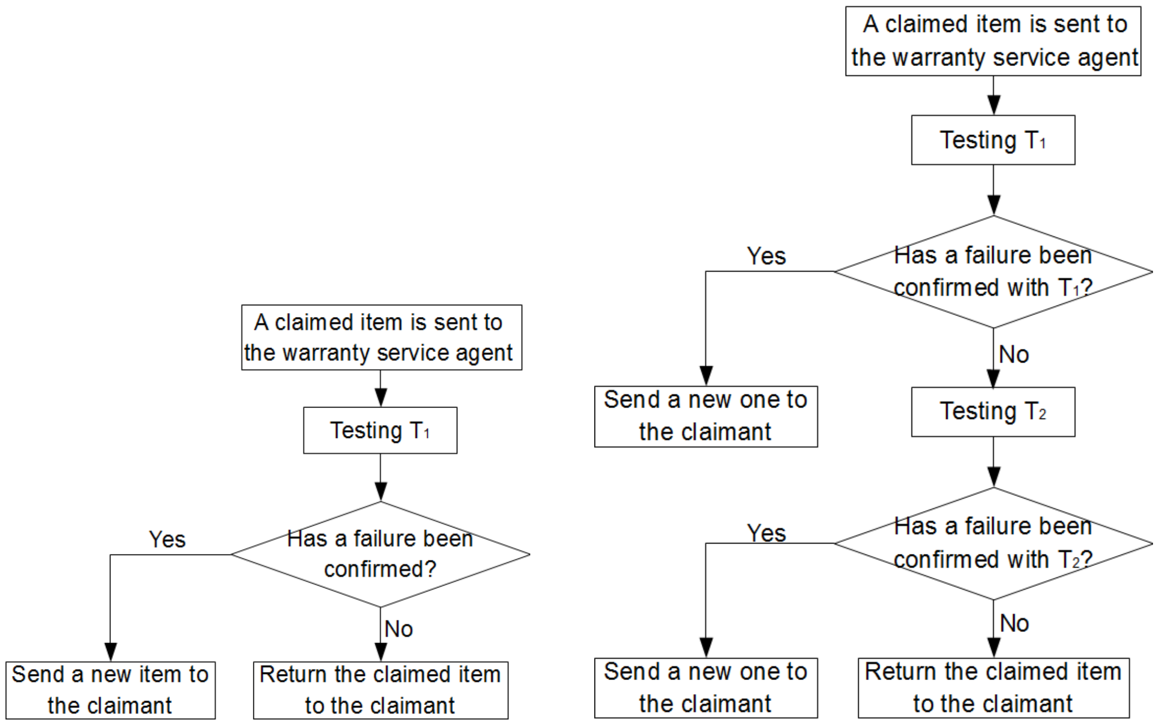
However, items with claim causes 2 or 3 may be misdiagnosed. As a result, items with claim cause 2 may be returned to the claimants, although new items should be sent to claimants. A new item may be sent to the claimant although her claim was reported due to claim cause 3.

From the assumptions in the preceding section, the cost distribution of diagnosing claimed items can be illustrated in Table 1. In Table 1, for example, the values in the cell in the 2nd column and the 2nd row means that the cost of implementing Return Policy 1 when the claim cause 2 is correctly identified is $c_n + c_a$, and the cost of implementing Return Policies 2 and 3 when the claim cause 2 is correctly identified is $c_n + c_a + c_{t1}$ and $c_n + c_a + c_{t1} + c_{t2}$, respectively. The values in the cell in the 2nd column and the 3nd row means that the cost of implementing Return Policy 1 when the claim cause 3 is incorrectly identified to be claim cause 2 is $c_n + c_a + c_{32}$, but Return Policies 2 and 3 do not mistakenly diagnose claim cause 3 to claim cause 2 and therefore does not incur any costs.

Table 1. Cost distribution

Actual / Diagnosed	Claim cause 2 (Actual)	Claim cause 3 (Actual)
Claim cause 2 (Diagnosed)	Return Policy 1: $c_n + c_a$ Return Policy 2: $c_n + c_a + c_{t1}$ Return Policy 3: $c_n + c_a + c_{t1} + c_{t2}$	Return Policy 1: $c_n + c_a + c_{32}$ Return Policy 2: not applicable Return Policy 3: not applicable
Claim cause 3 (Diagnosed)	Return Policy 1: not applicable Return Policy 2: $c_a + c_{t1} + c_{23}$ Return Policy 3: $c_a + c_{t1} + c_{t2} + c_{23}$	Return Policy 1: not applicable Return Policy 2: $c_a + c_{t1}$ Return Policy 3: $c_a + c_{t1} + c_{t2}$

Return Policy 1 is quite simply. Return Policy 2 and Return Policy 3 are also illustrated in Figure 1 (a) and Figure 1 (b), respectively.



(a). Return Policy 2 (b). Return Policy 3

Figure 1. Warranty claim handling procedure in Return Policy 2 and Return Policy 3

This following derives the expected cost of each return policy.

3.1. Expected Costs of the Three Return Policies

3.1.1 Expected Cost of Return Policy 1

Under Return Policy 1, new items are sent to warranty claimants regardless of the causes of the claims. A potential loss incurred with this Policy is to send new items to those claimants whose claims are due to claim cause 3, although the original claimed items should be returned to the claimants. We therefore have the following proposition.

Proposition 1. The expected cost of Return Policy 1 is given by

$$C_1(w) = (c_n + c_a)\, m_{123}(w) + \; c_{32}(1 - q_{X_{12}<X_3})\, m_{123}(w) \tag{1}$$

where $m_{123}(w)\big(= H_{123}(w) + \int_0^w m_{123}(w-t)dH_{123}(t)\big)$ that is the expected number of renewals within time interval (0,*w*), $H_{123}(t)\left(= 1 - \big(1 - F_1(t)\big)\big(1 - F_2(t)\big)\big(1 - F_3(t)\big)\right)$ that is the probability distribution of time to receive a claim due to one of the three claim causes, $q_{X_{12}<X_3}\left(=\int_0^w H_{12}(t)dF_3(t)\right)$ that is the probability of the occurrence of claim causes 1 and 2, and $H_{12}(t)\left(=1-\left(1-F_1(t)\right)\left(1-F_2(t)\right)\right)$ that is the probability distribution of time to receive a claim due to either of the claim causes 2 and 3.

Proof. Under Return Policy 1, claims due to one of the three claim causes result in renewals, hence, the three causes are three competing risks. As such, the probability distribution of time-to-renewal is $H_{123}(t)$. The expected number of warranty claims during period $(0,w)$ is $m_{123}(w)$, or the renewal function corresponding to the cumulative distribution function $H_{123}(t)$. $c_n + c_a$ is the sum of cost of sending a new item and administration cost per item. Hence, the total returns incurred due to returning new items upon any claim causes is $(c_n + c_a)\, m_{123}(w)$.

Under Return Policy 1, denote time to return a new item upon claim due to cause 3 by X_3 and time to return a new item upon claim due to causes 1 or 2 by X_{12}.

Apparently, $m_{123}(w)$ can be re-written as

$$m_{123}(w) = m_{123}(w)\Pr(X_{12} < X_3) + m_{123}(w)(1 - \Pr(X_{12} < X_3))$$

$$= m_{123}(w)\, q_{X_{12}<X_3} + m_{123}(w)(1 - q_{X_{12}<X_3}),$$

where $q_{X_{12}<X_3} = \Pr(X_{12} < X_3) = \int_0^\infty H_{12}(t)dF_3(t)$.

In the above equation, $m_{123}(w)(1 - q_{X_{12}<X_3})$ is the number of warranty claims due to claim cause 3, which incurs cost $c_{32}(1 - q_{X_{12}<X_3})m_{123}(w)$ of incorrectly classifying claim cause 3 to claim cause 2.

Hence, the total cost incurred in Return Policy 1 is $(c_n + c_a)\, m_{123}(w) + c_{32}(1 - q_{X_{12}<X_3})\, m_{123}(w)$. This completes the proof. ∎

The expected cost $C_1(w)$ of Return Policy 1 is the cost of returning new items upon claims due to any of the three claim causes. As claim cause 3 is the human error that is caused by the product users and that the warranty provider should not be responsible for, any additional cost relating to claim cause 3 should be considered. As such, $C_1(w)$ includes two elements: (1) cost of returning items due to all the claim causes, and (2) cost of wrongly sending a new item to the customer, resulting from misclassifying claim cause 3 to claim causes 1 or 2.

3.1.2 Expected Cost of Return Policy 2

Under Return Policy 2, Type I testing is carried out to detect known faults. New items are sent to the claimants whose claim causes are confirmed known faults. Otherwise, the original claimed items are returned to the claimants.

Proposition 2. The expected cost of Return Policy 2 is given by

$$C_2(w) = (c_n + c_a + c_{t1})m_1(w) + (c_a + c_{t1} + c_{23})m_1(w)\int_0^\infty \Lambda_2(t)dF_1(t)$$

$$(c_a + c_{t1})m_1(w)\int_0^\infty \Lambda_3(t)dF_1(t). \qquad (2)$$

Proof.

- Under Return Policy 2, the causes of any claimed items are diagnosed with Type I testing. New items will be sent to warranty claimants if claim cause 1 is confirmed, which incurs cost $(c_n + c_a + c_{t1})m_1(w)$, where $m_1(w)$ is the renewal function corresponding to the cumulative distribution function $F_1(t)$.
- If the causes of warranty claims are not detected or confirmed, the original claimed items will be returned. This essentially forms a renewal-reward process: claimed items due to claim cause 1 are renewed and the process is a renewal process, and within each inter-arrival period, the number of claimed items whose causes are not confirmed can be seen as a reward function depending on the length of the inter-arrival time. Since the occurrences of claim cause 1 and claim cause 2 are assumed to be statistically independent, according to Gallager (1995), the total expected number

of warranty claims due to claim cause 2 is $m_1(w)\int_0^\infty \Lambda_2(t)dF_1(t)$. Hence, the cost on returns, including administration cost and cost of Type I testing, due to claim cause 2 is given by $(c_a + c_{t1})m_1(w)\int_0^\infty \Lambda_2(t)dF_1(t)$.

- Claimed items may be due to cause 2, under which new items should be sent but the original claimed items are incorrectly returned to the claimants. Returning such items can cause potential or latent problems such as damaging manufacturer's reputation, and therefore incur cost $c_{23}m_1(w)\int_0^\infty \Lambda_2(t)dF_1(t)$.
- The original claimed items due to cause 3 are correctly returned to the claimants. Returning such products can incur cost $(c_a + c_{t1})m_1(w)\int_0^\infty \Lambda_3(t)dF_1(t)$, which includes administration cost and cost of Type I testing.

This completes the proof. ■

3.1.3 Expected Cost of Return Policy 3

Under Return Policy 3, a further testing, Type II testing, is conducted on those claims whose causes have not been identified with Type I testing.

Denote $F_T(t) = 1 - (1 - F_1(t))e^{-\rho\int_0^t \lambda_2(u)du}$, $m_T(t) = F_T(t) + \int_0^t m_T(t-u)dF_T(u)$, $F_{T2}(t) = 1 - e^{-\rho\int_0^t \lambda_2(u)du}$, and $q_{X_1<X_{T2}} = \Pr(X_1 < X_{T2}) = \int_0^\infty F_1(y)dF_{T2}(y)$. Then we have the following proposition.

Proposition 3. The expected warranty cost of Return Policy 3 is given by

$$C_3(w) = (c_n + c_a + c_{t1})m_T(w) + \left(1 - q_{X_1<X_{T2}}\right)c_{t2}m_T(w)$$
$$+ (c_a + c_{t1} + c_{t2} + c_{23})m_T(w)\int_0^\infty (1-\rho)\Lambda_2(t)dF_T(t) + (c_a + c_{t1}$$
$$+ c_{t2})m_T(w)\int_0^\infty (1-\rho)\Lambda_3(t)dF_T(t). \qquad (3)$$

Proof.

- An item is put in operation at time 0. If warranty on this item is claimed, the cause of this claim is checked with Type I testing. If either claim cause 1 or claim cause 2 is confirmed, then a new item will be returned to the customer. Otherwise, the original claimed item will be returned. Claim cause 1 can be detected and identified by Type I testing, whereas claim cause 2 can be correctly detected and identified with a probability ρ. That is, claim cause 2 may not be detected with a probability of $1-\rho$. If only the returns due to claim cause 2 is considered, according to (Block et al., 1985), the successive times on returning new items forms a renewal process with an inter-arrival distribution $1-e^{-\rho\int_0^t \lambda_2(u)du}$. Hence, if both claim causes 1 and 2 are considered, the successive times on returning new items forms a renewal process with an inter-arrival distribution $F_T(t)$ $\left(\text{ie.}, 1-(1-F_1(t))e^{-\rho\int_0^t \lambda_2(u)du}\right)$. The number of new items returned to the customers is $m_T(w)$. Hence, the cost is $(c_n+c_a+c_{t1})m_T(w)$.
- On the other hand, those items whose claim causes are not identified are returned to the customers. They may be diagnosed correctly (reveal the real claim cause correctly) or incorrectly (diagnosed claim causes 2 to claim cause 3, or claim cause 3 to claim cause 2). Among those items,

  (a) the number of items with claim cause 2, which are diagnosed correctly, is $\left(1-q_{X_1<X_{T2}}\right)m_T(w)$ and they incur cost $\left(1-q_{X_1<X_{T2}}\right)c_{t2}m_T(w)$ on Type II testing (the cost due to Type I testing on those items has already been included in the first term in Eq (3)),

  (b) the number of items with claim cause 2, which are incorrectly diagnosed as claim cause 3, is $m_T(w)\int_0^\infty(1-\rho)\Lambda_2(t)dF_1(t)$, which incurs a total cost of $(c_a+c_{t1}+c_{t2}+c_{23})m_T(w)\int_0^\infty(1-\rho)\Lambda_2(t)dF_1(t)$,

(c) the number of items with claim cause 3, which are correctly diagnosed as claim cause 3, is $m_T(w)\int_0^\infty (1-\rho)\Lambda_3(t)dF_1(t)$, which incurs a total cost of $(c_a + c_{t1} + c_{t2})m_T(w)\int_0^\infty (1-\rho)\Lambda_3(t)dF_1(t)$.

To sum up the different costs, one can obtain $C_{r3}(w,T)$, as shown in Eq. (3). ■

Remarks. In Eq. (3),

- $\rho = 0$ implies that the probability of correctly diagnosing claim causes 2 and 3 is 0 and there is therefore no need to conduct Type II testing,
- $\rho = 1$ implies that that each of claim causes 2 and 3 can be correctly diagnosed and new items are sent to the claimants who deserve the treatment, and
- if $\rho = 0$ and $c_{t2} = 0$, then $C_2(w) = C_3(w)$. Due to the following reason, both $\rho = 0$ and $c_{t2} = 0$ should hold to ensure that the expected costs of Policy 2 and Policy 3 are equal.
  (a) In the case when $\rho = 0$ and $c_{t2} \neq 0$, time on Type II testing still incurs cost although the probability of correctly diagnosing claim causes 2 and 3 is 0.
  (b) In the case when $c_{t2} = 0$ and $\rho \neq 0$, correctly diagnosing claim causes 2 and 3 is possible. Consequently, some items are handled correctly (ie., correctly returning new items or old items), which impacts cost.

3.2. Comparison of the expected costs on special cases

The preceding section derived the expected costs of the three return policies. Implementing Return Policy 1 is quite simply and straightforward, but it may incur the largest losses if new items are expensive. Implementing Return Policy 2 requires Type I testing and it can potentially damage the reputation of both the manufacturer and the retailer due to the fact that the original claimed items with claim causes 2 may be returned. Implementing Return Policy 3 is the most complicated but it can potentially benefit the manufacturer and/or the retailer as it maximises the chance to correctly respond the warranty claimants. An interesting question is to compare these costs and optimise the warranty periods, which are investigated below.

Denote

- $\theta_1 = \left(c_n + c_a + \left(\frac{\lambda_3}{\lambda_1+\lambda_3} + \frac{\lambda_3}{\lambda_2+\lambda_3} - \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)c_{32}\right)(\lambda_1 + \lambda_2 + \lambda_3)$,
- $\theta_2 = (c_n + c_a + c_{t1})\lambda_1 + (c_a + c_{t1} + c_{23})\lambda_2 + (c_a + c_{t1})\lambda_3$,

and

- $\theta_3 = (c_a + c_{t1})(\lambda_1 + \lambda_2) + c_n(\lambda_1 + \rho\lambda_2) + c_{t2}\lambda_2 + (1-\rho)c_{23}\lambda_2 + (1-\rho)(c_a + c_{t1} +$
  $+c_{t2})\lambda_3$.

The following Lemma can be derived from Propositions 1, 2, and 3.

Lemma 1. Assume $F_i(t) = 1 - e^{-\lambda_i t}$ (i=1,2,3). The expected costs of Return Policy k is given by

$$C_k(w) = \theta_k w, \tag{4}$$

where k=1,2,3.

Proof. Since $F_i(t) = 1 - e^{-\lambda_i t}$ (i=1,2,3), we have $H_{123}(w) = 1 - e^{-(\lambda_1+\lambda_2+\lambda_3)w}$, and $m_{123}(w) = (\lambda_1 + \lambda_2 + \lambda_3)w$.

Hence,

$$C_1(w) = (c_n + c_a)\, m_{123}(w) + c_{nf}\left(1 - q_{X_{12}<X_3}\right) m_{123}(w)$$

$$= \left(c_n + c_a + c_{nf}\left(1 - q_{X_{12}<X_3}\right)\right)(\lambda_1 + \lambda_2 + \lambda_3)w$$

Since

$$q_{X_{12}<X_3} = \int_0^\infty H_{12}(u)dF_3(u) = (1 - \frac{\lambda_3}{\lambda_1+\lambda_3} - \frac{\lambda_3}{\lambda_2+\lambda_3} + \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3})$$

Hence

$$C_1(w) = \theta_1 w.$$

Since $m_1(w) = \lambda_1 w$, $\Lambda_2(t) = \lambda_2 t$, $\Lambda_3(t) = \lambda_3 t$, $\int_0^\infty \Lambda_2(t) dF_1(t) = \frac{\lambda_2}{\lambda_1}$, and $\int_0^\infty \Lambda_3(t) dF_1(t) = \frac{\lambda_3}{\lambda_1}$, from Eq. (2), we have

$$C_2(w) = (c_n + c_a + c_{t1}) m_1(w) + (c_a + c_{t1} + c_{23}) m_1(w) \int_0^\infty \Lambda_2(t) dF_1(t)$$

$$+ (c_a + c_{t1}) m_1(w) \int_0^\infty \Lambda_3(t) dF_1(t)$$

$$= \theta_2 w.$$

Since $m_T(w) = \lambda_1 + \rho\lambda_2$, $\Lambda_2(t) = \lambda_2 t$, $\int_0^\infty \Lambda_2(t) dF_T(t) = \frac{\lambda_2}{\lambda_1 + \rho\lambda_2}$, $\int_0^\infty \Lambda_3(t) dF_T(t) = \frac{\lambda_3}{\lambda_1 + \rho\lambda_2}$, and

$$q_{X_1 < X_{T2}} = \Pr(X_1 < X_{T2}) = \int_0^\infty F_1(y) dF_{T2}(y) = 1 - \frac{\rho\lambda_2}{\lambda_1 + \rho\lambda_2}.$$

From Eq. (3), we have

$$C_3(w) = (c_n + c_a + c_{t1})(\lambda_1 + \rho\lambda_2) + \frac{\rho\lambda_2}{\lambda_1 + \rho\lambda_2} c_{t2}(\lambda_1 + \rho\lambda_2) + (c_a + c_{t1} + c_{t2} + c_{23})(\lambda_1$$

$$+ \rho\lambda_2) \frac{(1-\rho)\lambda_2}{\lambda_1 + \rho\lambda_2} + (c_a + c_{t1} + c_{t2})(\lambda_1 + \rho\lambda_2) \frac{(1-\rho)\lambda_3}{\lambda_1 + \rho\lambda_2}$$

$$= \theta_3 w.$$

This completes the proof. ■

Lemma 1 implies that the cost of each Return Policy is proportional to the length of warranty, which is evident.

As mentioned above, an interesting question is, among the three return policies, which policy is the cheapest? For general distributions $F_1(t)$ and $F_2(t)$, however, to derive simple close forms of $m_1(w)$, $m_{12}(w)$, $F_{12}^*(w, T)$, and $m_{12}^*(w, T)$ is not possible. Even if $F_1(t)$ is the Weibull distribution, for example, only approximation of its renewal function can be derived (see, Cui and Xie, 2003; Jiang, 2010, for example). Hence, we will only compare the three return policies for special cases of $F_i(t)$ (i=1,2,3).

Lemma 2. If $F_i(t) = 1 - e^{-\lambda_i t}$ (i=1,2,3), then we have

(a) If $\rho = 1$, $\lambda_3 = 0, c_{t1} = c_{t2} = 0$, then $C_1(w) = C_3(w)$;

(b) If $\rho = 1$, $\lambda_3 = 0, c_{t1} = c_{t2} = 0$, and $c_{23} > c_n$, then $C_2(w) > C_1(w)$ and $C_2(w) > C_3(w)$; and

(c) If $\rho c_{23} - \rho c_n - c_{t2}(1 + \frac{\lambda_3}{\lambda_2}) + \rho c_a \frac{\lambda_3}{\lambda_2} + \rho(c_{t1} + c_{t2})\frac{\lambda_3}{\lambda_2} > 0$, then $C_2(w) > C_3(w)$.

Proof. The proof can be easily completed based on the results of Lemma 1. ■

Remarks. From Lemma 2, we make the following remarks.

- From (a) and (b) of Lemma 2, $\lambda_3 = 0$ implies that there is no claim cause 3, $c_{t1} = c_{t2} = 0$ implies that neither Type I testing nor Type II testing incurs cost, and $\rho = 0$ implies that Type II testing can correctly reveal the claim cause, then we have the following results.
    - The expected cost incurred in Return Policy 1 equals to that in Return Policy 3. This is evident as there are only claim causes 1 and 2, both of which are caused due to the manufacturer and new items should therefore be sent on any claims. With either Return Policy 1 or Return Policy 3, new items are sent upon claims due to claim cause 1. If claims due to claim cause 2 are reported, with Return Policy 1, a new item will be sent to the claimant; with Return Policy 3, the claimed item will be tested with Type I testing and then Type II testing. Since the Type II testing can correctly reveal the claim cause, the problem that was diagnosed as NFF by Type I testing can be correctly detected. Consequently, a new item will be sent to the claimant. In other words, claims with either Return Policy 1 or Return Policy 3 will end up with returning new items to the claimants and the costs will only include administration cost and cost of returning new items.

    - if $c_{23} > c_n$ also holds, Return Policy 2 incurs more cost than both Return Policy 1 and Return Policy 3. Return Policy 2 returns a claimed item back to the claimant although the claim cause may be due to claim cause 2. If this may cause more cost than sending a new item to the claimant, then Return Policy 2 is more expensive than Return Policy 1 and Return Policy 3, which sends new items to the claimants.
- From (c), whether Return Policy 2 is more costly than Return Policy 3 is independent of λ_1 and of the actual values of λ_2 and λ_3, but depends on the ratio of λ_3 to λ_2.
- From (c), it can also been seen that $C_2(w) < C_3(w)$ if ρ=0. As ρ=0 indicates the probability of correctly detecting claim cause 2 is 0, spending time and cost on claim cause 2 is not necessary.

3.3. Sensitivity analysis

The preceding section 3.2 investigates the roles of some parameters for special cases. In this section, we conduct sensitivity analyses on different cost parameters without the assumption of the exponential distributions.

It can easily come to the following results.

- The costs of all the three return policies are increasing in c_n and c_a.
- The costs of Return Policies 2 and 3 are increasing in c_{t1} and c_{23}.
- The cost of Return Policy 1 is increasing in c_{32}, the costs of Return Policy 3 is increasing in c_{t2}.

As the major difference between the return policies lies in whether new items should be sent to the claimants, we further analyse the impact of C_n on the costs of return policies. Since $\frac{\partial C_1(w)}{\partial c_n} = m_{123}(w)$, $\frac{\partial C_2(w)}{\partial c_n} = m_1(w)$, $\frac{\partial C_3(w)}{\partial c_n} = m_T(w)$, and $m_{123}(w) \geq m_T(w) \geq m_1(w)$, we have $\frac{\partial C_1(w)}{\partial c_n} \geq \frac{\partial C_3(w)}{\partial c_n} \geq \frac{\partial C_2(w)}{\partial c_n}$. This implies that the expected cost of Return

Policy 1 is more sensitive to the change of c_n than the other two Return Policies, while the expected cost of Return Policy 2 is less sensitive to the change of c_n than the other two Return Policies.

Section 6 uses numerical examples to investigate the roles of ρ, C_{23}, and C_{t1}.

4. Optimisation of warranty periods under supply chain environments

In this section, we derive optimal warranty periods for the base warranty and the extended warranty, respectively. The following derivation is needed in this subsection. From Eqs. (1)---(3), we have

$$\frac{\partial C_1(w_i)}{\partial w_i} = \left(c_n + c_a + c_{32}(1 - q_{X_{12}<X_3})\right)\pi_1(w_i), \tag{5}$$

$$\frac{\partial C_2(w_i)}{\partial w_i} = \left(c_n + c_a + c_{t1} + (c_a + c_{t1})\int_0^\infty \left(\Lambda_2(t) + \Lambda_3(t)\right)dF_1(t) + c_{23}\int_0^\infty \Lambda_2(t)dF_1(t)\right)\pi_2(w_i) \tag{6}$$

$$\frac{\partial C_3(w_i)}{\partial w_i} = \Big(c_n + c_a + c_{t1} + \left(1 - q_{X_1<X_{T2}}\right)c_{t2} + (c_a + c_{t1} + c_{t2} + c_{23})\int_0^\infty (1-\rho)\Lambda_2(t)dF_T(t)$$

$$+ c_a\int_0^\infty (1-\rho)\Lambda_3(t)dF_T(t)\Big)\pi_3(w_i) \tag{7}$$

where

$$\pi_1(w_i) = f_{123}(w_i) + \int_0^\infty \pi_1(w_i - t)\, f_{123}(t)dt,$$

$$\pi_2(w_i) = f_1(w_i) + \int_0^\infty \pi_2(w_i - t)\, f_1(t)dt,$$

and

$$\pi_3(w_i) = f_T(w_i) + \int_0^\infty \pi_3(w_i - t)\, f_T(t)dt.$$

4.1. The supply chain context

We assume the following supply chain context. We take the assumptions used in (Chen et al., 2012), which assumed the manufacturer, as a Stackelberg leader, specified wholesale prices to two competing retailers, retailer 1 and retailer 2, who faced warranty period-dependent demand and had different sales costs and then analysed different strategies from both the manufacturer's and the retailers' perspective. They considered demands primarily influenced by extended warranty offered by retailers, provided the price differentiation between the retailers becomes insignificant to their customers at the time of purchase decision (Chen et al., 2012).

4.2. Period of the base warranty

Assume, under a supply chain environment, that the primary demand of a product is sensitive to the period of the base warranty. One can then define warranty period dependent demand as following:

$$D_1(w) = \alpha_0 + \alpha_1 w, \tag{8}$$

where $\alpha_0 (> 0)$ is the primary demand, and $\alpha_1 (> 0)$ is the consumers' sensitivity to warranty period.

The warranty provider's profit with Return Policy k is defined as

$$Q_{1,k}(w) = \beta_0 D_1(w) - C_k(w) D_1(w), \tag{9}$$

where $k = 1,2,3$, $\beta_0 D_1(w)$ =sales revenue−purchasing cost−sales cost, and $C_k(w)D_1(w)$ is the cost incurred due to warranty period service. Then, combine both Eqs. (8) and (9), we obtain

$$Q_{1,k}(w) = \big(\beta_0 - C_k(w)\big)(\alpha_0 + \alpha_1 w). \tag{10}$$

Proposition 4. If $\frac{\partial^2 C_k(w)}{\partial w^2} > 0$, the optimal warranty period w^* for Return Policy k satisfies

$$(\alpha_0 + \alpha_1 w^*) \left.\frac{\partial C_k(w)}{\partial w}\right|_{w=w^*} + \alpha_1 C_k(w^*) - \alpha_1 \beta_0 = 0. \tag{11}$$

Assume $F_i(t) = 1 - e^{-\lambda_i t}$ (i=1,2,3). The optimal warranty period for Return Policy k is given by

$$w^* = \frac{\beta_0}{2\theta_k} - \frac{\alpha_0}{2\alpha_1} \quad (12)$$

where k=1,2,3, respectively.

Proof. From Eq. (10), we have

$$\frac{\partial Q_{1,k}(w)}{\partial w} = \alpha_1\big(\beta_0 - C_k(w)\big) - \frac{\partial C_k(w)}{\partial w}(\alpha_0 + \alpha_1 w),$$

and

$$\frac{\partial^2 Q_{1,k}(w)}{\partial w^2} = -\frac{\partial^2 C_k(w)}{\partial w^2}(\alpha_0 + \alpha_1 w) - 2\alpha_1 \frac{\partial C_k(w)}{\partial w}.$$

From Eqs. (1)—(3), $\frac{\partial C_k(w)}{\partial w}$ is the derivative of a renewal function within time interval $(0,w)$. As any renewal function increases in w, $\frac{\partial C_k(w)}{\partial w} > 0$. Hence $\frac{\partial^2 Q_{1,k}(w)}{\partial w^2} < 0$ if $\frac{\partial^2 C_k(w)}{\partial w^2} > 0$. That is, $Q_{1,k}(w)$ is concave in w.

Let $\frac{\partial Q_{1,k}(w)}{\partial w} = 0$, one has

$$(\alpha_0 + \alpha_1 w)\frac{\partial C_k(w)}{\partial w} + \alpha_1 C_k(w) - \alpha_1\beta_0 = 0. \quad (13)$$

If $F_i(t) = 1 - e^{-\lambda_i t}$ (i=1,2,3), substitute $C_1(w)$, $C_2(w)$, and $C_3(w)$ to the above Eq. (13), one can derive the optimal warranty periods shown in Eq. (12).

This completes the proof. ■

Lemma 3. Assume $F_i(t) = 1 - e^{-\lambda_i t}$ (i=1,2,3). Then the minimum expected cost of $Q_{1,k}(w^*)$ is given by

$$Q_{1,k}(w^*) = \frac{(\alpha_0\theta_k - \alpha_1\beta_0)^2}{4\alpha_1\theta_k} + \alpha_0\beta_0. \quad (14)$$

Proof. Substitute w^* in Eq. (12) into Eq. (10), we can obtain $Q_{1,k}(w^*)$ in Eq (14). ■

4.3. Period of extended warranty

In this paper, we consider the following pricing strategy:

Manufacturer negotiates with both retailers simultaneously considering their sales cost and specifies the same wholesale price for both retailers.

One can then define warranty period dependent demand for retailer *j* as following

$$D_2(w_j) = \alpha_2 + \alpha_3 w_j - \alpha_4 w_{3-j} \tag{15}$$

where $\mathrm{j} = 1,2$, $\alpha_2(> 0)$ is the primary demand, $\alpha_3(> 0)$ represents the consumers' sensitivity to warranty period, and $\alpha_4(> 0)$ denotes the competitive factors, and $\alpha_4 < \alpha_3$. The retailer *j*'s profit with Return Policy *k* is defined as

$$Q_{2,k}(w_j) = \beta_1 D_2(w_j) - \delta_j D_2(w_j) - C_k(w_j) D_2(w_j) \tag{16}$$

where $j = 1,2$, $k = 1,2,3$, $\beta_1 D_2(w_j)$ is the difference between the retailer *j*'s sales revenue and purchasing cost, $\delta_i D_2(w_j)$ is the sales cost, and $C_k(w_j)D_2(w_j)$ is the costs incurred by warranty period service.

Then we have the following Proposition.

Proposition 5. The retailer *j*'s optimal warranty period w_j^* in Return Policy *k* satisfies

$$\alpha_3(\beta_1 - \delta_j) - (\alpha_2 + \alpha_3 w_j^* - \alpha_4 w_{3-j}^*) \left.\frac{\partial C_k(w_j)}{\partial w_j}\right|_{w_j = w_j^*} - \alpha_3 C_k(w_j^*) = 0, \tag{17}$$

where $\frac{\partial C_k(w_j)}{\partial w_j}$ (*k*=1,2,3) are given in Eqs. (5)---(7).

Assume $F_i(t) = 1 - e^{-\lambda_i t}$ (*i*=1,2,3), then the retailer *j*'s optimal extended warranty period w_j^* with Return Policy *k* is given by

$$w_j^* = \left(\frac{\beta_1}{\theta_k} - \frac{\alpha_2}{\alpha_3} - \frac{2\alpha_3\delta_j + \alpha_4\delta_{3-j}}{(2\alpha_3 + \alpha_4)\theta_k}\right)\frac{\alpha_3}{2\alpha_3 - \alpha_4}, \tag{18}$$

where *k*=1,2,3, respectively.

Proof. By mimicking the proof of Proposition 4, one can easily complete the proof. ■

5. Discussion

To derive the expected costs expressed in Eqs. (1)—(3), we need to obtain distribution functions $F_k(t)$, probability ρ, different costs c_{32}, c_{23}, c_a, c_n, c_{t1}, and c_{t2}. Their estimations are discussed below, respectively.

5.1. Estimation of the probability functions $F_k(t)$

In the preceding sections, we assume that $F_k(t)(k = 1,2,3)$ can be obtained, which is possible in practice. One may estimate them based on warranty data, which are comprised of claims data and supplementary data. Warranty claims data are the data collected during the servicing of items under warranty and supplementary data are additional data (such production and marketing related, items with no claims, etc.) that are needed for effective warranty management (Wu, 2013).

5.2. Estimation of the probability ρ

ρ is the probability of correctly diagnosing claim causes 2 and 3. Because time to detect unknown claim cause is uncertain, such a fault detection process can be regarded as a time-dependent stochastic process $\{X(t), t \geq 0\}$, where $X(t)$ is a random variable and is the time to successfully detect claim cause 2 at time ($t \geq 0$). One may regard the process of the ability to detect claim causes as a gamma process for the following reason. Time to successfully detecting the real cause is always positive; and it may become stochastically shorter over time. The learning process is monotonic in the sense that the probability of correctly detecting the real causes becomes larger with time. As such, a Gamma process can be used for modelling the learning process in which the detection of the real causes is supposed to take place gradually over time in a sequence of positive increments. In theory, a Gamma process $\{X(t), t \geqslant 0\}$ has the following three properties.

(1) The increment $X(t_i) - X(t_i - 1)$ for a given time interval $\Delta = t_i - t_{i-1}$ follows the Gamma distribution,

(2) The increments for any set of disjoint time intervals are independent random variables having the distributions described in property (1), and

(3) X(0) = 0 almost surely.

Let the probability density function of $X(t)$ in conformity with the definition of the gamma process, be given by $f_{X(t)}(x) = GA(x|v(t), u)$, with $GA(x|v(t), u) = \frac{u^{v(t)}}{\Gamma(v(t))} x^{v(t)-1} \exp\{-ux\} I_{(0,\infty)}(x)$, $E\big(X(t)\big) = \frac{v(t)}{u}$, and $\mathrm{Var}\big(X(t)\big) = \frac{v(t)}{u^2}$, where $I_A(x) = 1$ for $x \in A$ and $I_A(x) = 0$ otherwise.

At time t, denote the time when claim cause 2 is detected by time point T,

$$\Pr\{X(t) \le T\} = \int_0^T f_{X(t)}(x)\, dx = \frac{\gamma(v(t), Tu)}{\Gamma(v(t))}$$

where $\Gamma\big(v(t)\big) = \int_0^\infty \tau^{v(t)-1} e^{-\tau}\, d\tau$ and $\gamma(v(t), Tu) = \int_0^{Tu} \tau^{v(t)-1} e^{-\tau}\, d\tau$.

Then

$$p(t, T) = \frac{\partial \Pr\{X(t) \le T\}}{\partial t}. \tag{19}$$

The above method has been used in reliability engineering to model the deterioration process of reliability systems. Of cause, one needs to collect historical data for estimating $\Pr\{X(t) \le T\}$. Again, supplementary data can be used for this purpose.

5.3. Estimation of c_{23} and c_{t2}

Estimating c_{32}, c_a and c_n is not difficult. Below we discuss methods of estimating c_{23} and c_{t2}, respectively.

c_{23} is the cost of returning faulty items to users, which can result in profit losses. The losses can be larger if more claimed items with claim cause 2 are returned to the claimants, which is essentially similar to the situation that product costs are associated with its reliability, as the relationship proposed in Mettas (2000).

c_{t2} is the cost incurred in Type II testing. Type II testing might start from the first claim with claim causes 2 or 3, and then such effort might continue until all of the claim causes 2 and 3 are eventually detected and fixed or until a new model of products is launched to replace the old ones. In this case, the probability of successfully detecting and then fixing the causes depends on time. If we can set the time instant after the *n* products were sold to be 0, then the cumulative distribution function of time to the first failure (and then claim) is $F_{23}^{(n)}(t) = 1-((1-F_2(t))(1-F_2(t)))^n$. The probability that claim cause 2 or claim cause 3 occurs during the warranty period is given by $\int_0^w dF_{23}^{(n)}(t)$.

Proposition 6. The expected cost on detecting and fixing the cause of NFFs per unit time is given by

$$c_{t2} = \frac{C_{t2}}{n}\int_0^w \int_t^{T_n} \tau p(\tau, T) d\tau \, dF_{23}^{(n)}(t) \quad (20)$$

where T_n is an estimated time when the manufacturer might give up trying to diagnose the cause (or the time when a new model of products is launched), $p(\tau, T)$ can be estimated from Eq. (19), and C_{t2} is the total cost on diagnosing claim causes 2 and 3.

5.4. The expected number of warranty claims

The expected number of warranty claims of each return policy is another interesting quantity that can be required from time to time in practice. As can been seen, the expected numbers of warranty claims of the return policies have already been derived in the process of proving the first three Propositions.

6. Numerical examples

Section 4 discusses the three return policies for some special cases. In this section, we consider more complicated parameter settings and investigate the changes of the costs derived from the three policies, as we mentioned that it is unlikely to derive closed explicit forms for the renewal functions used in the expected costs for general inter-arrival distribution functions. As such, we use Monte Carlo simulation to generate random numbers with the parameters in Table 2 to estimate the expected cost values derived from the preceding sections. That is, we generate random numbers S_i as the time elapsed before an item fails (or is reported) for the "ith" time since the last time it failed (or was reported), and then count $\sup\{n: \sum_{i=1}^{n} S_i\}$ as the renewal functions in $C_1(w)$, $C_2(w)$, and $C_3(w)$. For each renewal function, we iterate this procedure for 5000 times and calculate the average of values $\sup\{n: \sum_{i=1}^{n} S_i\}$ to obtain a robust estimate of the renewal function.

Table 2. The distribution functions and the warranty period

$F_1(t) = 1 - \exp\left(-\left(\frac{t}{20}\right)^{1.1}\right)$	$F_3(t) = 1 - \exp\left(-\left(\frac{t}{28}\right)^{1.3}\right)$
$F_2(t) = 1 - \exp\left(-\left(\frac{t}{24}\right)^{1.2}\right)$	$w = 24$

6.1. The role of the probability ρ

ρ is the probability of correctly diagnosing claim causes 2 and 3. It is important to understand its role in Return Policy 3. In Figures 2, 3, and 4, ρ changes from 0.01 to 1, as shown in the X-axis, and the Y-axis represents the expected costs.

- If ρ changes from 0.01 to 1 with step 0.05 and let c_a=1, c_n=100, c_{32}=80, c_{23}=20, c_{t1}=4 and c_{t2}=5, then the changes of the expected costs $C_1(w)$, $C_2(w)$, and $C_3(w)$ are shown in Figure 2. From the figure, it can be found that Return Policy 2 is the cheapest one whereas Return Policy 1 is the most expensive one. The expected cost of Return Policy 3 increases slowly, and the other two return policies have stable costs. The increase of the expected cost of Return Policy 3 is due to the fact that more new items are required as a result of the correct diagnosis of claim cause 2, comparing to the small cost of misdiagnosing claim cause 2 to claim cause 3.

- If ρ changes from 0.01 to 1 with step 0.05 and let c_a=1, c_n=100, c_{32}=80, c_{23}=120, c_{t1}=4 and c_{t2}=5, then the changes of the expected costs $C_1(w)$, $C_2(w)$, and $C_3(w)$ are shown in Figure 3. From the figure, it can be found that Return Policy 1 is the cheapest before ρ changes to 0.21. The expected cost of Return Policy 3 dramatically decreases when ρ is larger than 0.25 and it then keeps the smallest one, whereas Return Policy 2 is the most expensive one when ρ is larger than 0.06.
- If ρ changes from 0.01 to 0.96 with step 0.05, let c_a=1, c_n=10, c_{32}=8, c_{23}=20, c_{t1}=5 and c_{t2}=10, then the changes of the expected costs $C_1(w)$, $C_2(w)$, and $C_3(w)$ are shown in Figure 4. In this case, Return Policy 1 remains the cheapest one whatever ρ is. The expected cost of Return Policy 3 decreases.

From the three examples, we can find that the expected cost of Return Policy 3 can increase or decrease if the probability of correctly diagnosing claim causes 2 and 3 increases. Each of the three return policies can be the cheapest one or the most expensive one, it depends on different costs of c_n, c_{32}, and c_{23}.

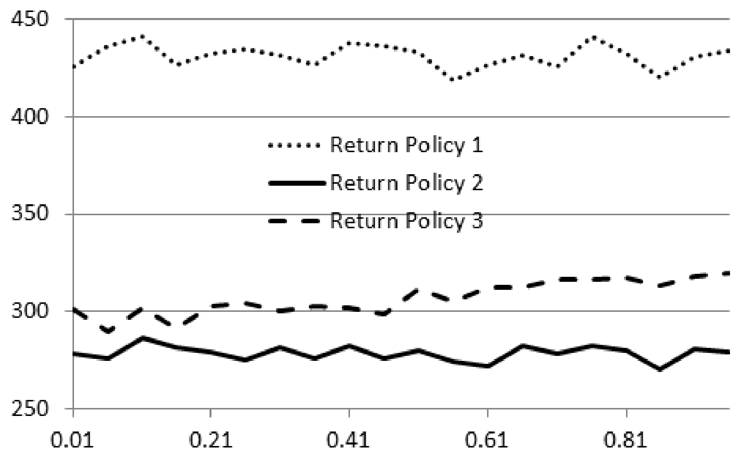


Figure 2. The expected costs of the three return policies when c_a=1, c_n=100, c_{32}=80, c_{23}=120, c_{t1}=4, and c_{t2}=5.

Figure 3. The expected costs of the three return policies when c_a=1, c_n=100, c_{32}=80, c_{23}=20, c_{t1}=4, and c_{t2}=5.

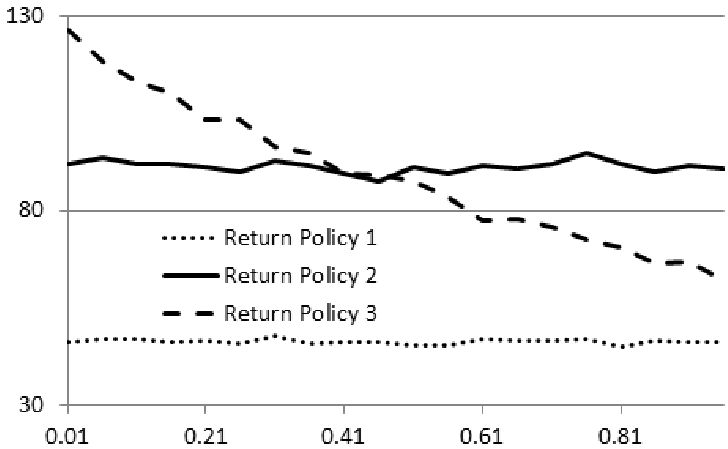


Figure 4. The expected costs of the three return policies when c_a=1, c_n=10, c_{32}=8, c_{23}=20, c_{t1}=5 and c_{t2}=10.

6.2. Dependence of the return policies on c_{23} and c_{t1}

c_{23} and c_{t1} are parameters used in the expected costs of Return Policy 2 and Return Policy 3, respectively. We therefore investigate their roles in the policies.

Figures 5 and 6 show the expected costs of the three return policies when c_{23} and c_{t1} increase. It can be seen that both the expected costs of Return Policy 2 and Return Policy 3 increase: the expected cost of Return Policy 2 increases faster than that of Return Policy 3.

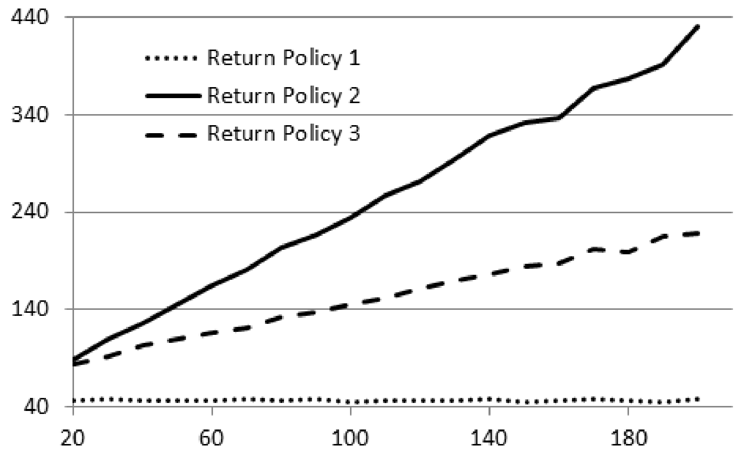


Figure 5. The expected costs of the three return policies when c_a=1, c_n=10, c_{32}=8, c_{t1}=5, and c_{t2}=10. c_{23} changes from 20 to 200, as shown in the X-axis. The Y-axis represents the expected costs.

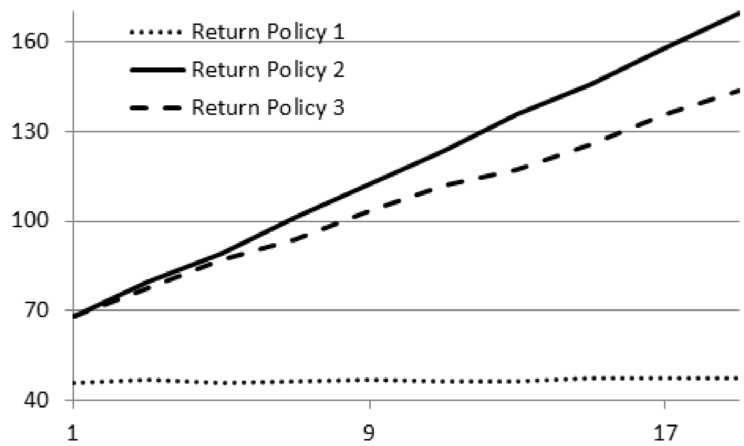


Figure 6. The expected costs of the three return policies when c_a=1, c_n=10, c_{32}=8, and c_{t2}=10, c_{t1} changes from 1 to 20, as shown in the X-axis. The Y-axis represents the expected costs.

7. Conclusions

This paper considered the fact that product returns can be due to other factors in addition to product failures. It proposed three warranty return policies, derived the expected costs of the policies and a testing method, respectively. It then compared the expected costs and derived optimal warranty periods under supply chain environments.

In estimating the number of warranty claims, traditionally, the renewal process is applied in the scenario when claimed items are not repairable and the nonhomogeneous Poisson process is used when the claimed items are repairable. This is the first paper that used the renewal-reward process to estimate the number of warranty claims. It is noted that this is the first paper that systematically studies and compares different solutions for warranty claims with the no-fault-found phenomenon. The paper also offers alternates for the industrialists to design different warrant policies.

Our future work will focus on developing new warranty policies. For example, if a customer continually returns an item whose failure mechanism has not been detected and confirmed, it may not wise to return the same item back to him/her. Instead, a new item should be returned. This can lead to develop a new return policy.

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Captions of the Figures

- Figure 1. Warranty claim handling procedure in Return Policy 2 and Return Policy 3
- Figure 2. The expected costs of the three return policies when c_a=1, c_n=100, c_{32}=80, c_{23}=120, c_{t1}=4, and c_{t2}=5.
- Figure 3. The expected costs of the three return policies when c_a=1, c_n=100, c_{32}=80, c_{23}=20, c_{t1}=4, and c_{t2}=5
- Figure 4. The expected costs of the three return policies when c_a=1, c_n=10, c_{32}=8, c_{23}=20, c_{t1}=5 and c_{t2}=10.
- Figure 5. The expected costs of the three return policies when c_a=1, c_n=10, c_{32}=8, c_{t1}=5, and c_{t2}=10. c_{23} changes from 20 to 200, as shown in the X-axis. The Y-axis represents the expected costs.
- Figure 6. The expected costs of the three return policies when c_a=1, c_n=10, c_{32}=8, and c_{t2}=10, c_{t1} changes from 1 to 20, as shown in the X-axis. The Y-axis represents the expected costs.