Extended Utility and DEA Models without Explicit Input

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Abstract. In this paper, we discuss the relationship between multi-attribute utility theory and DEA models without explicit inputs (DEA-WEI), including dual models and some theoretical analysis of DEA-WEI models. We then propose generic DEA-WEI models with quadratic utility terms. Finally, we provide illustrative examples to show that DEA-WEI with suitable quadratic utility terms are able to reflect some value judgments that the standard DEA models cannot.

Keywords. DEA**,** DEA without explicit input, value judgment, multiple attribute utility

1. Introduction

Since its introduction in 1978, DEA has been widely used in the performance analysis of many business and nonprofit evaluation procedures. One unique feature of DEA is that it allows assessed DMUs to assign their most favourable weights to maximise their scores in the assessments. Therefore, if a DMU is classified as inefficient under the weights that are the most favourable to this DMU, it can hardly be argued that its inefficiency is due to an unfair weight selection. There exist many DEA models, among the most well known of which include the CCR model (Charnes et al. 1978), the BCC model (Banker et al. 1984), the Additive model (Charnes et al. 1985), and the Cone Ratio model (Charnes et al. 1989). Excellent reviews on DEA theory and applications may be found in several recent books, e.g., Cooper et al. (2000, 2004, 2006), Cook et al. (2009). These DEA models are all formulated for desired inputs and outputs to measure the technical efficiency of DMUs.

In many applications, however, there are no explicit input data available. In practice, there may be two types of motivations to employ DEA models without explicit inputs. First, in some business and management studies, several ratio indicators, such as GDP per capita, the revenue-expenditure ratio, value-added per employee and profit per cost, are often used to measure performance. In this case, it is clearly difficult or sometimes impossible to reformulate the data into original inputs and outputs and then apply the classic DEA models to measure the performance of DMUs. Second, there are indeed many multi-criteria decision and evaluation problems that do not need to consider input (sometimes output) variables (such as the evaluation of national overall power). For more details, the readers are referred to the study by Liu et al. (2011) for a literature review.

The first systematic study on these DEA models is that by Lovell and Pastor (1999), who in their paper "DEA models without inputs." They attempt to demonstrate the following conclusions: "(i) a CCR model without inputs (or without outputs) is meaningless; (ii) a CCR model with a single constant input (or with a single constant output) coincides with the corresponding BCC model." In our recent work (Liu et al., 2011), systematic studies are conducted for this group of DEA models, which are called DEA models without explicit inputs (DEA-WEI models).

It is well known that the classic DEA is to measure the technical efficiency of the input-output system of the DMUs, while in general DEA-WEI does not reflect this. Therefore, it is important to clarify its theoretical foundation. This study is the first attempt in this direction. In this paper, we continue our investigation on DEA-WEI and link DEA-WEI models with multi-attribute utility theory. As one possible application, we show that it is useful to use nonlinear utility to reflect the value judgment of the decision-makers (DMs).

The paper is organised as follows: Section 2 presents the relationship between DEA-WEI models and multi-attribute utility theory. In Section 3, we study quadratic DEA-WEI models. Section 4 presents an empirical study of quadratic DEA-WEI models, and the conclusion is provided in Section 5.

2. The relationship between utility theory and DEA-WEI models

2.1 Extended utility function with variable weights

Multiple Attribute Utility Theory (MAUT) is one of the major analytical tools used in the field of decision analysis. An excellent review of MAUT is provided in studies by Keeney and Raiffa (1976) and von Winterfeldt and Edwards (1986).

General utility functions are to be estimated for any real application. Given $Y=(y_1, y_2, ..., y_n)$, $n \geq 2$, if y_i is "utility independent" of y_j for all $j \neq i$, then the following multi-linear utility function is appropriate (see Keeney and Raiffa 1976):

$$u(Y)=\sum_{i=1}^{n} w_i u_i(y_i)+\sum_{i=1}^{n-1}\sum_{j>i} w_{ij} u_i(y_i) u_j(y_j)+\sum_{i=1}^{n-2}\sum_{j>i}\sum_{m>j>i} w_{ijm} u_i(y_i) u_j(y_j) u_m(y_m)$$
$$+\cdots+w_{123...n} u_1(y_1) u_2(y_2)\cdots u_n(y_n)$$

where $u_i(y_i)$ is a single-attribute utility function scaled from 0 to 1, w_i is the fixed weight for attribute i, where $0 \leq w_i \leq 1$, and w_{ij}, w_{ijm}, $w_{123...n}$ represent the impact of the interactions between attributes on preferences. In applications, DMs must firstly determine the form of the utility (e.g., the order of the utility), and then the weights are to be estimated to apply the MAUT in applications.

For simplicity, if we let $f_r(Y)$ be the value of DMU$_j$'s partial utility function. The function may then be written as follows:

$$u(Y)=\sum_{i=1}^{s} w_i f_i(Y) \tag{1}$$

where $0 \leq w_i \leq 1$.

For simplicity, we may consider a multi-criteria problem with s criteria $(y_1, y_2, ..., y_s)$, where every criterion $y_r \geq 0$ and behaves such that "larger is better". In addition, there are n DMUs or alternatives, denoted by $Y_1, ..., Y_j, ..., Y_n$. Decision-makers (DM) must now assign weights to each attribute. There are two types of approaches used to identify weights: subjective approaches, such as the Analytic Hierarchy Process (Saaty 1980, 1986; Forman and GASS, 1999), and data-based approaches, such as the Entropy method (see Hwang and Yoon, 1981; Zeleny, 1982) and Principal Components Analysis. It is well documented (Fishburn, 1970) that in real applications, the most widely used form remains linear utility, although the additivity assumption can hardly be verified.

DEA-WEI is generally used for evaluation purposes, although as mentioned before, it may not reflect technical efficiencies of the DMUs. However it can be observed that the functional form of its objective function shares a family resemblance of the classic utility function, and thus now we will discuss the possible relationship between them. Unlike in the classic utility theory, here only the functional form of the objective function (i.e., what terms in U(Y) should be included in the evaluation) is determined by the DMs of the evaluation to reflect a subjective emphasis on the evaluation, whereas the (DMs of) DMUs determine the coefficients of the terms by DEA-style programming to present their own advantages. In this sense, the formulation can be regarded as an extended utility. This utility can only be understood as a pricewise function, unlike the standard utility functions, and each DMU uses its different marginal utilities at different parts (vertices), as the weights are no longer global but local. The classic DEA-WEI simply uses the linear terms, and all of the outputs are considered to be equally important in the DEA-style programming. However, it is possible to include higher-order terms to reflect some value judgments of DMs as is illustrated in the empirical studies, offered below, where the decision-makers for the evaluations chose to add a quadratic term in $u(Y)$ to reflect their subjective value judgments, whereas the coefficients are determined by DMUs to present their own advantages in evaluation.

Therefore it is possible to determine the weights from another point of view (DEA approach): we will allow the weights to differ between DMUs; that is, different DMUs may be assigned different weights so that $u(Y)$ in fact has a piecewise formulation, in which DMUs may obtain their optimal weights by themselves. If we use some truncations of $u(Y)$, then the model may be formulated as follows:

$$\begin{aligned} h^* = \max \; & \sum_{r=1}^{t} u_r f_r\left(Y_0\right) \\ \text{s.t.} \quad & \sum_{r=1}^{t} u_r f_r\left(Y_j\right) \le 1, \quad j = 1,\dots,n \\ & u_r \ge 0, \quad r = 1,\dots,t \end{aligned} \tag{2}$$

This approach is referred to as “utility DEA-WEI model” in this paper. For the simplest form, where $f_r\left(Y_j\right) = y_{rj}$ has been studied already, see the study by Liu et al. (2011). This maximisation formulation may decide the weight for each DMU. More generally, this approach may be formulated as follows:

$$h^* = \max u\, Y_0 \quad (\quad)$$
$$\text{s.t.} \quad w \in S,\ u(Y_j) \le 1,\ \ j = 1,\dots,n. \tag{3}$$

where w is the weight vector and S is the weight constraint set.

2.2 The dual models

We will now consider the dual model of DEA model (2). We first introduce the Envelopment form of utility DEA-WEI model:

$$\begin{aligned} \theta^* = \max \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j f_r\left(Y_j\right) \ge \theta f_r\left(Y_0\right), r = 1,\dots,t \\ & \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \ge 0,\ j = 1,\dots,n \end{aligned} \tag{4}$$

This model has also appeared in several applications, such as that by Yang and Kuo (2003), who apply a BCC model without inputs to solve the facilities layout performance frontiers problem. Lovell and Pastor (1999) regard this model as the output-oriented BCC model, with the inputs being assumed equal to unity. In our opinion, this argument is not quite precise. In fact, we find that because all the inputs have been assumed to be unity or because the data are index data, it is meaningless to consider the Production Possibility Set (PPS) and return to the scale problem as one would with the standard DEA models. Here we will show the relationship between models (2) and (4).

Theorem 1: The optimal value of model (2) is the reciprocal of that of model (4); that is, $h^* = 1/\theta^*$.

Proof: Then, the dual model of (2) is as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{j=1}^{n} \lambda_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} f_r\left(Y_j\right)\lambda_j \ge f_r\left(Y_0\right),\ r = 1,\dots,s \\ & \lambda_j \ge 0, \quad j = 1,\dots,n \end{aligned} \tag{5}$$

Notice, by the constraints of model (5), we may find that $\sum_{j=1}^{n}\lambda_j > 0$. If we let $t = \sum_{j=1}^{n}\lambda_j$ and $\lambda_j^{'} = \lambda_j / t$, then model (5) may be transformed to the following model:

$$\begin{aligned} &\text{Min} \quad t \\ &\text{s.t.} \sum_{j=1}^{n}\lambda_j^{'} f_r\left(Y_j\right) \geq (1/t)\, f_r\left(Y_0\right),\ r = 1,\ldots,s \\ &\sum_{j=1}^{n}\lambda_j^{'} = 1, \lambda_j^{'} \geq 0,\ j = 1,\ldots,n. \end{aligned} \tag{6}$$

If we let $\theta = 1/t$ and substitute the λ_j for $\lambda_j^{'}$, then model (6) may be transformed into model (4). Because the optimal value of model (6) equals those of models (5) and (2), and because the optimal value of model (4) equals the reciprocal of that of model (6), we may easily conclude that the optimal value of model (4) is the reciprocal of that of model (2).

3. Quadratic DEA-WEI models based on quadratic utility terms

Generally, utility functions are to be estimated for any real application. Although the linear truncation is the most widely used form in practice, linearity cannot reflect evidence enhancement (Yang et al., 1994, 2002), as illustrated below.

For example, suppose that there are two utility functions: one of them is the additive linear form $f_1(x) = x_1 + x_2$, and another is the quadratic item $f_2(x) = x_1 x_2$, where x_1, x_2 are two examination results (0-5 in 5 scale) for the same subject. When using the linear model, one may obtain reasonable overall scores so long as the examined score in either subject is very good. This is not the case for the nonlinear model – one would have very poor overall scores if either examined score is so. Therefore, if in some applications we must emphasise two or more indicators, one cannot simply use the standard DEA models directly. In this section, we will examine DEA-WEI models with quadratic terms.

Following the general form of a utility function, the generic quadratic DEA-WEI reads as follows:

$$
\begin{aligned}
h^* = \text{Max} \quad & \sum_{r=1}^{s} w_r f_r\left(Y_0\right) + \sum_{r=1}^{s}\sum_{k\geq r}^{s} a_{rk} f_{rk}\left(Y_0\right) \\
\text{s.t.} \quad & \sum_{r=1}^{s} w_r f_r\left(Y_j\right) + \sum_{r=1}^{s}\sum_{k\geq r}^{s} a_{rk} f_{rk}\left(Y_j\right) \leq 1, \\
& w_r \geq 0,\ j = 1,\ldots,n, \\
& a_{rk} \geq 0,\ r = 1,\ldots,s,\ \text{and } k = r,\ldots,s.
\end{aligned}
\tag{9}
$$

where $f_{rk}\left(Y_j\right) = y_{rj}\, y_{kj},\ j = 1,\ldots,n$. Sometimes the weights have further restrictions.

The meaning of the above quadratic items $a_{rk} f_{rk}\left(Y_0\right)$ may be viewed as the surface area of axis r, k, and a_{rk} represents half of the value of the sine of the angle between r, k and a_{rk} axes. Alternatively, we may rewrite (9) as follows:

$$
\begin{aligned}
\text{Max} \quad & W^T Y_0 + Y_0^T A Y_0 \\
\text{s.t.} \quad & W^T Y_j + Y_j^T A Y_j \leq 1,\ j = 1,\ldots,n \\
& W \geq 0,\ A = (a_{ij})_{s\times s} \geq 0, \\
& a_{ij} = 0,\ i > j.
\end{aligned}
\tag{10}
$$

Model (10) appears similar to the multiple objective quadratic-linear problem introduced by Rhode and Webber (1981). However, model (10) remains a linear programming model. Similarly, we may also obtain the dual form of the quadratic DEA-WEI as follows:

$$
\begin{aligned}
& \text{Max } \theta^* \\
\text{s.t.} \quad & \sum_{j=1}^{n} y_{rj}\lambda_j \geq \theta y_{r0},\ r = 1,\ldots,s \\
& \sum_{j=1}^{n} y_{rj}\, y_{kj}\lambda_j \geq \theta y_{r0}\, y_{k0},\ r = 1,\ldots,s,\ k = r,\ldots,s \\
& \theta \geq 1,\ r = 1,\ldots,s,\ k = r,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1,\quad \lambda_j \geq 0.
\end{aligned}
\tag{11}
$$

To increase the discriminatory power of the model, Kuosmanen and Post (2002) introduce a single second-order term sufficient to consider a wide range of non-convexities that only have an $x^T x$ item. Therefore, a unit matrix A is adopted in our model. In general, a non-complete DEA-WEI model is as follows:

$$
\begin{aligned}
&\text{Max } \theta^* \\
&\text{s.t.} \quad \sum_{j=1}^{n} y_{rj}\lambda_j \ge \theta y_{r0},\ r = 1,\ldots,s \\
&\qquad \sum_{j=1}^{n} a_{rk} y_{rj} y_{kj}\lambda_j \ge \theta a_{rk} y_{r0} y_{k0},\ r = 1,\ldots,s,\ k = r,\ldots,s \\
&\qquad \theta \ge 1,\ r = 1,\ldots,s \\
&\qquad a_{rk} = 0 \text{ or } 1,\ \theta \ge 1,\ r = 1,\ldots,s,\ k = r,\ldots,s \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \ge 0.
\end{aligned}
\tag{12}
$$

Note that here if $a_{rk} = 0$, the corresponding term will not appear in the DEA model. In addition, similar to the DEA-WEI models, there are radial and non-radial measures in the quadratic DEA-WEI models. Obviously, Model 12 is quadratic DEA-WEI model with radial measurement. To eliminate indicator slack, we may use the quadratic DEA-WEI model with Russell measurement (output-based) as follows (Model 13), in which Pareto preference is implied.

$$
\begin{aligned}
&\theta^* = \text{Max} \quad \left(\sum_{r=1}^{s}\theta_r + \sum_{r=1}^{s}\sum_{k\ge r}^{s} a_{rk}\theta_{rk}\right) \Big/ \left(\sum_{r=1}^{s}\sum_{k\ge r}^{s} a_{rk} + s\right) \\
&\text{s.t.} \quad \sum_{j=1}^{n} y_{rj}\lambda_j \ge \theta_r y_{r0},\ r = 1,\ldots,s \\
&\qquad \sum_{j=1}^{n} a_{rk} y_{rj} y_{kj}\lambda_j \ge \theta_{rk} a_{rk} y_{r0} y_{k0},\ r = 1,\ldots,s,\ k = r,\ldots,s \\
&\qquad \theta_r \ge 1,\ r = 1,\ldots,s \\
&\qquad a_{rk} = 0 \text{ or } 1,\ \theta_{rk} \ge 1,\ r = 1,\ldots,s,\ k = r,\ldots,s \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \ge 0.
\end{aligned}
\tag{13}
$$

When all the quadratic terms disappear, we have the DEA-WEI model with Russell measurement (output-based), following Liu et al. (2011):

$$
\begin{aligned}
\theta^* = \max & \sum_{r=1}^{s} \theta_r / s \\
s.t. & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \theta_r y_{r0},\ r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0,\ j = 1,\ldots,n \\
& \theta_r \geq 1,\ r = 1,\ldots,s
\end{aligned}
\qquad (14)
$$

As we know, Russell measurement is more discriminating than radial measurement at estimating inefficiencies, so quadratic DEA-WEI models with the Russell measure may be more discriminative, as will be seen later.

4. Empirical studies

In this section, we present two illustrative examples of quadratic DEA-WEI models, including an example of the economic performance of 27 Chinese cities and an example of institutes' evaluation practices in the Chinese Academy of Sciences (CAS) in 2006.

Example 1: We adopt data in this example from Charnes et al. (1989) to evaluate relative efficiencies in the economic performance of Chinese cities. In this example, we use the data of 27 Chinese cities in 1984. There are 9 index indicators as follows:

$$I_{1j} = GIOV / Labor\ ;\ I_{2j} = PT / Labor\ ;\ I_{3j} = RS / Labor$$

$$I_{4j} = GIOV / WF\ ;\ I_{5j} = PT / WF\ ;\ I_{6j} = RS / WF$$

$$I_{7j} = GIOV / INV\ ;\ I_{8j} = PT / INV\ ;\ I_{9j} = RS / INV$$

where (1) Labour denotes the number of labour force, (2) GIOV denotes the annual gross industrial output value, (3) PT denotes the annual total profit payment and tax turnover, (4) RS denotes the total volume of retail sales, (5) WF denotes annual working capital, and (6) INV denotes investment in production capacity enlargement.

The formula used to standardise these indexes is $y_{rj} = y_{rj} \Big/ \max_j y_{rj}, r = 1...9$. In this example, to illustrate our idea, we suppose that indexes I_{1j} and I_{2j} are of key importance for the DMs and thus must both be high for a city to exhibit excellent performance. Therefore, DMs choose to include the quadratic item $I_{1j}*I_{2j}$ in the utility function. The standardised indexes are presented in Table A in the appendix.

Here we use quadratic DEA-WEI models with radial and Russell measures (Model 12, Model 13, respectively) and a DEA-WEI model with Russell measurement (Model 14) to evaluate these 27 Chinese cities. Table A shows the standardised indexes and efficiency scores of 27 Chinese cities in Models 12, 14 and 13.

First, from Table A it is clear that the efficiency scores in Model 13 are often much smaller than those in Model 12, which may demonstrate that the quadratic DEA-WEI with Russell measurement is more discriminative.

Second, comparing the efficiency scores using the quadratic Model 13 with those using the linear Model 14, we find that the efficiency scores of 12 DMUs (Beijing, Tianjin, Wuhan, Guangzhou, Nanjing, Taiyuan, Dalian, Qingdao, Lanzhou, Jinan, Fushun, Kunming) of the 27 Chinese cities increase because the values of the quadratic term ($I_{1j}*I_{2j}$) of these DMUs are relatively high. On the other hand, the efficiency scores of 5 DMUs (Shenyang, Haerbin, Chongqing, Xi'an, Changchun) decrease due to the relatively low values of quadratic term ($I_{1j}*I_{2j}$). Taking Dalian and Changchun as examples, Changchun is more efficient than Dalian in Model 14 but less efficient than Dalian in Model 13. This difference exists because Dalian is far more efficient than Changchun on the quadratic term ($I_{1j}*I_{2j}$; Dalian 0.3430; Changchun 0.1101). See Table A for the details.

To understand those changes more clearly, we look into the changes for the peers on the frontier by comparing the reference points of evaluated DMUs in Model 14 and Model 13 as shown in Table 1 below:

Table 1: The DMUs whose reference points have changed

DMU		Model 14			Model 13 ($I_{1j}*I_{2j}$)		
		Efficiency scores	Rank	Reference points	Efficiency scores	Rank	Reference points
3	Shenyang	0.2341	17	$LAMDA_{22}$=0.4552 $LAMDA_{27}$=0.5448	0.2278	19	$LAMDA_{21}$=1.0000
4	Wuhan	0.2450	15	$LAMDA_{21}$=0.1014 $LAMDA_{22}$=0.8986	0.2579	14	$LAMDA_{21}$=1.0000
5	Guangzhou	0.1787	25	$LAMDA_{21}$=0.3834 $LAMDA_{22}$=0.0935 $LAMDA_{27}$=0.5231	0.1916	25	$LAMDA_{21}$=0.9599 $LAMDA_{27}$=0.0401
6	Haerbin	0.2063	20	$LAMDA_{22}$=1.0000	0.1939	24	$LAMDA_{21}$=1.0000
7	Chongqing	0.2336	18	$LAMDA_{22}$=1.0000	0.2317	17	$LAMDA_{21}$=1.0000
9	Xi'an	0.1736	26	$LAMDA_{22}$=0.2734 $LAMDA_{27}$=0.7266	0.1691	26	$LAMDA_{21}$=1.0000
11	Changchun	0.2464	14	$LAMDA_{22}$=1.0000	0.2282	18	$LAMDA_{21}$=1.0000
14	Qingdao	0.3571	12	$LAMDA_{21}$=0.4368 $LAMDA_{22}$=0.5414	0.3753	12	$LAMDA_{21}$=0.5476 $LAMDA_{22}$=0.4524
16	Jinan	0.2805	13	$LAMDA_{22}$=1.0000	0.2826	13	$LAMDA_{21}$=1.0000

19	Kunming	0.1883	23	$LAMDA_{21}$=0.3448 $LAMDA_{22}$=0.2499 $LAMDA_{27}$=0.4054	0.2001	23	$LAMDA_{21}$=0.6557 $LAMDA_{27}$=0.3443
26	Yichang	0.4678	11	$LAMDA_{22}$=1.0000	0.4547	11	$LAMDA_{21}$=0.1564 $LAMDA_{22}$=0.8436

From Table 1, we observe 11 DMUs (Shenyang, Wuhan, Guangzhou, Haerbin, Chongqing, Xi'an, Changchun, Qingdao, Jinan, Kunming and Yichang) whose peer mixtures have changed due to the nonlinear terms. In Table A, we find that the value of the product term ($I_{1j}*I_{2j}$) of DMU_{21} is the largest (-0.8414), indicating that DMU_{21} gains more weight in the peer mixtures of non-efficient DMUs in Model 13, which may demonstrate that the direction of the performance improvement of non-efficient DMUs shifts due to the quadratic term ($I_{1j}*I_{2j}$), as is clearly observed in Table 1.

Example 2: In this example, we conduct another comparative empirical study, applying the quadratic DEA-WEI model with Russell measurement (Model 13) to evaluate the efficiencies of 12 basic research institutes in the Chinese Academy of Sciences (CAS) in 2006.

Since 2005, CAS headquarter has built up Comprehensive Quality Evaluation (CQE) system for institutes’ evaluation in CAS. The results of the evaluation are expressed as multi-dimensional feedback data, used as the tools to provide a basis of comprehensive analysis and decision-making and to provide institutes with targeted evaluation information and diagnostic comments.

In the framework of CQE, the basic research institutes in CAS are monitored using several quantitative indicators. In this paper, we use the same index indicators proposed in Liu et al. (2011) for 12 basic research institutes in CAS; that is,

y_{1j} =SCI Pub./staff; y_{2j} =SCI Pub./Res.Expen.; y_{3j} =High Pub./staff;

y_{4j} =High Pub./Res.Expen.; y_{5j} =Exter.Fund./staff; y_{6j} =Grad.Enroll./staff.

where (1) SCI Pub. denotes publications including the international papers indexed by the Science Citation Index, (2) High Pub. denotes high-quality papers published in top research journals, (3) Exter. Fund denotes external research funding, (4) Grad. Enroll. denotes graduate students’ enrolment, (5) Staff denotes the number of full-time research staff and (6) Res. Expen. denotes total research expenditures. In this evaluation, the CAS wishes to emphasise the importance of training graduates and obtaining external funding for the sustainable development of its institutes.

Consequently, the CAS has chosen to add a quadratic term $y_{5j}*y_{6j}$ in the utility function to reflect its emphasis on training and external grant, Therefore, we add the quadratic term $y_{5j}*y_{6j}$ in Model 13.

The formula used to standardise these indexes is $y_{rj} = y_{rj} \Big/ \max_j y_{rj}, r = 1 \ldots 6$. See Table 2 for details.

Table 2: Index data of 12 basic research institutes in CAS in 2006[1]

DMU	y_{1j}	y_{2j}	y_{3j}	y_{4j}	y_{5j}	y_{6j}	$y_{5j}*y_{6j}$
$Unit_1$	1	0.8178	0.8330	0.4915	0.4041	1	0.4041
$Unit_2$	0.2618	0.4021	0.4604	0.5102	0.1928	0.5720	0.1103
$Unit_3$	0.5184	0.8756	0.8205	1	0.2416	0.6550	0.1583
$Unit_4$	0.3813	0.3031	0.8738	0.5011	0.3867	0.2723	0.1053
$Unit_5$	0.1494	0.1765	0.3055	0.2605	0.2961	0.2783	0.0824
$Unit_6$	0.3605	0.5608	0.1376	0.1544	0.1122	0.2296	0.0258
$Unit_7$	0.4106	0.4533	0.9135	0.7277	0.3501	0.3928	0.1375
$Unit_8$	0.3268	0.3141	0.5117	0.3549	0.3704	0.1945	0.0720
$Unit_9$	0.5333	0.6120	0.6950	0.5755	0.5333	0.1869	0.0997
$Unit_{10}$	0.3487	0.3149	0.2606	0.1698	0.3956	0.5390	0.2132
$Unit_{11}$	0.8217	1	1	0.8782	0.5670	0.5187	0.2941
$Unit_{12}$	0.5846	0.3852	0.7065	0.3359	1	0.8517	0.8517

Here we use the quadratic DEA-WEI model with Russell measurement (Model 13) to evaluate 12 CAS basic research institutes listed above and compare the results with those from the standard DEA model 13. In addition, we compare the above results with those of Liu et al. (2011) in the table below, in which a matrix DEA-WEI Model (Model 15 in Liu et al. (2011), denoted by Model-LIU in this paper) is used in CAS research evaluation, which combines the six indexes into three new equally important indexes. Model-LIU has been shown by Liu et al. (2011) to be the most suitable DEA model for the evaluation of institutes, as it considers the CAS strategic value judgments. Table 3 below shows the efficiency scores (Column 4, Column 6) of 12 basic research institutes in 2006 evaluated using the quadratic DEA-WEI model with Russell measurement (Model 13) and the DEA-WEI model with Russell measurement (Model 14). The data in Column 2 show evaluation results using Model-LIU.

Table 3: The efficiency scores of 12 basic research institutes in 2006

DMU	Model-LIU		Model 13 ($y_{5j}*y_{6j}$)		Model 14	
	Efficiency Scores	Rank	Efficiency Scores	Rank	Efficiency Scores	Rank
$Unit_1$	1.0000	1	1.0000	1	1.0000	1

[1] Note: These data were derived from these institutes in the period of Jan. 01, 2005 – Dec. 31, 2005

$Unit_2$	0.5013	9	0.3768	9	0.4522	8
$Unit_3$	0.8591	4	1.0000	1	1.0000	1
$Unit_4$	0.6888	7	0.4361	7	0.5037	7
$Unit_5$	0.3498	11	0.2644	11	0.2779	11
$Unit_6$	0.3250	12	0.1408	12	0.2466	12
$Unit_7$	0.7245	6	0.5618	5	0.6330	5
$Unit_8$	0.5136	8	0.2912	10	0.4024	9
$Unit_9$	0.7478	5	0.4395	6	0.5576	6
$Unit_{10}$	0.4791	10	0.3840	8	0.3565	10
$Unit_{11}$	1.0000	1	1.0000	1	1.0000	1
$Unit_{12}$	1.0000	1	1.0000	1	1.0000	1

To understand the rank changes more clearly, we again examine the changes in the peers on the frontier by comparing the reference points of evaluated DMUs in Model 14 and Model 13, as shown in Table 4 below:

Table 4: The efficiency scores and reference points of 12 basic research institutes

DMU	Model 14			Model 13 (y_{5j}*y_{6j})		
	Efficiency Scores	Rank	Reference points	Efficiency Scores	Rank	Reference points
$Unit_1$	1.0000	1	$LAMDA_1$=1.0000	1.0000	1	$LAMDA_1$=1.0000
$Unit_2$	0.4522	8	$LAMDA_1$=0.1106 $LAMDA_{11}$=0.8894	0.3768	9	$LAMDA_{11}$=0.3214 $LAMDA_{12}$=0.6786
$Unit_3$	1.0000	1	$LAMDA_3$=1.0000	1.0000	1	$LAMDA_3$=1.0000
$Unit_4$	0.5037	7	$LAMDA_1$=0.7557 $LAMDA_{11}$=0.2443	0.4361	7	$LAMDA_{11}$=0.5699 $LAMDA_{12}$=0.4301
$Unit_5$	0.2779	11	$LAMDA_{11}$=1.0000	0.2644	11	$LAMDA_{12}$=1.0000
$Unit_6$	0.2466	12	$LAMDA_{11}$=1.0000	0.1408	12	$LAMDA_{11}$=0.2855 $LAMDA_{12}$=0.7145
$Unit_7$	0.6330	5	$LAMDA_1$=0.3891 $LAMDA_{11}$=0.6110	0.5618	5	$LAMDA_{11}$=0.7226 $LAMDA_{12}$=0.2774
$Unit_8$	0.4024	9	$LAMDA_1$=1.0000	0.2912	10	$LAMDA_{11}$=0.0350 $LAMDA_{12}$=0.9650
$Unit_9$	0.5576	6	$LAMDA_1$=0.5838 $LAMDA_{11}$=0.2743 $LAMDA_{12}$=0.1419	0.4395	6	$LAMDA_{11}$=0.4418 $LAMDA_{12}$=0.5582
$Unit_{10}$	0.3565	10	$LAMDA_1$=0.0422 $LAMDA_{11}$=0.9578	0.3840	8	$LAMDA_1$=0.0422 $LAMDA_{11}$=0.9578
$Unit_{11}$	1.0000	1	$LAMDA_{11}$=1.0000	1.0000	1	$LAMDA_{11}$=1.0000
$Unit_{12}$	1.0000	1	$LAMDA_{12}$=1.0000	1.0000	1	$LAMDA_{12}$=1.0000

From efficiency scores in Column 2 and Column 5 in Table 4, we find that $Unit_1$, $Unit_3$, $Unit_{11}$ and $Unit_{12}$ are efficient institutes among 12 basic research institutes in CAS in 2006 when using Model 13 and Model 14. With regard to the 8 inefficient institutes, we compare the efficiency scores using the quadratic Model 13 with those using the linear Model 14 and find that the efficiency scores of 7 institutes ($Unit_2$, $Unit_4$, $Unit_5$, $Unit_6$, $Unit_7$ $Unit_8$ and $Unit_9$) decrease because the values of the quadratic term of these institutes are relatively low. On the other hand, the efficiency score of $Unit_{10}$ increases because the value of the quadratic term of $Unit_{10}$ is the highest

(-0.2132) among those 8 inefficient institutes. In addition, the peer mixtures of $Unit_2$, $Unit_4$, $Unit_5$, $Unit_6$, $Unit_7$, $Unit_8$ and $Unit_9$ are shifted to increase the weighted product term ($y_{5j}*y_{6j}$). Because the value of the quadratic term of $Unit_{12}$ is the largest one (0.8517), DMU_{12} gains more weight in the peer mixture of non-efficient DMUs in Model 13, which may demonstrate that the direction of the performance improvement of non-efficient Units shifts due to the quadratic term ($y_{5j}*y_{6j}$).

Let us note that Model-LIU designed in Liu et al. (2011) is considered to be the most suitable DEA model for CAS performance evaluation. It uses sophisticate matrix preference to reflect the CAS strategic value judgment. It is interesting to see that here a simple quadratic DEA-WEI model (model 13) produces almost identical results with Model-LIU and that the Pearson correlation coefficient is 0.95.

5. Conclusions and Discussions

In this paper, we address a research framework of utility DEA-WEI models and discuss the following subjects: (1) We show the relationship between utility theory and DEA-WEI models, including dual models and theoretical analysis of utility DEA-WEI model, etc. (2) We propose generic quadratic DEA-WEI models, including corresponding models with radial and Russell measurements. (3) We provide illustrative examples to test the features of quadratic DEA-WEI models to reflect the value judgment of DMs, including the illustrative examples of the economic performance evaluation of 27 Chinese cities and institutes' evaluation practices in the Chinese Academy of Sciences (CAS) in 2006. Overall, we conclude that by using the utility DEA-WEI model it is possible to reflect some value judgments that cannot be reflected by simply using the standard DEA models.

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References

[1] Banker, R.D., Charnes, A., Cooper, W.W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science 30 (9), 1078-1092.

[2] Charnes, A., Cooper, W.W., Golany, B., Seiford, L., Stutz, J. (1985). Foundations of data envelopment analysis for Pareto–Koopmans efficient empirical production functions. Journal of Econometrics 30, 91–107.

[3] Charnes, A., Cooper, W.W., Li, S.L. (1989). Using DEA to evaluate Relative

efficiencies in the Economic Performance of Chinese Cities. Socio-economic Planning Science 23, 325-344.

[4] Charnes, A., Cooper W.W., Rhodes E. (1978) Measuring the efficiency of decision making units. European journal of Operation Research 2, 429–444.

[5] Cook, W.D., Seiford, L.M. (2009). Data Envelopment Analysis (DEA)-Thirty years on. European Journal of Operational Research 192, 1-17.

[6] Cooper, W.W., Seiford, L.M., Tone, K. (2000). Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References. Kluwer Academic Publishers, Boston, MA.

[7] Cooper, W.W., Seiford, L.M., Zhu, J. (2004). Handbook on Data Envelopment Analysis. Kluwer Academic Publishers, Boston, MA.

[8] Cooper, W.W., Seiford, L.M., Tone, K. (2006). Introduction to Data Envelopment Analysis and its Uses. Springer Science.

[9] Despotis, D.K. (2005a). Measuring human development via data envelopment analysis: the case of Asia and the Pacific. Omega -The International Journal of Management Science 33, 385-390.

[10] Despotis, D.K. (2005b). A reassessment of the human development index via data envelopment analysis. Journal of the Operational Research Society 56, 969-980.

[11] Fishburn, P.C. (1970). Utility Theory for Decision Making. New York: Wiley.

[12] Forman, E.H., GASS, S.I. (1999). The Analytic Hierarchy Process—An Exposition. Operations Research 49, 469-486.

[13] Halkos, G.E., Salamouris D.S. (2004) Efficiency measurement of the Greek commercial banks with the use of financial ratios: a data envelopment analysis approach. Management Accounting Research 15, 201-224.

[14] Hwang, C.L., Yoon, K. (1981). Multiple Attribute Decision Making: Methods and Applications. Springer-Verlag, Berlin.

[15] Keeney, R.L., Raiffa, H. (1976). Decisions with Multiple Objectives. Wiley, New York.

[16] Kuosmanen, T., Post, T. (2002). Quadratic data envelopment analysis. Journal of the Operational Research Society 53, 1204-1214.

[17] Liu, W.B., Zhang, D.Q., Meng, W., Li, X.X., Xu, F. (2011). A study of DEA models without explicit inputs. Omega-The International Journal of Management Science 39, 472-480.

[18] Lovell, C.A.K., Pastor, J.T. (1999). Radial DEA models without inputs or without outputs. European Journal of Operational Research 188, 46-51.

[19] Manandhar, R., Tang, J.C.S. (2002). The evaluation of bank branch performance using data envelopment analysis: a framework. Journal of High Technology Management Research 13, 1–17.

[20] Martin, J.C., Roman, C. (2006). A Benchmarking Analysis of Spanish Commercial Airports, A Comparison Between SMOP and DEA Ranking Methods. Networks and Spatial Economics 6, 111-134.

[21] Saaty, T.L. (1980). The analytic hierarchy process: planning, priority setting,

resource allocation, McGraw-Hill International Book Company, New York.

[22] Saaty, T.L. (1986). Axiomatic foundation of the analytic hierarchy process. Management Science 32(7), 841–855.

[23] Stanford, R.E. (2004). A Frontier Analysis Approach for Benchmarking Hospital Performance in the Treatment of Acute Myocardial Infarction. Health Care Management Science 7, 145-154.

[24] Sutton P.P. and Green, R.H. (2002). A data envelopment approach to decision analysis. Journal of the Operational Research Society 53, 1215–1224.

[25] Takamura, Y., Tone, K. (2003). A comparative site evaluation study for relocating Japanese government agencies out of Tokyo. Socio-Economic Planning Sciences 37, 85-102.

[26] Von Winterfeldt, D., Edwards, W. (1986). Decisional Analysis and Behavioral Research. Cambridge University Press, New York.

[27] Yang, J.B., Singh, M.G. (1994). An Evidential Reasoning Approach for Multiple Attribute Decision Making with Uncertainty. IEEE Transactions on Systems,Man,a, nd Cybernetics 24(1), 1-18.

[28] Yang, J.B., Xu, D.L. (2002). On the evidential reasoning algorithm for multiattribute decision analysis under uncertainty. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans 32(3), 289-304.

[29] Yang, T., Kuo, C. (2003). A hierarchical AHP/DEA methodology for the facilities layout design problem. European Journal of Operational Research 147, 128-136.

[30] Zeleny, M. (1982). Multiple Criteria Decision Making, McGraw-Hill, New York.

Appendix

Table A: Standardised indexes and efficiency scores of 27 Chinese cities

	DMU	I_{1j}	I_{2j}	I_{3j}	I_{4j}	I_{5j}	I_{6j}	I_{7j}	I_{8j}	I_{9j}	$I_{1j}*I_{2j}$	Model 12		Model 14		Model 13 ($I_{1j}*I_{2j}$)	
												scores	rank	scores	rank	scores	rank
1	Beijing	0.4752	0.6998	0.4207	0.4249	0.6247	0.4169	0.0605	0.1019	0.0417	0.3325	0.8263	13	0.1924	21	0.2058	20
2	Tianjin	0.5819	0.7222	0.3214	0.4629	0.5737	0.2834	0.0760	0.1077	0.0327	0.4202	0.7863	15	0.1870	24	0.2014	22
3	Shenyang	0.4055	0.3472	0.3053	0.3933	0.3363	0.3282	0.1087	0.1063	0.0637	0.1408	0.5102	25	0.2341	17	0.2278	19
4	Wuhan	0.4362	0.5189	0.2975	0.3819	0.4535	0.2886	0.1083	0.1472	0.0575	0.2263	0.6006	21	0.2450	15	0.2579	14
5	Guangzhou	0.4643	0.5137	0.5249	0.3999	0.4418	0.5010	0.0577	0.0730	0.0508	0.2385	0.7384	17	0.1787	25	0.1916	25
6	Haerbin	0.3184	0.3038	0.3073	0.2662	0.2535	0.2847	0.0889	0.0969	0.0668	0.0967	0.4338	27	0.2063	20	0.1939	24
7	Chongqing	0.3894	0.3953	0.3409	0.3547	0.3595	0.3441	0.0969	0.1124	0.0660	0.1539	0.5473	23	0.2336	18	0.2317	17
8	Nanjing	0.4876	0.5850	0.3623	0.4731	0.5667	0.3895	0.0838	0.1149	0.0485	0.2853	0.7338	18	0.2203	19	0.2352	16
9	Xi'an	0.3448	0.3041	0.3149	0.2870	0.2527	0.2905	0.0711	0.0716	0.0505	0.1048	0.4413	26	0.1736	26	0.1691	26
10	Chengdu	0.3970	0.4299	1.0000	0.3582	0.3874	1.0000	0.0905	0.1120	0.1774	0.1707	1.0000	1	1.0000	1	1.0000	1
11	Changchun	0.3131	0.3517	0.4238	0.2805	0.3146	0.4207	0.0920	0.1181	0.0969	0.1101	0.5570	22	0.2464	14	0.2282	18
12	Taiyuan	0.3633	0.4118	0.2391	0.3808	0.4310	0.2778	0.0466	0.0604	0.0239	0.1496	0.5402	24	0.1153	27	0.1251	27
13	Dalian	0.5140	0.6672	0.3989	0.4271	0.5536	0.3673	0.0893	0.1325	0.0539	0.3430	0.7611	16	0.2408	16	0.2561	15
14	Qingdao	0.6148	0.6549	0.4880	0.5853	0.6224	0.5148	0.1469	0.1788	0.0907	0.4026	0.8748	12	0.3571	12	0.3753	12
15	Lanzhou	0.4647	0.7898	0.2987	0.3109	0.5277	0.2215	0.0705	0.1369	0.0353	0.3670	0.8106	14	0.1909	22	0.2036	21
16	Jinan	0.4454	0.4709	0.3844	0.4344	0.4586	0.4155	0.1159	0.1400	0.0778	0.2097	0.6606	19	0.2805	13	0.2826	13
17	Fushun	0.4670	0.8415	0.2529	0.4961	0.8927	0.2977	0.1164	0.2397	0.0491	0.3930	0.9240	11	0.5272	10	0.5492	10
18	Anshan	0.4846	1.0000	0.2648	0.4853	1.0000	0.2939	0.1006	0.2371	0.0428	0.4846	1.0000	1	1.0000	1	1.0000	1
19	Kunming	0.3780	0.5253	0.3521	0.3516	0.4879	0.3629	0.0643	0.1020	0.0466	0.1985	0.6565	20	0.1883	23	0.2001	23
20	Suzhou	0.8260	0.3854	0.5616	0.9752	0.4543	0.7349	0.7710	0.4110	0.4079	0.3183	1.0000	1	1.0000	1	1.0000	1
21	Hangzhou	0.9380	0.8971	0.6815	0.6176	0.5898	0.4973	0.9152	1.0000	0.5174	0.8414	1.0000	1	1.0000	1	1.0000	1
22	Ningbo	0.7183	0.4762	0.5770	1.0000	0.6620	0.8903	1.0000	0.7575	0.6251	0.3420	1.0000	1	1.0000	1	1.0000	1
23	Wuxi	1.0000	0.6294	0.4802	0.9018	0.5668	0.4799	0.8825	0.6346	0.3297	0.6294	1.0000	1	1.0000	1	1.0000	1

24	Changzhou	0.9363	0.6441	0.4863	0.9157	0.6290	0.5271	0.7577	0.5955	0.3062	0.6031	1.0000	1	1.0000	1	1.0000	1
25	Nantong	0.6488	0.2499	0.5870	0.9518	0.3660	0.9544	0.7550	0.3322	0.5315	0.1621	1.0000	1	1.0000	1	1.0000	1
26	Yichang	0.5126	0.2851	0.2248	0.4225	0.2346	0.2054	0.9867	0.6270	0.3368	0.1461	0.9867	10	0.4678	11	0.4547	11
27	Changsha	0.3150	0.2393	0.4332	0.3875	0.2939	0.5906	0.9343	0.8111	1.0000	0.0754	1.0000	1	1.0000	1	1.0000	1