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Global Financial Crisis and Multiscale Systematic Risk: Evidence from Selected European Markets

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Abstract. There is tremendous interest among financial analysts, researchers, policy makers and the general public regarding the impact of the recent United States subprime crisis on the global financial markets ensued by a prolonged and deep global recession. In this paper, we are investigating the impact of the crisis on the stock markets of selected European markets within the framework of Capital Asset Pricing Model. The behavior and performance of the CAPM during the pre-crisis, crisis, and two post-crisis periods provides a convenient and powerful framework for an empirical assessment of the impact of the crisis on the European stock markets. Given the mixed results regarding the inference about the CAPM and betas, and the multi-scale nature of the systematic risk, in this paper, we have employed a recent and powerful method to estimate the systematic risk of CAPM using wavelet analysis to examine the meteor shower effects of the global financial crisis on selected European stock markets. Our results support the CAPM at medium scales, however, the behavior of beta is different for the two groups. Finally, the *VaR* was estimated at different time-scales for the four time-periods. Our results indicate that for all periods the risk is concentrated at higher frequencies (lower scales) of the data. Moreover, the *VaR* was increased for all countries during the crisis and the two post-crisis periods however the difference between the two groups is evident.

Keywords: wavelet analysis, CAPM, value at risk, global financial crisis, systematic risk

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1. Introduction

There is tremendous interest among financial analysts, researchers, policy makers and the general public regarding the impact of the recent United States subprime crisis on the global financial markets ensued by a prolonged and deep global recession. The global financial crisis in 2008-2009 triggered by the subprime crisis led to a progressive deterioration of the investment situation and financial climate around the globe, in general, and European economies in particular.

Although the major financial US institutions, such as New Century Financial, US holding of HSBC, and the world's top five investment banks suffered huge losses in the subprime mortgage and collateralized debt obligation (CDO) transactions by summer 2007, the world financial system observed a period of relative calm with some optimism regarding the outcome of the ongoing crisis until the eight months of 2008. The subprime mortgage crisis eventually erupted when first, major US financial firms, such as Lehman Brothers and AIG and then European financial institutions, such as Northern Rock, Fortis, Dexia, and a number of Icelandic banks showed signs of insolvency.[1] The crisis exposed the inherent vulnerabilities, systemic risks and a catalogue of regulatory failures in the global financial services industries. The crisis then expanded in magnitude, and a full-scale turmoil ensued in financial markets, buffeting many developed and emerging economies. The meltdown of the subprime crisis of 2007 exerted a meteor showers effect across the world's stock market by the fourth quarter of the 2008. In the last quarter of 2008, the stock markets of both developed and emerging economies experienced large decline in prices of securities.

In this paper, we are investigating the impact of the crisis on the stock markets of selected European markets, such as France, Germany, Greece, Italy, Netherlands, Portugal, Spain and the United Kingdom within the framework of Capital Asset Pricing Model (CAPM). The stock exchanges of these countries represent major exchanges within the European Union (EU) in terms of both market capitalisation and trading volume. The behavior and performance of the CAPM during the pre-crisis, crisis, and two post-crisis periods provides a convenient and powerful framework for an empirical assessment of the impact of the crisis on the European stock markets.

Since the seminal contribution made by Sharpe (1964) and Lintner (1965), the notion and significance of the CAPM has spawned considerable research at both theoretical and empirical levels that span almost six decades. According to CAPM, in a perfect capital market, the excess return of a stock or a portfolio of stocks (return over the riskless rate of return) should move in proportion to the market premium (market return over the riskless rate of return). The proportionality factor known as 'beta' (β) captures the 'systematic risk' of the market. Although early research during the 1970s is supportive to the theoretical prediction of the CAPM, later studies during the 1980s and 1990s yield mixed results. Empirical research aimed at testing the validity of the CAPM progressed and expanded through several distinct strands. Gençay *et al.* (2005) succinctly summarized those issues as: the stability of beta over time, borrowing constraints, the impact of structural change and regime switches, the effect of world markets and volatility, non-synchronous data issues, time horizons of investors and the impact of return interval.

Previous studies suggest that empirical validity of CAPM appears to depend on the return interval chosen albeit with mixed results. For example, studies of Kothari

[1] The mortgage financial crisis usually starts from the August 1, 2007 and ends until July 31, 2009.

et al. (1995), and Handa *et al.* (1993) show that βs from annual returns produce stronger relation between beta and average return than βs from monthly return. Frankfurter *et al.* (1994) contend that the mean and variance of β increases from daily returns to yearly returns. A study by Brailsford & Faff (1997) suggests that CAPM is rejected when daily returns data is used, while CAPM is accepted when weekly returns data is used. In contrast, Fama & French (1996) show that annual and monthly βs produce the same inference about β premium.[2]

Given the mixed results regarding the inference about the CAPM and βs, and the multi-scale nature of the systematic risk, in this paper, we have employed a recent and powerful method to estimate the systematic risk of CAPM using wavelet analysis (WA) to examine the meteor shower effects of the global financial crisis on selected European stock markets. WA provides a powerful and appropriate platform to investigate the multi-scale behaviour of beta at different time horizons in frequency domain framework.

Our analysis is also motivated by Fernandez (2006), Gencay *et al.* (2005), Masih *et al.* (2010), and Norsworthy *et al.* (2000), among others, who advocate the incorporation of different time scales using a framework of the WA in the empirical reassessment of CAPM. As Masih *et al.* (2010) contend, the security market consists of thousands of traders and investors with different time horizons and strategies in their mind regarding the investment decision. Owing to different decision-making time horizons and strategies, among investors, the true dynamics of the relationship between stock returns and risk factors is likely to vary depending on the time horizon of the investors. In addition, even if investors agree on a well-diversified portfolio to be the proxy of market portfolio, their perception and measurement of the portfolio risk will not be the same. In this circumstance, financial analysts need to examine the behaviour of systematic risk using a framework of different time scales or horizons in decision making process. Furthermore, Fernandez (2006) recommends the use of wavelet method as a suitable alternative to GARCH and GARCH-in-mean models to study the time-varying beta and time-varying risk premium. The wavelet approach provides a robust result under the conditions of structural break, discontinuity, non-normality and time-varying volatility.

The rest of the paper is organised as follows. Section 2 presents the model with a discussion of the methodology. In Section 3 the dataset is described. A quick introduction of wavelet analysis is provided in Section 4. In Section 5 the computation of the wavelet variance and covariance is presented. The estimation of the Value-at-Risk at different time-scales are presented in Section 6. Finally, in Section 7 we conclude.

2. Estimation of the Capital Asset Pricing Model

The choice of optimal portfolio in investment decision emanates from the consumption-saving-investment decision of representative investor. The choice of the optimal portfolio is a function of both the risk-return possibility curve that is available in the market and the investor's utility function. The OHR is obtained by setting the

[2] Several explanations are offered for the interval bias of systematic risk, such as infrequent trading, delays in information processing, increase of standard error of the beta as the return interval is lengthened, disproportionate move of covariance relative to the variance estimate in the measurement of beta, and seasonality. Masih *et al.* (2010) furnished a good discussion on the issue.

investor's subjective marginal rate of substitution (MRS) between risk and return equal to the slope of the risk-return possibility curve.

Both life-cycle and permanent income hypotheses utilize and inter-temporal optimization framework where our finitely-lived representative household is faced with the following problem of maximization[3]:

$$E_t \sum_{t=1}^{\infty} \left(1+\delta\right)^{-(s-t)} u\left(c_s\right) \tag{1}$$

subject to

$$E_t w_s = E_t \left[\left(1+r\right) w_{t-1} + y_s - c_s\right], \quad s = t, t+1, \ldots, \infty \tag{2}$$

where E_t denotes mathematical expectations, conditional on all information available at *t*; δ signifies rate of subjective time preference; $u(\cdot)$ implies single-period utility function; variables *c*, *w*, *r* and *y* denote constant-price consumer expenditure, real value of non-human wealth, real rate of interest, and real labor income, respectively.[4]

Solving the first-order condition for a constrained optimization problem from the corresponding lagrangean function, and after certain manipulation, we may derive the following stochastic Euler equation (see Gausden & Whitfield (2000)):

$$E_t u'\left(c_{t+1}\left(1+\delta\right) / \left(1+r\right)\right) E_t u'\left(c_t\right) \tag{3}$$

Equation (3) asserts that optimal consumption decision requires marginal utilities of adjacent periods to be proportional to one another. Assuming that consumer can allocate his wealth among *n*-1 risky assets with an $r_{i,t}$ rate of return and a riskless asset with a rate of return $r_{f,t}$, the resulting first order condition may be rewritten as:

$$E\left[u'\left(c_{t+1}\right)\right] E\left[r_{i,t} - r_{f,t}\right] + Cov\left[u'\left(c_{t+1}\right), r_{i,t}\right] = 0 \tag{4}$$

at equilibrium, the return from asset *i* must satisfy the following equation

$$E\left[r_{i,t}\right] = r_{f,t} - \frac{Cov\left[u'\left(c_{t+1}\right), r_{i,t}\right]}{E\left[u'\left(c_{t+1}\right)\right]} \tag{5}$$

If we further assume that the return of a benchmark market portfolio (proixed by market index) is inversely related with the marginal utility of consumption in the next period, so that:

$$u'\left(c_{t+1}\right) = -\gamma r_{m,t} \tag{6}$$

for some positive γ.

[3] This part draws extensively from Gencay *et al* (2005) and Gausden and Winfield (2000).

[4] In the model specification, variables are expressed in real terms as we assume that consumer does not suffer from money illusion.

It follows that $Cov\left[u'\left(c_{t+1}\right), r_{i,t}\right] = \gamma Cov\left[r_{i,t}, r_{m,t}\right]$ and allows us to rewrite equation (5) after certain manipulation as[5]:

$$E\left[r_{i,t}\right] = r_{f,t} + \frac{Cov\left[r_{i,t}, r_{m,t}\right]}{\sigma_m^2}\left[E\left[r_{m,t}\right] - r_{f,t}\right] \tag{7}$$

Equation (7) in estimable form yields the widely presented testing equation for the Capital Asset Pricing Model:

$$r_{i,t} - r_{f,t} = \beta\left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t} \tag{8}$$

From equation (8) the variance of the return on asset i is estimated by:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2 \tag{9}$$

3. Data description

In this section we will focus on estimating the CAPM at different time-scales. We are investigating the impact of the crisis on the stock markets of eight European markets. The selected markets are distinguished in two groups. The first group consists of four countries that at the moment they face a lot of uncertainty and they are under a rescue program and under the supervision of the International Monetary Fund (IMF) and/or the European Central Bank (ECB). These countries are: Portugal, Italy, Greece and Spain. On the other hand, the second group consists of four countries where traditionally their economies are considered strong and stable. These countries are: Germany, Netherlands, UK and France. The selected countries represent major exchanges within the EU in terms of both market capitalisation and trading volume.

Our data set includes the daily values of the main stock index in each country from 01/06/2005 to 10/09/2012 as well as the daily stock prices of the stocks that constitute each index. The eight indices are the following: AEX25 from Netherlands, FTSE/ATHEX 20 from Greece, CAC 40 from France, DAX 30 from Germany, FTSE 100 from UK, IBEX 35 from Spain, MIB 40 from Italy and PSI 20 from Portugal.

In our analysis, only the stocks that survive for the whole sample period are analyzed. Hence, this results to 23 stocks from Netherlands, 19 from Greece, 37 from France, 87 from UK, 26 from Germany, 32 from Italy and 15 from Portugal.

As it is already mentioned, in this study we estimate the beta of a risky asset at different time-frequencies. Moreover, in this study we repeat our analysis in different time-periods in order to obtain an estimate of the impact of the crisis in the systematic risk in these markets.

More precisely, our data set is split in different 4 periods. The first data set corresponds to the pre-crisis period and includes daily stock values from 01/06/2005-31/07/2007. The second data set represents the crisis period and it is the dataset ranges from 01/08/2007-30/09/2009. The third data set represent the post-crisis period in USA and the beginning of the crisis in Europe, 01/10/2009-30/11/2011. Finally, there is fourth data set from 01/12/2011-10/09/2012 that represents the current situation in Europe. Finally, our analysis is repeated in the whole time period, from

[5] For a detailed derivation, see Gencay *et al.* (2005).

01/06/2005-10/09/2012, in order to have a complete evaluations and empirical assessment of the impact of the crisis on the European stock markets

Daily return series for each stock as well as the market index were collected from each stock market. This results to 564 values for the first sample, 566 for the second, 565 for the third, 203 for the fourth resulting to 1898 values. The daily stock returns $r_{i,t}$ are calculated using the log-returns formula:

$$r_{i,t} = \ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) \tag{10}$$

where $S_{i,t}$ is the price of the stock i at day t. Similar the return of the market index is estimated by:

$$r_{m,t} = \ln\left(\frac{S_{m,t}}{S_{m,t-1}}\right) \tag{11}$$

where $S_{m,t}$ is the value of the index at day t.

4. Wavelet Analysis

The attempt to understand complicated time-series by breaking them into basic pieces that are easier to understand is one of the central themes in Fourier analysis. In the framework of Fourier series, complicated periodic functions are written as the sum of simple waves mathematically represented by sines and cosines. More precisely, Fourier Transform (FT) breaks down a signal into a linear combination of constituent sinusoids of different frequencies; hence the FT is decomposition on a frequency by frequency basis.

Fourier analysis performs excellent in the analysis of periodic signals. However, in transforming to the frequency domain, time information is lost. When looking at a FT of a signal, it is impossible to tell when a particular event took place. This is a serious drawback if the signal properties change a lot over time, i.e., if they contain nonstationary or transitory characteristics: drift, trends, abrupt changes, or beginnings and ends of events. These characteristics are often the most important part of a time-series, and FT is not suited to detecting them, Zapranis & Alexandridis (2006).

Trying to overcome the problems from the classical FT, Gabor applied the FT in small time “windows”, Mallat (1999). Window Fourier Transform (WFT) or Short-Time Fourier Transform (STFT) is an extension of the FT where a symmetric window is used to localize signals in time. The STFT represents a sort of compromise between the time- and frequency-based views of a signal.

Fourier analysis is inefficient in dealing with local behavior of signals. On the other hand Windowed Fourier Analysis is an inaccurate and inefficient tool for analyzing regular time behavior that is either very rapid or very slow relatively to the size of the window, Kaiser (1994).

In Grossmann & Morlet (1984) instead of the constant window used in WFT, waveforms of shorter duration at higher frequencies and waveforms of longer duration at lower frequencies were used as windows. This method is called WA. WA is an extension of the FT. The fundamental idea behind wavelets is to analyze according to scale. Low scale represents high frequency while high scales represent low frequency. The wavelet transform (WT) not only is localized in both time and frequency but also overcomes the fixed time-frequency partitioning. This means that the WT has good frequency resolution for low-frequency events and good time resolution for high-frequency events. Hence, the WT can be used to analyze time series that contain nonstationary dynamics at many different frequencies, Daubechies (1992).

WA has proved to be a valuable tool for analyzing a wide range of time-series and has already been used with success in time-series analysis, image processing, signal de-noising, density estimation, signal and image compression and time-scale decomposition. Wavelet techniques are being used in finance, for detecting the properties of quick variation of values.WA is often regarded as a “microscope” in mathematics, Cao *et al.* (1995), and it is a powerful tool for representing nonlinearities, Fang & Chow (2006).

The daily return time-series are represented by local information such as frequency, duration, intensity and time-position and by global information such as the mean states over different time periods. Both global and local information is needed for a correct analysis of the daily return time-series.

In addition, wavelets have the ability to decompose a signal or a time-series in different levels. As a result, this decomposition brings out the structure of the underlying signal as well as trends, periodicities, singularities or jumps that cannot be observed originally.

WA decomposes a general function or signal into a series of (orthogonal) basis functions, called wavelets, with different frequency and time locations. More precisely, WA decomposes time-series and images into component waves of varying durations, called wavelets. These wavelets are localized variations of a signal, Walker (2008). As illustrated in Donoho & Johnstone (1994) the wavelet approach is very flexible in handling very irregular data series. Ramsey (1999) also comments that WA has the ability to represent highly complex structures without knowing the underlying functional form, which is of great benefit in economic and financial research. A particular feature of the analyzed signal can be identified with the positions of the wavelets into which it is decomposed. Recently an increasing number of studies apply WA in order to analyze financial time series, Alexandridis & Zapranis (2012), Fernandez (2006), Fernandez (2005), Gençay *et al.* (2003), (2005), He *et al.* (2012), In & Kim (2006a), In & Kim (2006b), (2007), Kim & In (2005), (2007), Maharaj *et al.* (2011), Masih *et al.* (2010), Norsworthy *et al.* (2000), Rabeh & Mohamed (2011), Ramsey (1999), Rua & Nunes (2012), Yousefi *et al.* (2005), Zapranis & Alexandridis (2008), (2009), (2011).

A wavelet ψ is a waveform of effectively limited duration that has an average value of zero. The WA procedure adopts a particular wavelet function, called a *mother wavelet*. A *wavelet family* is a set of orthogonal basis functions generated by dilation and translation of a compactly supported *scaling function*, ϕ (or *father wavelet*), and a *wavelet function*, ψ (or mother wavelet).

The father wavelets φ and mother wavelets ψ satisfy:

$$\int \varphi(t)dt = 1 \tag{12}$$

$$\int \psi(t)dt = 0 \tag{13}$$

The wavelet family consists of *wavelet children* which are dilated and translated forms of a mother wavelet:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \tag{14}$$

where, a is the *scale* or *dilation* parameter and b is the *shift* or *translation* parameter.

The value of the scale parameter determines the level of stretch or compression of the wavelet. The term $1/\sqrt{a}$ normalizes $\|\psi_{a,b}(t)\| = 1$. In most cases, we will limit our choice of a and b values by using a discrete set, because calculating wavelet coefficients at every possible scale is computationally intensive. Temporal analysis is performed with a contracted high-frequency version of the mother wavelet, while frequency analysis is performed with a dilated, low-frequency version of the same mother wavelet. In other words, while Fourier analysis consists of breaking up a signal into sine waves of various frequencies, WA is the breaking up of a signal into shifted and scaled versions of the original (or *mother*) wavelet, Misiti *et al.* (2009).

4.1. Maximal Overlap Discrete Wavelet Transform

In this study we use the Maximal Overlap Discrete Wavelet Transform (MODWT). The MODWT is an extension of the classical discrete wavelet transform (DWT) that has many desirable properties, Gençay *et al.* (2002), Percival & Walden (2000), In & Kim (2007). First, the MODWT can handle any sample size of the data. Second, the MODWT does not suffer from sensitivity to the choice of a starting point for a time series. More precisely, in MODWT both wavelet and scaling coefficients are invariant to circularly shifting the original time series. Third, the detains and smooth coefficients of a MODWT MRA are associated with zero phase filters. Hence, it is possible to align features in the MRA with the original time-series. Finally, the wavelet variance estimator is asymptotically more efficient that the same estimator based on the DWT. However, on the other hand the MODWT is more computational expensive than the classical DWT.

So far, the MODWT was successfully applied in many studies in finance. In In & Kim (2006a) and In & Kim (2006b) the MODWT was applied in the estimation of the hedge ratio while it was used in the estimation of the International CAPM in In & Kim (2007). The estimation of the systematic risk was studied in Gençay *et al.* (2002), Gençay *et al.* (2005), Masih *et al.* (2010) and Rabeh & Mohamed (2011). In Maharaj *et al.* (2011) a comparison is made of developed and emerging equity market return volatility at different time scales. In Kim & In (2007) the relationship between changes in stock prices and bond yields in the G7 countries was studied. Finally, in Kim & In (2005) the relationship between stock returns and inflation is examined using the MODWT.

In this study the LA8 (Least Asymmetric of length 8) wavelet transform filter is used. Our analysis is performed in 5 levels of the decomposition and the reflection method was used for the boundary conditions.

A time-series $f(t)$ can be written as a linear combination of wavelet functions as follows:

$$f(t) \approx \sum_k s_{J,k}\varphi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \quad (15)$$

where J is the number of scales and k indicates the k^{th} coefficient. Following the notations from Fernandez (2006) the wavelet transform coefficients $s_{J,k}, d_{J,k}, \ldots, d_{1,k}$ can be approximated by the following integrals:

$$s_{J,k} \approx \int \varphi_{J,k}(t) f(t) dt, \;\; j = 1,2,\ldots,J \quad (16)$$

$$d_{J,k} \approx \int \psi_{J,k}(t) f(t) dt, \;\; j = 1,2,\ldots,J \quad (17)$$

The functions $\varphi_{j,k}$ and $\psi_{j,k}$ are the approximating wavelet functions and are given by:

$$\varphi_{j,k} = 2^{-j/2}\varphi\left(\frac{t-2^j k}{2^j}\right) \quad (18)$$

$$\psi_{j,k} = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \quad (19)$$

By setting

$$S_J(t) = \sum_k s_{J,k}(t)\varphi_{J,k}(t) \quad (20)$$

$$D_J(t) = \sum_k d_{J,k}(t)\psi_{J,k}(t) \quad (21)$$

the original time-series can be reconstructed:

$$f(t) \approx S_J(t) + D_J(t) + D_{J-1}(t) + \cdots + D_1(t) \quad (22)$$

This reconstruction is known as Multi-resolution analysis (MRA). MRA is applied in order to reconstruct the original time-series from the wavelet and scaling coefficients. The elements of S_J are related to the scaling coefficients at the maximal scale and therefore represent the smooth components of $f(t)$. The elements of D_j are the detail (or rough) coefficients of $f(t)$ at scale j.

5. Computation of Wavelet Variance and Covariance

In order to estimate the wavelet-variance, the variance must be split it in various parts, each one representing the variance at each scale. This wavelet-variance analysis shows us which scales are contributing significantly to the overall variability of the time-series, Percival & Walden (2000). Suppose a stationary process X, then the variance σ_X^2 is given by:

$$\sigma_X^2 = \sum_{j=1}^{\infty} \upsilon_x^2\left(\tau_j\right) \tag{23}$$

where $\upsilon_x^2\left(\tau_j\right)$ is the wavelet variance for scale τ_j. As it is mentioned in Fernandez (2006) and Masih *et al.* (2010), equation (23) is analogous to the relationship between the variance of a stationary process and its spectral density function.

$$\sigma_X^2 = \int_{-1/2}^{1/2} S_X\left(f\right)df \tag{24}$$

Following Gençay *et al.* (2002), an unbiased estimator of the wavelet variance is given by:

$$\hat{\upsilon}_X^2\left(\tau_j\right) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^2 \tag{25}$$

where $\tilde{d}_{j,t}^2$ is the MODWT wavelet coefficients at scale τ_j, n is the sample size, $L_j = \left(2^j - 1\right)\left(L-1\right)+1$ is the length of the scale τ_j wavelet filter and $\tilde{N}_j = N - L_j + 1$ is the number of the MODWT coefficients unaffected by the boundary and L is the width of the wavelet filter.

Similarly, an unbiased estimator of the wavelet-covariance between two time-series X and Y is given by:

$$\hat{\upsilon}_{XY}^2\left(\tau_j\right) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^{(X)}\tilde{d}_{j,t}^{(Y)} \tag{26}$$

Since, the wavelet variance and wavelet covariance are known, under the CAPM the wavelet beta estimator for asset i at scale j is defined as:

$$\hat{\beta}_i\left(\tau_j\right) = \frac{\hat{\upsilon}_{R_iR_m}^2\left(\tau_j\right)}{\hat{\upsilon}_{R_m}^2\left(\tau_j\right)} \tag{27}$$

where $\hat{\upsilon}_{R_iR_m}^2\left(\tau_j\right)$ is the wavelet covariance of asset i and the market portfolio at scale j, and $\hat{\upsilon}_{R_m}^2\left(\tau_j\right)$ is the wavelet variance of the market portfolio at scale j. Following Fernandez (2005) and Masih *et al.* (2010) the wavelet R^2 estimator for asset i at scale j is given by:

$$R_i^2\left(\tau_j\right) = \hat{\beta}_i^2\left(\tau_j\right)\frac{\hat{\upsilon}_{R_m}^2\left(\tau_j\right)}{\hat{\upsilon}_{R_i}^2\left(\tau_j\right)} \tag{28}$$

In Table 1 to Table 5 the beta and R^2 at each scale j are presented. Furthermore, in the last two columns of Table 1 to Table 5 the beta and R^2 from the raw data are presented. The average beta and the average R^2 at each scale is presented for each country.

In Table 1 the results for the pre-crisis period are presented. It is clear that the linear relationship between an individual stock and the market portfolio becomes stronger as the scale increases. However, in most cases a slight decrease is observed at scale 5. In other words the maximum values of beta and R^2 are observed in scales 3 and 4. Our results are in line with Gençay et al. (2005), Masih et al. (2010) and Fernandez (2006). The results for all countries are similar. The mean betas in each scale is around 1 and increases in higher scales. Similarly, the R^2 increases as the scale increases. The R^2 ranges from 0.12 at scale 1 in Portugal to 0.47 in scale 5 in Spain. The lower values of R^2 are observed in Portugal, Netherlands and Greece while the highest values are observed in Spain, Germany and France.

Our analysis in Table 2 reflects the results during the in-crisis period. The results are similar as in the pre-crisis period. However, a closer inspection of Table 2 reveals that both betas and R^2 are increased for every country. The lower values of R^2 were observed in Germany and UK, 0.37 and 0.38 respectively, while the R^2 for the remaining countries are over 0.40 and up to 0.53 in Spain. In addition, in contrast to the remaining countries, the beta in Greece decreases 0.83 at scale 1 to 0.81 at scale 4 and then goes up to 0.88 at scale 5. For the remaining countries the maximum beta is observed at scales 3 and 4 while the minimum, usually, at scale 1.

Next, we focus on Table 3 where the results during the post-crisis period are presented. This period reflects the end of the American crisis and the beginning of the European crisis. Our results indicate that the betas in almost every country are almost 1 for each scale although a slight increase is observed at higher scales. The R^2 was increased in each country and it is 0.51, 0.47, 0.61,0.50, 0.43, 0.60, 0.56, 0.48 in Netherlands, Greece, France, Germany, UK, Spain, Italy and Portugal respectively. Again the maximum betas were observed at scales 3 and 4 while the minimum at scale 1. For all countries the R^2 increases from scale 1 to scale 3 and then starts to decrease until scale 5.

Table 4 presents the results of our analysis in the last time-period which reflects the current situation in Europe. The results are similar as in Table 3. However, a slight increase is observed in the beta values of Netherlands, Greece, France, Spain and Italy. On the other hand, the betas in Germany, UK and Portugal remained the almost the same. On the contrary, the R^2 was reduced for every country with an exception of Portugal. The maximum betas were observed at scales 3 for France, UK, Spain, and Italy; at scale 4 for Greece and Portugal; at scale 5 for Netherlands and Germany. For all countries the R^2 increases as we move from lower scales to mid-scales and then it decreases at higher scales.

Finally, in Table 5 the beta and R^2 is estimated for each country for the whole sample. Our results indicate that beta increases at higher scales. The beta from the raw data ranges from 0.88 in Greece to 1.06 in France while the R^2 from 0.36 in Portugal to 0.53 in Spain. Again the maximum beta were observed at scales 3 and 4 with an exception of Greece, Spain, Italy and Portugal where the beta was maximized at scale 5. A similar behavior is observed for the R^2.

6. Value-at-Risk at different time-scales

In this section we focus on the estimation of the Value-at-Risk (VaR). VaR is a very popular measure that describes the market risk. VaR measures the amount that an investor can lose with a given probability over a certain time horizon.

From the CAPM we have that the variance of the excess return of stock i and the covariance of the returns of stocks i and j is given by:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2, \quad i = 1, 2, ..., k \tag{29}$$

and

$$\sigma_{i,j} = \beta_i \beta_j \sigma_m^2, \quad i = 1, 2, ..., k, \; i \neq j \tag{30}$$

where $E\left(\varepsilon_i^2\right) = \sigma_{\varepsilon_i}^2$ and $E\left(\varepsilon_i \varepsilon_j\right) = 0, \; \forall i \neq j$.

Following Fernandez (2005) and Fernandez (2006) the variance-covariance of the excess returns can be written in a matrix form as:

$$\Omega = \mathrm{BB}\sigma_m^2 + \mathrm{E} \tag{31}$$

where

$$\mathrm{B} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix} \tag{32}$$

and

$$\mathrm{E} = \begin{pmatrix} \sigma_{\varepsilon_1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{\varepsilon_k}^2 \end{pmatrix} \tag{33}$$

For simplicity we assume an equally weighted portfolio of k assets where ω is vector that contains the portfolio weights, i.e. a $k \times 1$ vector which each element is $1/k$. Hence, the $(1-a)\%$ Value-at-Risk, $VaR(a)$, for a portfolio with initial value V_0 is given by:

$$VaR(a) = V_0 \Phi^{-1}\left(1-a\right)\sqrt{\omega'\left(\mathrm{BB}'\sigma_m^2 + \mathrm{E}\right)\omega} \tag{34}$$

or similarly for an equally weighted portfolio by:

$$VaR(a) = V_0 \Phi^{-1}(1-a) \sqrt{\sigma_m^2 \left(\sum_{i=1}^{k} \beta_i / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2} \tag{35}$$

where $\Phi^{-1}(1-a)$ is the inverse cumulative distribution function of the standard normal distribution at the probability level $1-a$. In our analysis we set $a = 0.05$.

The above equation can be used in order to estimate the $VaR(a)$ at different time-scales. More precisely, if only the variance and beta components of the *j*-scale are used, then the $VaR(a)$ at *j*-scale can be estimated. Hence, we have that the $VaR(a)$ at *j*-scale is given by:

$$VaR_{\tau_j}(a) = V_0 \Phi^{-1}(1-a) \sqrt{\sigma_m^2(\tau_j) \left(\sum_{i=1}^{k} \beta_i(\tau_j) / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2(\tau_j)} \tag{36}$$

and the noise variance at each scale can be estimated by rearranging (29) which results to:

$$\sigma_\varepsilon^2(\tau_j) = \sigma_i^2(\tau_j) - \beta_i^2(\tau_j) \sigma_m^2(\tau_j) \tag{37}$$

Hence, an approximation of the $VaR(a)$ above all scales is given by:

$$VaR(a) \approx V_0 \Phi^{-1}(1-a) \sqrt{\sum_{j=1}^{J-1} \left(\sigma_m^2(\tau_j) \left(\sum_{i=1}^{k} \beta_i(\tau_j) / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2(\tau_j) \right)} \tag{38}$$

The difference between equations (36) and (38) should be negligible, Fernandez (2006). Hence, from (36) and (38) we have that

$$\frac{\sum_{j=1}^{J-1} \left(\sigma_m^2(\tau_j) \left(\sum_{i=1}^{k} \beta_i(\tau_j) / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2(\tau_j) \right)}{\sigma_m^2 \left(\sum_{i=1}^{k} \beta_i / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2} \approx 1 \tag{39}$$

Hence, the ratio

$$\frac{\sigma_m^2(\tau_j) \left(\sum_{i=1}^{k} \beta_i(\tau_j) / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2(\tau_j)}{\sigma_m^2 \left(\sum_{i=1}^{k} \beta_i / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_{\varepsilon_i}^2} \tag{40}$$

is an estimate of the contribution of scale *j* to total Value-at-Risk of an equally weighted portfolio, Fernandez (2006), Masih *et al.* (2010).

In Table 6 to Table 9 the $VaR(a)$ at different time scales for an equally weighted portfolio is presented for the four different time-periods while in Table 10 the $VaR(a)$ at different time scales for an equally weighted portfolio is presented for the whole time period. The initial value of the portfolio was 1 unit of the specific market's currency invested in 1-day horizon at the 95% confidence interval.

As we can see from Table 6 to Table 10 the $VaR(a)$ declines monotonically as we move to higher scales. In other word, the $VaR(a)$ is higher at lower scales. Similarly, the contribution of the $VaR(a)$ is higher at lower scales and decreases as we move to higher scales. In other words, potential losses of the portfolio is higher when we focus on lower scales. Finally, we can observer that the total $VaR(a)$ estimated from the raw data and the total $VaR(a)$ estimated from the recomposed data are very close.

Our results are similar to the ones presented in Fernandez (2006) and Masih *et al.* (2010) and suggest that risk is concentrated at the lower scale of the data. In all time samples, Scale 1 contributes with more that 41% to the total $VaR(a)$ while in some cases reaches up to 55%.

A closer inspection of Table 6 reveals that the total $VaR(a)$ is relatively low for all countries. More precisely, the lower values are observed in Portugal and Italy, 0.009 and 0.012 respectively. The higher value is estimated for Greece and it is 0.0163. However, these values are not significantly different than the ones observed in France and Germany, 0.0145 and 0.0137 respectively.

In Table 7 the $VaR(a)$ is estimated during the crisis time-period. A closer inspection of Table 7 reveals that the $VaR(a)$ was grown threefold almost for every country. Again, the lower values were observed in Portugal and Italy, 0.0257 and 0.0280 respectively, while the higher values were observed in France and Netherlands, 0.0364 and 0.0351. Our results, from Table 6 and Table 7 indicate that the countries that are now in crisis and under a rescue plan were performing similar or in some cases better that the countries with stronger and more stable economies.

In Table 8 the $VaR(a)$ estimated in the post-crisis can be found. This period is also the same as when the European crisis started. The effects can be found in the estimation of $VaR(a)$ in Greece, which was further increased. On the contrary the $VaR(a)$ from the remaining countries were decreased or remained stable.

The results of our analysis between 01/12/2011-10/09/2012 are presented in Table 9. During this time Greece was in deep crisis while Spain and Italy were under a rescue plan. This can be reflected from the estimated $VaR(a)$ in each country. For Greece the $VaR(a)$ is 0.0507 while for Spain and Italy is 0.0306 and 0.0324. On the other hand the $VaR(a)$ for Germany, UK, Netherlands and France is 0.02, 0.0161, 0.0223 and 0.0262 respectively. Surprisingly, the estimated $VaR(a)$ for Portugal, another country with financial problems, is 0.0211.

Finally, in Table 10 the $VaR(a)$ is estimated for the whole time-period, from 01/06/2005 to 10/09/2012. Our results indicate that the $VaR(a)$ is similar for all countries and around 0.022 with an exception of Greece which is 0.0338.

7. Conclusions

In this study the we examined how the global financial crisis affected the systematic crisis in selected European markets. Furthermore, a multi-scale analysis of the systematic risk was presented. More precisely, in this study the CAPM was estimated in at different time-scale for 8 European countries - four countries heavily affected by the financial crisis and four countries that traditionally their economies are considered strong and stable. Furthermore, our analysis was repeated in four time-periods defined as pre-crisis, crisis and two post crisis periods.

Our results indicate that in most cases the maximum betas at observed at scales 3 and 4 supporting the CAPM at medium time horizons. Moreover, in our analysis, the results from the two post-crisis samples, indicate that changes of both the betas and R^2 varies between the two groups of the European markets.

Finally, the VaR was estimated at different time-scales for the four time-periods. Our results indicate that for all periods the risk is concentrated at higher frequencies (lower scales) of the data. Moreover, the VaR was increased for all countries during the crisis and the two post-crisis periods however the difference between the two groups are evident. The VaR was stable in last period for Germany, Netherlands, France and UK while it was significantly increased for Greece, Spain Portugal and Italy.

Table 1. Beta and R^2 computed from recomposed crystals of each index. Pre Crisis Period

	Beta at each scale					R^2 at each scale					Raw Data	
	1	2	3	4	5	1	2	3	4	5	Beta	R^2
AEX												
Mean	0.93	0.95	0.95	0.97	0.95	0.29	0.28	0.34	0.32	0.28	0.94	0.29
SD	0.23	0.25	0.25	0.26	0.37	0.16	0.16	0.16	0.16	0.16	0.22	0.15
Skew.	0.23	0.22	0.26	0.69	0.72	1.60	1.45	1.34	0.77	0.89	0.17	1.72
Kurtosis	2.71	1.76	2.10	2.52	3.61	4.42	4.34	4.26	3.30	3.92	2.28	4.97
ATHEX												
Mean	0.89	0.82	0.91	0.98	0.87	0.28	0.28	0.32	0.35	0.34	0.88	0.29
SD	0.32	0.26	0.27	0.31	0.50	0.18	0.17	0.18	0.18	0.24	0.28	0.17
Skew.	-0.26	0.47	0.06	0.08	-0.08	0.75	0.91	0.62	0.62	0.41	0.01	0.85
Kurtosis	2.55	3.71	2.26	2.17	1.95	3.46	3.46	3.00	2.96	2.52	2.45	3.51
CAC 40												
Mean	1.01	1.00	1.00	1.06	1.04	0.39	0.36	0.40	0.39	0.33	1.01	0.38
SD	0.24	0.22	0.26	0.32	0.39	0.15	0.14	0.16	0.16	0.16	0.23	0.14
Skew.	0.29	-0.17	-0.20	0.87	0.16	0.66	0.36	0.40	0.38	0.03	0.02	0.68
Kurtosis	2.68	2.73	2.43	3.86	2.82	2.71	2.58	2.39	2.00	2.10	2.54	2.67
DAX 30												
Mean	0.84	0.87	0.92	0.98	0.97	0.31	0.33	0.41	0.38	0.36	0.87	0.34
SD	0.18	0.16	0.25	0.27	0.33	0.12	0.13	0.16	0.14	0.18	0.18	0.12
Skew.	0.22	-0.03	0.04	0.54	0.67	0.82	0.98	0.18	0.30	0.35	0.22	0.82
Kurtosis	2.32	2.15	2.32	2.80	3.29	3.70	4.24	2.45	2.31	2.19	2.09	3.52
FTSE 100												
Mean	1.03	1.03	1.11	1.10	1.02	0.31	0.29	0.35	0.33	0.29	1.05	0.31
SD	0.38	0.34	0.44	0.45	0.54	0.12	0.11	0.14	0.15	0.17	0.37	0.11
Skew.	1.27	1.15	0.97	0.48	1.03	0.30	0.26	0.02	0.07	0.34	1.16	0.19
Kurtosis	4.65	4.18	3.79	2.71	3.89	2.36	2.42	2.11	2.51	2.23	4.29	2.19
IBEX 35												
Mean	1.02	1.04	1.07	1.17	1.11	0.41	0.39	0.42	0.45	0.47	1.05	0.41
SD	0.23	0.24	0.34	0.41	0.39	0.16	0.15	0.16	0.15	0.17	0.25	0.15
Skew.	0.41	0.54	0.34	1.06	0.84	0.63	0.53	-0.16	-0.09	-0.05	0.55	0.56
Kurtosis	2.24	2.73	3.09	3.64	2.77	2.78	3.00	2.64	2.89	2.39	2.67	2.84
MIB												
Mean	0.89	0.92	0.95	1.00	1.00	0.31	0.26	0.31	0.29	0.34	0.92	0.30
SD	0.31	0.27	0.33	0.42	0.33	0.15	0.12	0.15	0.14	0.15	0.29	0.13
Skew.	-0.22	-0.54	0.01	0.89	0.17	0.29	0.25	0.54	0.19	-0.10	-0.34	0.30
Kurtosis	2.93	2.79	2.29	4.51	2.39	2.73	2.62	3.15	1.84	2.19	2.53	2.84
PSI-20												
Mean	0.73	0.73	0.81	0.96	0.94	0.12	0.12	0.18	0.26	0.21	0.78	0.14
SD	0.41	0.49	0.38	0.44	0.61	0.11	0.12	0.15	0.15	0.16	0.38	0.12
Skew.	-0.12	0.08	-0.37	0.70	0.36	1.14	1.18	1.04	-0.05	0.11	-0.26	1.04
Kurtosis	2.59	2.00	3.91	2.44	2.20	2.80	2.99	3.23	1.63	1.40	2.67	2.65

Table 2. Beta and R^2 computed from recomposed crystals of each index. In Crisis Period

	Beta at each scale					R^2 at each scale					Raw Data	
	1	2	3	4	5	1	2	3	4	5	Beta	R^2
AEX												
Mean	0.93	0.95	1.05	1.09	1.02	0.45	0.46	0.44	0.41	0.48	0.96	0.45
SD	0.37	0.42	0.50	0.57	0.43	0.13	0.15	0.15	0.17	0.13	0.40	0.13
Skew.	1.21	1.01	0.52	0.42	0.76	0.44	-0.18	-0.62	-0.45	-0.25	1.01	0.10
Kurtosis	3.76	3.62	2.73	2.26	2.54	2.75	2.68	2.65	2.04	1.79	3.42	2.71
ATHEX												
Mean	0.83	0.82	0.81	0.81	0.88	0.42	0.41	0.39	0.45	0.44	0.83	0.42
SD	0.31	0.32	0.35	0.35	0.36	0.20	0.21	0.22	0.21	0.22	0.31	0.20
Skew.	0.31	0.33	-0.30	0.67	0.04	0.54	0.41	0.20	0.40	0.32	0.33	0.50
Kurtosis	2.22	2.60	2.93	3.29	1.49	2.09	2.31	2.63	2.77	2.22	2.37	2.30
CAC 40												
Mean	1.01	1.07	1.16	1.16	1.14	0.52	0.54	0.48	0.49	0.45	1.05	0.52
SD	0.28	0.32	0.42	0.46	0.38	0.13	0.13	0.14	0.17	0.13	0.31	0.12
Skew.	0.37	-0.03	-0.13	-0.08	0.08	0.22	-0.13	-0.84	-0.76	-0.73	0.11	0.07
Kurtosis	2.50	2.37	2.28	2.33	3.10	2.43	2.42	3.21	2.95	4.36	2.35	2.50
DAX 30												
Mean	0.84	0.87	1.03	0.93	0.93	0.35	0.37	0.39	0.41	0.42	0.88	0.37
SD	0.28	0.30	0.39	0.42	0.41	0.14	0.16	0.17	0.19	0.20	0.30	0.15
Skew.	0.03	-0.02	-0.13	0.08	0.13	-0.27	0.01	-0.11	-0.31	-0.43	-0.02	-0.17
Kurtosis	2.16	2.11	2.43	2.56	2.65	2.04	2.04	2.60	2.44	2.46	2.16	2.10
FTSE 100												
Mean	0.96	0.97	1.03	1.06	0.99	0.39	0.40	0.35	0.34	0.35	0.98	0.38
SD	0.36	0.40	0.49	0.64	0.50	0.11	0.13	0.13	0.14	0.16	0.39	0.11
Skew.	1.01	1.03	0.80	1.67	1.21	0.14	0.04	-0.08	-0.01	0.26	0.99	0.21
Kurtosis	2.97	3.19	3.01	7.38	3.86	3.24	2.46	2.71	2.54	2.55	2.92	2.96
IBEX 35												
Mean	0.95	0.93	0.95	0.94	1.00	0.55	0.52	0.53	0.46	0.37	0.95	0.53
SD	0.23	0.28	0.31	0.35	0.39	0.16	0.18	0.18	0.19	0.21	0.25	0.17
Skew.	-0.09	0.04	0.43	0.56	0.21	0.43	0.06	0.21	0.37	0.45	0.07	0.36
Kurtosis	2.18	2.23	2.36	2.60	1.88	2.29	2.04	2.01	2.66	2.37	2.19	2.26
MIB												
Mean	0.80	0.80	0.86	0.93	0.95	0.40	0.41	0.42	0.42	0.47	0.82	0.41
SD	0.30	0.30	0.37	0.35	0.37	0.17	0.17	0.18	0.17	0.17	0.30	0.16
Skew.	0.44	0.32	0.21	-0.05	0.81	0.35	0.23	-0.38	-0.26	0.12	0.28	0.13
Kurtosis	3.43	3.33	2.93	3.14	4.57	2.55	2.90	2.69	2.66	2.29	3.47	2.87
PSI-20												
Mean	0.91	0.94	0.95	0.96	1.01	0.41	0.38	0.41	0.37	0.36	0.94	0.40
SD	0.20	0.25	0.29	0.30	0.31	0.12	0.14	0.12	0.12	0.11	0.22	0.11
Skew.	-0.97	-0.87	-0.31	-0.73	0.75	-0.43	0.35	0.13	-0.90	0.65	-0.80	0.08
Kurtosis	3.61	3.19	1.99	2.42	2.48	3.51	3.45	1.76	2.64	3.19	2.91	3.25

Table 3. Beta and R^2 computed from recomposed crystals of each index. Post Crisis Period

	Beta at each scale					R^2 at each scale					Raw Data	
	1	2	3	4	5	1	2	3	4	5	Beta	R^2
AEX												
Mean	1.01	1.04	1.11	1.11	1.13	0.46	0.53	0.59	0.57	0.58	1.05	0.51
SD	0.44	0.41	0.44	0.45	0.43	0.15	0.14	0.16	0.16	0.16	0.43	0.14
Skew.	0.82	0.93	0.15	0.08	-0.07	-0.04	-0.31	-0.77	-0.58	-0.72	0.61	-0.32
Kurtosis	3.58	3.88	2.47	2.26	2.13	2.18	2.22	2.32	2.25	3.36	3.19	2.20
ATHEX												
Mean	0.85	0.87	0.86	0.92	0.87	0.45	0.49	0.49	0.45	0.42	0.86	0.47
SD	0.44	0.46	0.34	0.37	0.35	0.21	0.20	0.16	0.18	0.18	0.42	0.19
Skew.	0.67	0.73	0.29	0.36	0.27	0.42	0.27	0.21	0.10	0.08	0.65	0.34
Kurtosis	2.13	2.16	1.65	1.90	1.78	2.21	2.03	2.10	2.09	1.88	2.07	2.14
CAC 40												
Mean	1.04	1.05	1.09	1.05	1.06	0.58	0.64	0.65	0.62	0.60	1.05	0.61
SD	0.32	0.34	0.37	0.35	0.37	0.14	0.13	0.13	0.15	0.17	0.33	0.13
Skew.	0.39	0.67	0.22	0.32	0.08	-0.46	-0.38	-0.73	-0.54	-0.50	0.41	-0.48
Kurtosis	3.04	3.16	2.54	2.61	2.16	2.65	2.59	3.21	2.50	2.41	2.91	2.79
DAX 30												
Mean	0.91	0.92	0.95	0.94	0.90	0.45	0.55	0.59	0.54	0.52	0.92	0.50
SD	0.29	0.28	0.32	0.31	0.27	0.14	0.15	0.16	0.16	0.18	0.28	0.15
Skew.	-0.32	-0.19	-0.07	0.03	-0.09	-0.75	-0.69	-0.62	-0.38	0.02	-0.27	-0.67
Kurtosis	2.30	2.63	2.11	2.18	1.93	2.88	3.09	2.32	2.50	1.95	2.23	2.76
FTSE 100												
Mean	0.98	1.02	1.04	1.00	0.99	0.40	0.47	0.50	0.46	0.46	1.00	0.43
SD	0.41	0.39	0.49	0.46	0.49	0.15	0.14	0.17	0.18	0.20	0.41	0.15
Skew.	0.85	0.68	0.75	0.56	0.83	0.36	-0.15	-0.26	-0.10	-0.26	0.74	0.10
Kurtosis	3.16	2.71	2.71	2.55	3.40	2.60	2.54	2.32	2.33	2.14	2.78	2.40
IBEX 35												
Mean	0.89	0.92	0.99	0.98	1.02	0.56	0.65	0.68	0.63	0.63	0.92	0.60
SD	0.22	0.22	0.25	0.25	0.24	0.18	0.14	0.13	0.15	0.17	0.22	0.16
Skew.	0.61	0.59	0.23	0.10	0.16	0.73	0.49	0.25	-0.23	-0.30	0.45	0.63
Kurtosis	2.87	2.69	1.84	2.31	2.96	2.38	2.53	2.46	2.44	2.07	2.54	2.47
MIB												
Mean	0.91	0.92	0.95	0.92	0.93	0.53	0.59	0.61	0.57	0.50	0.92	0.56
SD	0.30	0.30	0.31	0.26	0.36	0.15	0.16	0.16	0.16	0.20	0.29	0.15
Skew.	0.43	0.30	0.06	0.02	0.28	0.06	-0.27	-0.45	0.01	-0.27	0.32	-0.02
Kurtosis	3.23	3.25	2.78	2.77	2.63	2.35	2.34	2.19	1.94	2.20	3.19	2.26
PSI-20												
Mean	0.95	0.99	0.94	0.93	1.05	0.45	0.52	0.51	0.47	0.50	0.97	0.48
SD	0.31	0.35	0.31	0.35	0.36	0.15	0.16	0.16	0.19	0.13	0.31	0.15
Skew.	-1.14	-0.85	-1.38	-1.01	0.19	-1.64	-1.99	-2.04	-1.38	-0.81	-1.23	-2.13
Kurtosis	4.81	4.23	4.87	4.93	1.94	6.32	7.37	7.27	4.28	2.58	5.05	8.08

Table 4. Beta and R^2 computed from recomposed crystals of each index. Forc. Crisis Period

	Beta at each scale					R^2 at each scale					Raw Data	
	1	2	3	4	5	1	2	3	4	5	Beta	R^2
AEX												
Mean	1.22	1.21	1.22	1.17	1.25	0.44	0.44	0.47	0.33	0.31	1.21	0.43
SD	0.58	0.57	0.58	0.72	0.78	0.18	0.18	0.19	0.20	0.24	0.57	0.17
Skew.	0.22	0.43	0.60	1.43	0.15	-0.38	-0.24	-0.13	0.40	0.40	0.32	-0.32
Kurtosis	1.88	2.11	2.38	5.19	2.06	2.98	2.36	2.16	2.29	1.90	2.02	2.66
ATHEX												
Mean	0.97	1.00	1.03	1.14	1.09	0.42	0.46	0.41	0.46	0.67	1.01	0.44
SD	0.58	0.60	0.60	0.76	0.41	0.18	0.18	0.20	0.25	0.17	0.57	0.17
Skew.	0.95	0.84	0.46	0.69	-0.17	0.47	0.46	-0.23	-0.27	-1.24	0.87	0.43
Kurtosis	2.56	2.49	2.18	2.59	2.83	2.16	2.19	2.28	1.81	4.80	2.55	2.17
CAC 40												
Mean	1.13	1.16	1.17	1.11	1.13	0.54	0.57	0.56	0.43	0.40	1.14	0.54
SD	0.38	0.40	0.48	0.63	0.58	0.15	0.15	0.17	0.21	0.22	0.40	0.15
Skew.	0.15	0.26	0.52	1.05	0.15	-0.25	-0.84	-0.68	-0.15	-0.45	0.30	-0.38
Kurtosis	2.13	2.25	2.67	4.15	2.35	1.99	3.36	2.83	1.92	1.91	2.39	2.08
DAX 30												
Mean	0.88	0.97	0.95	0.93	1.04	0.42	0.53	0.54	0.35	0.40	0.92	0.46
SD	0.34	0.33	0.35	0.42	0.47	0.17	0.15	0.19	0.19	0.18	0.33	0.16
Skew.	0.50	0.36	-0.16	0.06	0.63	-0.07	-0.09	-0.45	0.28	-0.38	0.19	-0.17
Kurtosis	3.35	2.51	2.31	2.10	2.91	2.20	2.46	2.35	1.88	1.88	2.64	2.28
FTSE 100												
Mean	1.02	1.11	1.11	0.97	0.94	0.41	0.40	0.39	0.31	0.31	1.05	0.39
SD	0.46	0.52	0.58	0.59	0.59	0.16	0.17	0.19	0.19	0.19	0.48	0.16
Skew.	0.76	0.53	0.33	0.78	0.91	-0.12	-0.05	-0.16	0.48	0.16	0.65	-0.03
Kurtosis	3.24	2.51	2.25	2.72	3.64	2.14	2.08	2.01	2.32	1.91	2.78	2.07
IBEX 35												
Mean	0.99	0.97	1.04	0.92	0.92	0.54	0.59	0.61	0.44	0.49	0.99	0.56
SD	0.26	0.28	0.33	0.41	0.43	0.16	0.17	0.17	0.25	0.25	0.27	0.16
Skew.	-0.12	-0.01	0.74	-0.12	0.20	0.89	0.58	0.34	0.17	0.00	-0.02	0.86
Kurtosis	2.15	1.58	4.05	1.70	1.93	2.96	2.49	2.31	2.18	2.00	1.97	2.78
MIB												
Mean	1.01	1.05	1.08	0.98	1.08	0.52	0.54	0.55	0.44	0.41	1.04	0.52
SD	0.41	0.44	0.51	0.50	0.59	0.16	0.15	0.17	0.20	0.24	0.44	0.15
Skew.	0.46	0.32	0.40	0.58	0.32	0.04	-0.02	-0.36	-0.37	-0.04	0.38	0.05
Kurtosis	2.10	1.87	2.32	3.83	1.94	2.16	1.98	2.67	2.32	1.80	2.02	2.23
PSI-20												
Mean	0.97	0.94	0.96	1.21	1.04	0.25	0.30	0.33	0.41	0.42	0.99	0.29
SD	0.53	0.51	0.51	0.82	0.75	0.14	0.16	0.15	0.20	0.28	0.54	0.15
Skew.	0.19	0.32	0.83	0.58	0.03	0.00	-0.24	-0.57	-0.53	-0.01	0.41	-0.20
Kurtosis	2.11	2.31	4.06	2.59	1.55	1.90	2.16	2.75	2.59	1.50	2.54	2.18

Table 5. Beta and R^2 computed from recomposed crystals of each index. All Period

	Beta at each scale					R^2 at each scale					Raw Data	
	1	2	3	4	5	1	2	3	4	5	Beta	R^2
AEX												
Mean	0.97	0.99	1.07	1.08	1.04	0.42	0.44	0.45	0.41	0.44	1.00	0.43
SD	0.37	0.40	0.45	0.48	0.38	0.13	0.14	0.14	0.14	0.12	0.39	0.13
Skew.	0.99	0.98	0.39	0.31	0.42	0.18	-0.06	-0.58	-0.40	-0.17	0.83	-0.04
Kurtosis	3.43	3.68	2.59	2.19	2.31	2.18	2.43	2.55	2.20	1.88	3.23	2.36
ATHEX												
Mean	0.87	0.87	0.88	0.92	0.95	0.40	0.42	0.40	0.42	0.46	0.88	0.41
SD	0.39	0.41	0.35	0.39	0.33	0.19	0.19	0.17	0.18	0.15	0.39	0.18
Skew.	0.64	0.71	0.34	0.53	0.28	0.68	0.69	0.48	0.57	0.21	0.65	0.69
Kurtosis	2.07	2.11	1.93	1.97	1.89	2.46	2.29	2.79	2.54	2.00	2.05	2.49
CAC 40												
Mean	1.03	1.06	1.12	1.11	1.09	0.52	0.54	0.52	0.49	0.44	1.06	0.52
SD	0.28	0.29	0.37	0.39	0.33	0.12	0.12	0.13	0.14	0.12	0.30	0.12
Skew.	0.17	0.01	-0.04	0.08	0.10	0.02	0.04	-0.48	-0.16	-0.04	0.07	0.03
Kurtosis	2.24	2.19	2.20	2.27	2.53	2.19	2.10	2.70	2.09	2.61	2.19	2.21
DAX 30												
Mean	0.86	0.90	0.98	0.94	0.93	0.37	0.42	0.45	0.42	0.40	0.90	0.40
SD	0.26	0.27	0.31	0.34	0.29	0.13	0.14	0.15	0.17	0.16	0.27	0.14
Skew.	-0.13	-0.13	-0.24	0.10	-0.06	-0.26	-0.12	-0.31	-0.18	-0.04	-0.13	-0.20
Kurtosis	2.23	2.34	2.39	2.45	2.26	2.16	2.31	2.55	2.47	2.22	2.30	2.26
FTSE 100												
Mean	0.97	1.00	1.04	1.05	0.98	0.37	0.39	0.37	0.35	0.34	0.99	0.37
SD	0.36	0.37	0.46	0.51	0.46	0.11	0.11	0.12	0.13	0.15	0.38	0.11
Skew.	0.98	0.89	0.70	1.03	1.19	0.39	0.09	-0.20	0.08	0.47	0.90	0.27
Kurtosis	3.09	2.90	2.66	4.01	3.97	3.05	2.56	2.42	2.25	2.76	2.91	2.74
IBEX 35												
Mean	0.94	0.94	0.98	0.97	1.01	0.52	0.55	0.56	0.48	0.45	0.95	0.53
SD	0.20	0.22	0.26	0.26	0.25	0.16	0.16	0.14	0.15	0.16	0.22	0.15
Skew.	-0.07	-0.02	0.09	0.33	0.09	0.95	0.75	0.66	0.91	0.76	-0.02	0.93
Kurtosis	2.26	2.25	1.90	2.35	1.76	2.91	2.72	2.72	3.52	2.88	2.14	2.96
MIB												
Mean	0.87	0.88	0.93	0.93	0.96	0.43	0.46	0.47	0.44	0.43	0.89	0.45
SD	0.27	0.28	0.33	0.29	0.30	0.15	0.15	0.15	0.15	0.13	0.28	0.15
Skew.	0.41	0.38	0.23	0.09	0.63	0.47	0.31	-0.12	0.13	0.19	0.33	0.34
Kurtosis	3.61	3.73	3.25	3.30	3.99	2.65	2.68	2.65	2.33	1.95	3.65	2.74
PSI-20												
Mean	0.92	0.94	0.94	0.99	1.03	0.35	0.36	0.37	0.37	0.37	0.95	0.36
SD	0.24	0.28	0.27	0.30	0.26	0.11	0.13	0.12	0.13	0.10	0.25	0.11
Skew.	-1.39	-1.31	-1.05	-0.82	-0.34	-1.15	-0.79	-0.79	-0.64	-0.59	-1.29	-1.10
Kurtosis	5.02	4.57	3.50	3.03	2.29	5.02	4.41	3.30	2.89	2.54	4.41	4.96

Table 6. Value At Risk (VaR) at different time scales for equally weighted portfolio. Pre Crisis Period.

	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR
	AEX		ATHEX		CAC		DAX	
Scale1	0.0091	50.68%	0.0112	47.06%	0.0106	53.94%	0.0095	49.20%
Scale2	0.0062	23.58%	0.0081	24.63%	0.0070	23.45%	0.0066	23.99%
Scale3	0.0051	15.80%	0.0067	16.74%	0.0054	13.84%	0.0055	16.69%
Scale4	0.0034	6.94%	0.0047	8.45%	0.0036	6.30%	0.0037	7.31%
Scale5	0.0022	3.00%	0.0029	3.11%	0.0023	2.46%	0.0023	2.81%
Total	0.0128		0.0163		0.0145		0.0135	
Total Raw	0.0130		0.0166		0.0147		0.0137	
	FTSE		IBEX		MIB		PSI	
Scale1	0.0092	52.44%	0.0101	49.88%	0.0090	55.33%	0.0060	43.92%
Scale2	0.0061	23.04%	0.0070	24.42%	0.0057	22.22%	0.0045	24.74%
Scale3	0.0050	15.41%	0.0053	13.65%	0.0044	13.34%	0.0036	16.10%
Scale4	0.0033	6.62%	0.0040	8.05%	0.0029	5.90%	0.0030	11.34%
Scale5	0.0020	2.49%	0.0028	3.99%	0.0022	3.21%	0.0018	3.91%
Total	0.0127		0.0142		0.0120		0.0090	
Total Raw	0.0129		0.0144		0.0122		0.0093	

Table 7. Value At Risk (VaR) at different time scales for equally weighted portfolio. In Crisis Period.

	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR
	AEX		ATHEX		CAC		DAX	
Scale1	0.0246	51.18%	0.0224	49.49%	0.0258	51.81%	0.0204	49.22%
Scale2	0.0180	27.47%	0.0164	26.71%	0.0191	28.48%	0.0152	27.33%
Scale3	0.0125	13.18%	0.0113	12.67%	0.0123	11.78%	0.0109	14.06%
Scale4	0.0080	5.48%	0.0090	8.01%	0.0087	5.95%	0.0074	6.53%
Scale5	0.0056	2.69%	0.0056	3.12%	0.0050	1.98%	0.0049	2.85%
Total	0.0344		0.0318		0.0358		0.0291	
Total Raw	0.0351		0.0328		0.0364		0.0297	
	FTSE		IBEX		MIB		PSI	
Scale1	0.0223	52.74%	0.0230	54.41%	0.0197	49.49%	0.0178	48.11%
Scale2	0.0164	28.67%	0.0158	25.72%	0.0145	26.72%	0.0131	25.84%
Scale3	0.0103	11.23%	0.0113	13.08%	0.0104	13.77%	0.0103	16.14%
Scale4	0.0071	5.43%	0.0071	5.22%	0.0072	6.61%	0.0068	6.97%
Scale5	0.0043	1.94%	0.0039	1.57%	0.0052	3.41%	0.0044	2.93%
Total	0.0307		0.0312		0.0280		0.0257	
Total Raw	0.0311		0.0319		0.0287		0.0265	

Table 8. Value At Risk (VaR) at different time scales for equally weighted portfolio. Post Crisis Period.

	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR
	AEX		ATHEX		CAC		DAX	
Scale1	0.0156	43.55%	0.0270	49.40%	0.0188	46.04%	0.0148	43.36%
Scale2	0.0123	27.36%	0.0212	30.54%	0.0150	29.31%	0.0123	29.67%
Scale3	0.0098	17.40%	0.0138	12.84%	0.0109	15.40%	0.0094	17.38%
Scale4	0.0067	8.05%	0.0087	5.12%	0.0070	6.42%	0.0059	6.81%
Scale5	0.0045	3.64%	0.0056	2.11%	0.0047	2.83%	0.0038	2.78%
Total	0.0236		0.0384		0.0277		0.0225	
Total Raw	0.0239		0.0389		0.0280		0.0229	
	FTSE		IBEX		MIB		PSI	
Scale1	0.0132	44.52%	0.0173	41.94%	0.0187	47.07%	0.0156	45.63%
Scale2	0.0108	29.47%	0.0150	31.52%	0.0146	28.89%	0.0130	31.48%
Scale3	0.0080	16.50%	0.0110	17.07%	0.0107	15.52%	0.0086	13.81%
Scale4	0.0050	6.50%	0.0068	6.57%	0.0068	6.16%	0.0055	5.67%
Scale5	0.0034	3.01%	0.0045	2.90%	0.0042	2.36%	0.0043	3.41%
Total	0.0198		0.0267		0.0273		0.0231	
Total Raw	0.0200		0.0270		0.0276		0.0234	

Table 9. Value At Risk (VaR) at different time scales for equally weighted portfolio. Forc Crisis Period.

	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR
	AEX		ATHEX		CAC		DAX	
Scale1	0.0164	54.29%	0.0345	46.37%	0.0187	51.22%	0.0135	45.67%
Scale2	0.0114	26.37%	0.0264	27.10%	0.0141	29.15%	0.0113	31.93%
Scale3	0.0084	14.35%	0.0176	12.08%	0.0100	14.63%	0.0082	17.01%
Scale4	0.0042	3.61%	0.0142	7.85%	0.0050	3.63%	0.0036	3.32%
Scale5	0.0026	1.38%	0.0130	6.60%	0.0031	1.36%	0.0029	2.07%
Total	0.0223		0.0507		0.0262		0.0200	
Total Raw	0.0226		0.0523		0.0266		0.0204	
	FTSE		IBEX		MIB		PSI	
Scale1	0.0120	55.63%	0.0210	47.04%	0.0227	48.94%	0.0132	41.30%
Scale2	0.0083	26.64%	0.0167	29.95%	0.0174	28.93%	0.0108	27.82%
Scale3	0.0055	11.86%	0.0125	16.64%	0.0131	16.37%	0.0081	15.65%
Scale4	0.0032	4.02%	0.0062	4.12%	0.0064	3.87%	0.0067	10.57%
Scale5	0.0022	1.85%	0.0046	2.26%	0.0045	1.89%	0.0044	4.66%
Total	0.0161		0.0306		0.0324		0.0206	
Total Raw	0.0163		0.0313		0.0331		0.0211	

Table 10. Value At Risk (VaR) at different time scales for equally weighted portfolio. All Crisis Period.

	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR	VaR	Contribution to VaR
	AEX		ATHEX		CAC		DAX	
Scale1	0.0175	49.69%	0.0231	48.76%	0.0194	50.37%	0.0154	47.35%
Scale2	0.0129	27.14%	0.0175	28.05%	0.0146	28.46%	0.0119	28.24%
Scale3	0.0094	14.46%	0.0118	12.85%	0.0099	13.26%	0.0088	15.51%
Scale4	0.0061	6.01%	0.0086	6.76%	0.0066	5.84%	0.0056	6.29%
Scale5	0.0041	2.71%	0.0063	3.58%	0.0039	2.07%	0.0036	2.62%
Total	0.0248		0.0330		0.0273		0.0224	
Total Raw	0.0253		0.0338		0.0277		0.0228	
	FTSE		IBEX		MIB		PSI	
Scale1	0.0155	51.09%	0.0180	49.04%	0.0173	49.17%	0.0140	46.30%
Scale2	0.0116	28.23%	0.0137	28.14%	0.0129	27.52%	0.0109	28.02%
Scale3	0.0078	12.79%	0.0099	14.82%	0.0094	14.71%	0.0080	15.13%
Scale4	0.0052	5.72%	0.0061	5.70%	0.0060	5.90%	0.0056	7.28%
Scale5	0.0032	2.17%	0.0039	2.30%	0.0040	2.70%	0.0037	3.27%
Total	0.0218		0.0257		0.0246		0.0206	
Total Raw	0.0220		0.0262		0.0251		0.0212	

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