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A replacement policy for a repairable system with its repairman having multiple vacations

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Abstract

This paper considers a replacement policy for a repairable system with a repairman, who can have multiple vacations. If the system fails and the repairman is on vacation, it will wait for repair until the repairman is available. Assuming that the system can not be repaired “as good as new” and a repair upon failure can be performed immediately with a probability of p , we optimise replacement policy using geometric processes. The explicit expression of the expected cost rate is derived, and the corresponding optimal policy can be determined analytically or numerically. Finally, a numerical example is given to illustrate the theoretical results of the model.

Keywords: Geometric process; Multiple vacation; Replacement policy; Maintenance policy

1. Introduction

A repairable system is a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, rather than the replacement of the entire system (Ascher and Feingold, 1984). Repair models developed upon successive inter-failure times have been employed in many applications such as the optimisation of

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24 maintenance policies, decision making and whole life cycle cost analysis. With different repair levels,
25 repair can be broken down into three categories (Yamez et al, 2002): perfect repair, normal repair and
26 minimal repair. A perfect repair can restore a system to an “as good as new” state, a normal repair is
27 assumed to bring the system to any condition, and a minimal repair, or imperfect repair, can restore the
28 system to the exact state it was before failure. Example models for perfect, normal, and minimal repair
29 are renewal process (RP) models or homogeneous Poisson process (HPP) models, generalised renewal
30 processes, and non-homogeneous Poisson process (NHPP) models, respectively. On the basis of the
31 relationship between failure intensities and time, repair models fall into three categories: models with a
32 constant failure intensity (e.g. HPP models), models with an operating-time dependent failure intensity
33 (e.g. NHPP models) and models with a repair-time dependent failure intensity (e.g. geometric processes
34 (GP) models (Lam, 1988)).

35 In reality, the survival times of a system after each repair can become shorter and shorter due to
36 various reasons such as ageing and deterioration. The working times and repair times can be modeled by
37 geometric processes as many authors have studied (Lam, 1988; Wu and Clements-Croome, 2005; Zhang
38 and Wang, 2007). The geometric process introduced by Lam (1988) defines an alternative to the non-
39 homogeneous Poisson process: a sequence of random variables $\{X_k, k=1,2,\dots\}$ is a geometric process if
40 the distribution function of X_k is given by $F(a^{k-1}t)$ for $k=1,2,\dots$ and a is a positive constant. Wang and
41 Pham (1996) later refer the geometric process as a quasi-renewal process. Finkelstein (1993) develops a
42 model: he defines a general deteriorating renewal process such that $F_{k+1}(t) \leq F_k(t)$. Wu and Clements-
43 Croome (2006) extend the geometric process by replacing its parameter a^{k-1} with $a_1 a^{k-1} + b_1 b^{k-1}$, where
44 $a > 1$ and $0 < b < 1$. The geometric process has been applied to reliability analysis and maintenance policy
45 optimisation for various systems by authors; for example, Wu and Clements-Croome (2005), Castro and
46 Pérez-Ocón (2006), Zhang and Wang (2007), and Braun et al (2008).

47 The existing research mainly concentrates on the reliability analysis or maintenance optimisation with
48 a consideration of the behaviours of repairable systems themselves. Little work has been conducted to

49 consider reliability analysis for a system where the repairman might take a sequence of vacations of
50 random durations and a repair on a failure is a normal repair. Here we emphasize that the durations of
51 vacations can be different. Such a vacation policy is called a multiple vacation policy, which has
52 attracted attention in queuing theory (for example, Lee, 1988; Krishna et al, 1998; Chang and Choi,
53 2005).

54 The applications of such situations where a repairman can take multiple vocations can be found in
55 practice. In some situations, a repairman can have two roles: one for caring a system and one for other
56 duties, which can happen in a small/median firm that wants to use the repairman effectively. If the
57 repairman is assigned to look after only one system, he might have plenty of idle time. In this paper,
58 vocation can mean period when the repairman is on other duties. The repairman can periodically check
59 the status of the system: if the system fails, he repairs it; if the system is working, he goes back to the
60 other duties. Allocating the manpower of the repairman in such a way is more realistic and more
61 profitable than simply assigning him a single role of being a repairman.

62 This paper presents the formulations of the expected long-run profit per unit time for a repairable
63 system with a repairman. We assume that the repairman takes multiple vacations. When the system fails,
64 the repairman will be called in to bring the system back to a certain state. The time to repair is composed
65 of two different periods: waiting and real repair periods. The waiting time starts from the component's
66 failure to the start to repair, and the real repair time is the time between the start to repair and the
67 completion of the repair. Both the working and real repair times are assumed to be a type of stochastic
68 processes: geometric processes, and the waiting times are subject to a renewal process. The probability
69 that a failed system can be immediately repaired is assumed to be p . The expected long-run profit per
70 unit time is derived and a numerical example is given to illustrate the theoretical results of the model.

71 The paper is structured as follows. The coming section introduces geometric processes defined by
72 Lam (1988), and assumptions. Sections 3 and 4 derives the expected long-run profit per unit time, and

73 discusses special cases, respectively. Section 5 offers numerical examples. Concluding remarks are
74 offered in the last section.

75 2. Definitions and Model Assumptions

76 This section first borrows the definition of geometric processes from Lam (1988), and then makes
77 assumptions for model development.

78 2.1 Definition

79 **Definition 1** Assume ξ, η are two random variables. For arbitrary real number α , there is

$$80 \quad P(\xi \geq \alpha) > P(\eta \geq \alpha)$$

81 then ξ is called stochastically bigger than η . Similarly, if

$$82 \quad P(\xi \geq \alpha) < P(\eta \geq \alpha)$$

83 then ξ is called stochastically smaller than η .

84 **Definition 2** (Lam, 1988) Assume that $\{X_n, n=1,2,\dots\}$ is a sequence of independent non-negative
85 random variables. If the distribution function of X_n is $F(a^{n-1}t)$, for some $a>0$ and all, $n=1,2, \dots$, then
86 $\{X_n, n=1,2,\dots\}$ is called a geometric process.

87 Obviously,

88 if $a > 1$, then $\{X_n, n=1,2,\dots\}$ is stochastically decreasing,

89 if $a < 1$, then $\{X_n, n=1,2,\dots\}$ is stochastically increasing, and

90 if $a = 1$, $\{X_n, n=1,2,\dots\}$ is a renewal process.

91 2.2 Assumptions

92 The following assumptions are assumed to hold in what follows.

93 A. At time $t=0$, the system is new.

- 94 B. The system starts to work at time $t=0$, and it is maintained by a repairman. The repairman takes his
 95 first vacation after the system has started. After his vacation ends, there will be two situations.
- 96 (a) If the system has failed and is waiting for repair, the repairman will repair it. He will then take
 97 his second vacation after the repair is completed.
- 98 (b) If the system is still working, the repairman will take his second vacation. This operating policy
 99 continues until a replacement takes place.
- 100 C. After the repairman finishes his vacation, the probability that he can immediately repair the failed
 101 system is p . Denote V_n as the waiting time after the n th failure occurs, where $\{V_n, n=1,2,\dots\}$ are
 102 independently and identically distributed with distribution $S(t)$ ($t \geq 0$) and $\tau = EV_n < +\infty$.
- 103 D. The time interval from the completion of the $(n-1)$ th repair to that of the n th repair of the system is
 104 called the n th cycle of the system, where $n = 1,2, \dots$. Denote the working time and the repair time of
 105 the system in the n th cycle ($n = 1,2, \dots$) as X_n and Y_n , respectively. Denote the length of the i th
 106 vacation during the n th cycle as $\{Z_n^i, n=1,2,\dots\}$. Denote the cumulative distribution functions of
 107 X_n , Y_n , Z_n^i and $F_n(x)$ as $G_n(y)$, and $H_n(z)$, respectively, where $F_n(x) = F(a^{n-1}x)$,
 108 $G_n(y) = G(b^{n-1}y)$, and $H_n(z) = H(d^{n-1}z)$. Denote $E(X_1) = \lambda$, $E(Y_1) = \mu$, and $E(Z_1^1) = \gamma$.
- 109 E. X_n , Y_n , Z_n^i , and V_n ($i=1,2,\dots$ and $n = 1,2, \dots$) are statistically independent.
- 110 F. When a replacement is required, a brand new but identical component will be used, and the length
 111 of a replacement time is negligible.
- 112 G. The following costs are considered:
- 113 • C_1 : repair cost per unit time;
 - 114 • C_2 : reward per unit time when the system is working;
 - 115 • C_3 : cost incurred for a replacement;

- C_4 : reward per unit of the repairman when he is taking vacation or other duties, which can produce profits for the firm;
- C_5 : cost per unit time when the system is waiting for repair; and
- C_6 : cost per unit time incurred in the waiting time after the system has failed.

3. Expected profit under replacement policy N

Denote η_n the times of vacations of the repairman during the n th cycle of the system. A typical progress is given in Figure 1.

Figure 1 here

Figure 1. A typical progress of the system

Let T_1 be the time before the first replacement, T_n be the time between the $(n-1)$ th and n th replacement with $n=2,3,\dots$. The process $\{T_n, n=1, 2,\dots\}$ forms a renewal process. Denote $P(N)$ as the expected long-run profit per unit time under replacement policy N, then we have

$$P(N) = \lim_{t \rightarrow \infty} \frac{\text{Expected profit within } [0, t]}{t}$$

Since $\{T_n, n=1,2,\dots\}$ is a renewal process, the time between two adjacent replacements is the length for a replacement. Hence

$$P(N) = \frac{\text{expected profit within a replacement cycle}}{\text{expected length of a cycle}} = \frac{ER}{EW} \quad (1)$$

Lemma 1. The probability of η_n is given by

$$P(\eta_n = m) = \int_0^{+\infty} [S_{m-1}(t) - S_m(t)] dF(a^{n-1}t), \quad m = 1, 2, \dots, n = 1, 2, \dots, N$$

and

$$E\eta_n = \int_0^{+\infty} \left[\sum_{m=1}^{\infty} S_m(t) \right] dF(a^{n-1}t).$$

where $S_m(t)$ is the cumulative distribution function of $\sum_{i=1}^m Z_n^i$.

Proof According to the law of total probability, we have

$$\begin{aligned} P(\eta_n = m) &= P\left[\sum_{i=1}^{m-1} Z_n^i < X_n < \sum_{i=1}^m Z_n^i\right] = \int_0^{+\infty} P\left[\sum_{i=1}^{m-1} Z_n^i < t < \sum_{i=1}^m Z_n^i, X_n \leq t\right] dF(a^{n-1}t) \\ &= \int_0^{+\infty} [S_{m-1}(t) - S_m(t)] dF(a^{n-1}t), \end{aligned}$$

and

$$\begin{aligned}
E\eta_n &= \sum_{m=1}^{\infty} mP(\eta_n = m) = \sum_{m=1}^{\infty} m \int_0^{+\infty} [S_{m-1}(t) - S_m(t)]dF(a^{n-1}t) \\
&= \int_0^{+\infty} \sum_{m=1}^{\infty} m [S_{m-1}(t) - S_m(t)]dF(a^{n-1}t) = \int_0^{+\infty} [\sum_{m=1}^{\infty} S_m(t)]dF(a^{n-1}t). \quad \dagger
\end{aligned}$$

From the assumptions, the length of a replacement cycle is given by

$$\begin{aligned}
W &= \sum_{n=1}^N \sum_{k=1}^{\eta_n} Z_n^k + \sum_{i=1}^{N-1} [Y_i I\{A_i\} + (Y_i + V_i) I\{B_i\}] \\
&= \sum_{n=1}^{N-1} Y_n + \sum_{n=1}^N \sum_{k=1}^{\eta_n} Z_n^k + \sum_{i=1}^{N-1} V_i I\{B_i\}.
\end{aligned}$$

where $I\{A\} = 1$ if event A occurs, otherwise 0. Denote $A_i = \{\text{the system can be repaired immediately after the } i\text{th failure}\}$, and $B_i = \{\text{the system can not be repaired immediately after the } i\text{th failure}\}$.

Hence,

$$\begin{aligned}
E[\sum_{k=1}^{\eta_n} Z_n^k] &= E[E(\sum_{k=1}^{\eta_n} Z_n^k | \eta_n)] = \sum_{m=1}^{\infty} [\sum_{k=1}^m E(Z_n^k)]P(\eta_n = m) \\
&= \sum_{m=1}^{\infty} \sum_{k=1}^m \frac{\gamma}{d^{n-1}} P(\eta_n = m) = \frac{\gamma}{d^{n-1}} \sum_{m=1}^{\infty} mP(\eta_n = m) = \frac{\gamma}{d^{n-1}} E(\eta_n) \\
&= \frac{\gamma}{d^{n-1}} \int_0^{+\infty} [\sum_{m=1}^{\infty} S_m(t)]dF(a^{n-1}t), \text{ and}
\end{aligned}$$

$$E[\sum_{i=1}^{N-1} V_i I\{B_i\}] = \sum_{i=1}^{N-1} E[V_i I\{B_i\}] = (N-1)(1-p)\tau.$$

The expected time for a replacement is

$$\begin{aligned}
EW &= \sum_{n=1}^{N-1} EY_n + \sum_{n=1}^N E[\sum_{k=1}^{\eta_n} Z_n^k] + \sum_{i=1}^{N-1} E[V_i I\{B_i\}] \\
&= \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \sum_{n=1}^N \frac{\gamma}{d^{n-1}} \int_0^{+\infty} [\sum_{m=1}^{\infty} S_m(t)]dF(a^{n-1}t) + (N-1)(1-p)\tau \quad (2)
\end{aligned}$$

and the profit within a cycle is

$$\begin{aligned}
R &= C_2 \sum_{n=1}^N X_n + C_4 \sum_{n=1}^N \sum_{k=1}^{\eta_n} Z_n^k - C_1 \sum_{n=1}^{N-1} Y_n - C_5 \sum_{n=1}^N (\sum_{k=1}^{\eta_n} Z_n^k - X_n) - C_6 E[\sum_{i=1}^{N-1} V_i I\{B_i\}] - C_3 \\
&= (C_2 + C_5) \sum_{n=1}^N X_n + (C_4 - C_5) \sum_{n=1}^N \sum_{k=1}^{\eta_n} Z_n^k - C_1 \sum_{n=1}^{N-1} Y_n - C_6 E[\sum_{i=1}^{N-1} V_i I\{B_i\}] - C_3
\end{aligned}$$

The expected profit within a cycle is given by

$$\begin{aligned}
ER &= (C_2 + C_5) \sum_{n=1}^N \frac{\lambda}{a^{n-1}} + (C_4 - C_5) \sum_{n=1}^N \frac{\gamma}{d^{n-1}} \int_0^{+\infty} [\sum_{m=1}^{\infty} S_m(t)]dF(a^{n-1}t) - C_1 \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} \\
&\quad - C_6 (N-1)(1-p)\tau - C_3 \quad (3)
\end{aligned}$$

162 If we consider equations (1), (2) and (3), we obtain the expected long-run profit per unit time as

$$163 \quad P(N) = \frac{(C_2 + C_3) \sum_{n=1}^N \frac{\lambda}{a^{n-1}} + (C_4 - C_5) \sum_{n=1}^N \frac{\gamma}{d^{n-1}} \int_0^{\infty} [\sum_{m=1}^{\infty} S_m(t)] dF(a^{n-1}t) - C_1 \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - C_6(N-1)(1-p)\tau - C_3}{\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \sum_{n=1}^N \frac{\gamma}{d^{n-1}} \int_0^{\infty} [\sum_{m=1}^{\infty} S_m(t)] dF(a^{n-1}t) + (N-1)(1-p)\tau} \quad (4)$$

164 4. Special cases

165 We assume that the cumulative distribution functions of X_n , Y_n , Z_n^i , and V_n are

$$166 \quad F_n(t) = F(a^{n-1}t) = 1 - \exp\left(-\frac{a^{n-1}}{\lambda}t\right)$$

$$167 \quad G_n(t) = G(b^{n-1}t) = 1 - \exp\left(-\frac{b^{n-1}}{\mu}t\right),$$

$$168 \quad H_n(t) = H(d^{n-1}t) = 1 - \exp\left(-\frac{d^{n-1}}{\gamma}t\right)$$

169 and

$$170 \quad S(t) = 1 - \exp\left(-\frac{t}{\tau}\right)$$

171 respectively, where $t \geq 0$.

172 As we assume that Z_n^i ($i = 1, 2, \dots, m$) are mutually independent, the probability density function of

173 $\sum_{i=1}^m Z_n^i$ is a hypo-exponential distribution (Ross, 1997).

174 **Lemma 2.** Assume that random variables V_1, V_2, \dots, V_n are independently and identically
175 distributed with an exponential distribution of parameter λ_0 , then the probability density function of

176 $\sum_{i=1}^n V_i$ is

$$177 \quad \phi_n(t) = \frac{\lambda_0 (\lambda_0 t)^{n-1}}{(n-1)!} e^{-\lambda_0 t} \quad (4)$$

178 Denote the cumulative distribution function of $\sum_{i=1}^n V_i$ as $\Phi_n(t)$, then

$$179 \quad \sum_{n=1}^{\infty} \Phi_n(t) = \lambda_0 t \quad (5)$$

180 **Proof.** From Ross (1997), we have $\phi_n(t) = \frac{\lambda_0 (\lambda_0 t)^{n-1}}{(n-1)!} e^{-\lambda_0 t}$. Then

$$181 \quad \sum_{n=1}^{\infty} \Phi_n(t) = \sum_{n=1}^{\infty} \int_0^t \phi_n(t) dt = \int_0^t \lambda_0 \left(\sum_{n=1}^{\infty} \frac{(\lambda_0 t)^{n-1}}{(n-1)!} \right) e^{-\lambda_0 t} dt = \int_0^t \lambda_0 e^{\lambda_0 t} e^{-\lambda_0 t} dt = \lambda_0 t. \quad \bar{\dagger}$$

182 **Theorem 1.** The expected long-run profit per unit time is given by

$$P(N) = \frac{(C_2 + C_4) \sum_{n=1}^N \frac{\lambda}{a^{n-1}} - C_1 \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - C_6(N-1)(1-p)\tau - C_3}{\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \sum_{n=1}^N \frac{\lambda}{a^{n-1}} + (N-1)(1-p)\tau}$$

There exists an optimal N^* that maximizes the value $P(N)$.

Proof. Since Z_n^i ($i = 1, 2, \dots, m$) are independently and identically distributed with an exponential

distribution of parameter $\frac{d^{n-1}}{\gamma}$, then the probability distribution of $\sum_{i=1}^m Z_n^i$ is a gamma distribution with

scale parameter $\frac{\gamma}{d^{n-1}}$ and shape parameter m , the probability density function of $\sum_{i=1}^m Z_n^i$ is given by

$$f_m(t) = \frac{1}{(m-1)!} \left(\frac{d^{n-1}}{\gamma}\right)^m t^{m-1} e^{-\frac{d^{n-1}}{\gamma}t} \quad (t \geq 0)$$

Hence, the cumulative distribution function of $\sum_{i=1}^m Z_n^i$ is given by

$$S_m(t) = \int_0^t f_m(u) du$$

Hence

$$\begin{aligned} \sum_{m=1}^{\infty} S_m(t) &= \int_0^t \sum_{m=1}^{\infty} \left[\frac{1}{(m-1)!} \left(\frac{d^{n-1}}{\gamma}\right)^m u^{m-1} e^{-\frac{d^{n-1}}{\gamma}u} \right] du \\ &= \int_0^t \left[\sum_{m=1}^{\infty} \frac{\left(\frac{d^{n-1}u}{\gamma}\right)^{m-1}}{(m-1)!} \right] \left(\frac{d^{n-1}}{\gamma}\right) e^{-\frac{d^{n-1}}{\gamma}u} du \\ &= \int_0^t \left[\frac{d^{n-1}}{\gamma} \right] du = \frac{d^{n-1}}{\gamma} t \end{aligned}$$

Then,

$$\sum_{n=1}^N \frac{\gamma}{d^{n-1}} \int_0^{+\infty} \left[\sum_{m=1}^{\infty} S_m(t) \right] dF(a^{n-1}t) = \sum_{n=1}^N \int_0^{\infty} t dF(a^{n-1}t) = \sum_{n=1}^N EX_n = \sum_{n=1}^N \frac{\lambda}{a^{n-1}}$$

Hence, the expected long-run profit per unit time is given by

$$P(N) = \frac{(C_2 + C_4) \sum_{n=1}^N \frac{\lambda}{a^{n-1}} - C_1 \sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} - C_6(N-1)(1-p)\tau - C_3}{\sum_{n=1}^{N-1} \frac{\mu}{b^{n-1}} + \sum_{n=1}^N \frac{\lambda}{a^{n-1}} + (N-1)(1-p)\tau} \quad (6)$$

Since $a > 1, 0 < b < 1$, the expected long-run profit per unit time is monotonously increasing when the number N is small, and the expected long-run profit per unit time is monotonously decreasing when the

201 number N is large. $\lim_{N \rightarrow \infty} P(N) = -C_1$. Therefore, there exists a maximum value in $P(N)$, or we can find
 202 the optimum replacement policy N^* , which maximizes the value of $P(N^*)$.
 203 This proves the theorem.

204 5. Numerical examples

205 In this section, we will give examples to demonstrate the theoretical results of our model.

206 5.1 Sensitivity analysis for the repair times influencing the profit

207 If we set $a = 1.1, b = 0.98, \lambda = 100, \mu = 1, C_1 = 20, C_2 = 500, C_3 = 5000, C_4 = 200, C_6 = 100, \tau = 0.2,$
 208 and $p = 0.8$, then the optimum number for a replacement will be $N=8$, and the corresponding expected
 209 long-run profit per unit time is 535.09. The change of value $P(N)$ with repair times N is shown in Figure
 210 2. The value $P(N)$ increases rapidly when repair times changes from 1 to 8, and then decreases slowly
 211 when repair times increases. This indicates that the expected long-run profit per unit time is more
 212 sensitive to big values of N^* . In case it is not possible to undertake a replacement when repair times
 213 reaches $N^*(=8)$, we can replace the system after more repairs have been conducted, rather than less. This
 214 is because larger N^* ($3 < N^* < 13$, say) tends to have greater profit, whereas smaller N^* might not have
 215 good profits ($N^* < 4$).

216 Figure 2 here

217 Figure 2 The expected long-run profit per unit time $P(N)$ against repair times N .

a	N^*	$P(N^*)$	a	N^*	$P(N^*)$	a	N^*	$P(N^*)$
1.01	17	665.03	1.18	6	459.75	1.35	5	367.5
1.02	15	650.42	1.19	6	452.3	1.36	5	363.7
1.03	13	634.58	1.2	6	445.19	1.37	5	360.03
1.04	12	618.46	1.21	6	438.4	1.38	4	356.47
1.05	11	602.68	1.22	6	431.9	1.39	4	353.03
1.06	10	587.58	1.23	6	425.68	1.4	4	349.69
1.07	10	573.27	1.24	6	419.72	1.41	4	346.46
1.08	9	559.78	1.25	5	414.01	1.42	4	343.33

1.09	9	547.07	1.26	5	408.52	1.43	4	340.29
1.1	8	535.09	1.27	5	403.24	1.44	4	337.34
1.11	8	523.79	1.28	5	398.17	1.45	4	334.47
1.12	8	513.12	1.29	5	393.3	1.46	4	331.69
1.13	7	503.02	1.3	5	388.6	1.47	4	328.98
1.14	7	493.46	1.31	5	384.07	1.48	4	326.35
1.15	7	484.38	1.32	5	379.7	1.49	4	323.8
1.16	7	475.76	1.33	5	375.49	1.5	4	321.31
1.17	7	467.56	1.34	5	371.43			

Table 1: The expected long-run profit per unit time against the values of a and N^* .

5.2 Sensitivity analysis for parameters a and N

If we keep the values of parameters in Section 5.1, apart from the parameter a , we obtain results shown in Table 1. Table 1 shows how the optimum repair times N^* and the expected long-run profit per unit time change when parameter a changes from 1.01 to 1.5. From Table 1, we have the following results.

- We can see that the optimum N^* is sensitive to a small change of parameter a when a is smaller than 1.1: the optimum N^* change from 17 to 9. The optimum N^* becomes stable when a is larger than 1.1: it changes from 8 to 7 when a changes from 1.11 to 1.21. The N^* remains even more stable when a is larger than 1.21.
- The expected long-run profit per unit time for smaller a , for example, changing from 1.01 to 1.05, changes faster than that for larger a . As we can image, smaller a 's are more profitable than larger a 's. This is because they require fewer replacements and earn greater profit.

Figure 3 shows all of the changes over parameter a and repair times N , which gives a visual description on the changes of the expected long-run profits, parameter a and failure times N .

Figure 3 here

Figure 3 The expected long-run profit per unit time $P(N)$ against repair times N and parameter a .

6. Conclusions

235 Searching an optimal replacement point for a system maintained by a repairman with multiple vocations
236 is of interest and importance. This paper derived the expected long-run profit per unit time for such a
237 system. We also considered a special scenario where the working times, real repair times, and vacation
238 times are geometric processes. A numerical example is given to illustrate the theoretical results of the
239 model.

240

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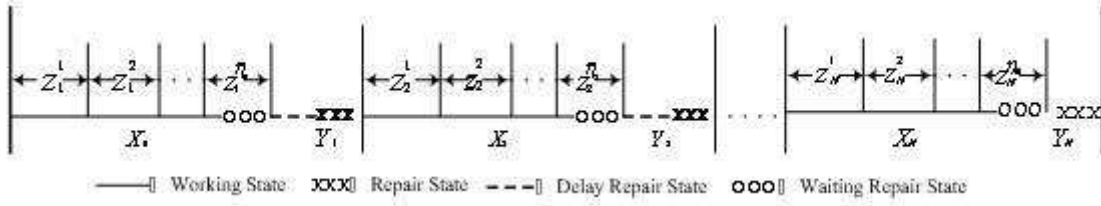
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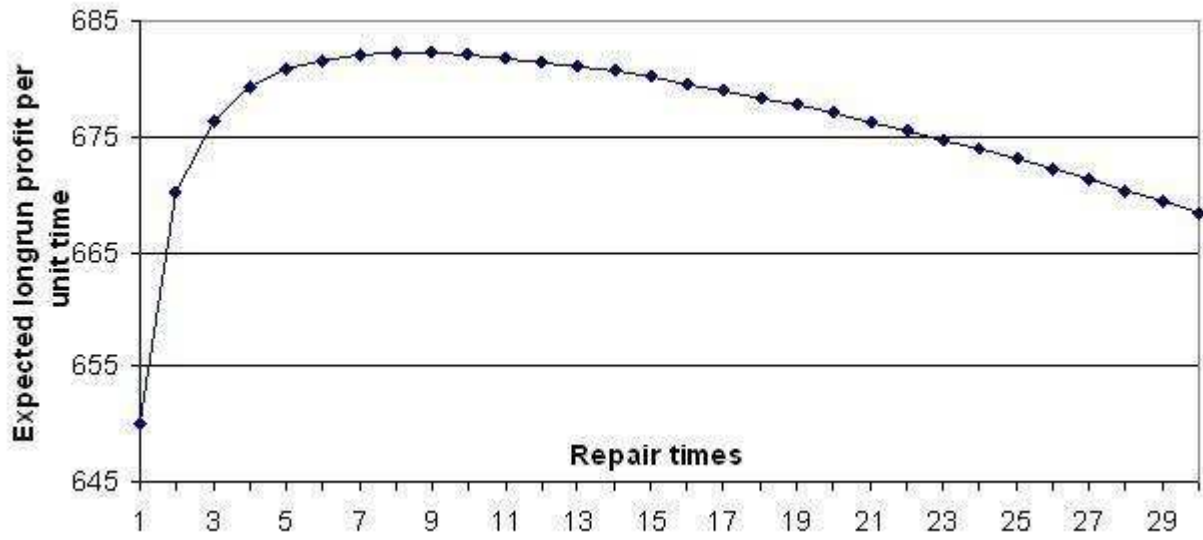
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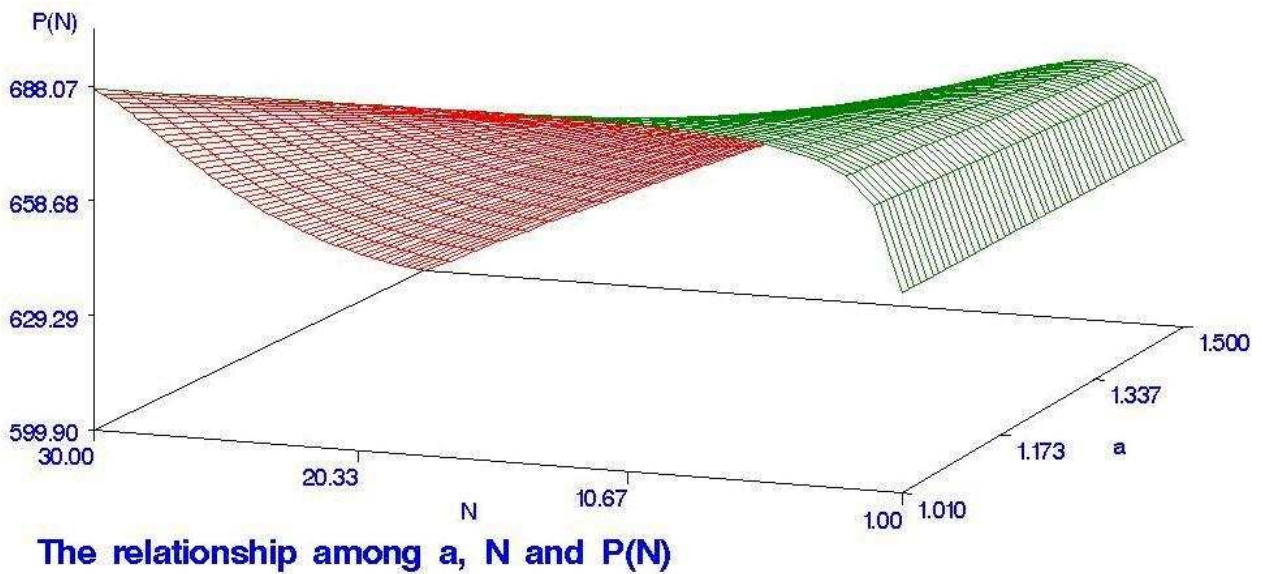
275

276 Figure 1



277

278 Figure 2



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280 Figure 3