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Forecasting warranty claims for recently launched products

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Abstract

Forecasting warranty claims for recently launched products that have short histories of claim records is vitally important for manufacturers in preparing their fiscal plans. Since the amount of historical claim data for such products is not large enough, developing forecasting models with good performance has been a difficult problem.

The objective of this paper is to develop an algorithm for forecasting the number of warranty claims of recently launched products. A two-phase modelling algorithm is developed: in Phase I, we estimate the upper and the lower bounds of the warranty claim rates of the reference products that have been in the market for a longer time; in Phase II, we build forecasting models for the recently launched products and assume that their future claim rates are subject to the bound constraints derived from Phase I. Based on this algorithm, we use the NHPP (non-homogeneous Poisson process) and the constrained maximum likelihood estimation to build forecasting models on artificially generated data as well as warranty claim data collected from an electronics manufacturer. The results show that the proposed algorithm outperforms commonly used NHPP models.

Keywords: Non-homogenous Poisson process (NHPP), warranty claims, forecasting, constrained maximum likelihood estimation.

1 Introduction

In warranty management, forecasting the number of warranty claims is vitally important for manufacturers in preparing their fiscal plans. Starting with

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Kalbfleisch et al. [1], considerable research on forecasting warranty claims has been conducted (see [1–13,21], for example). A recently published review paper on warranty data analysis can be found in [14], which includes comments on different types of warranty forecasting techniques. In existing literature, however, there has been found little research on warranty claim forecasting for products that have short histories of claims (for example, those products that have only 3-month or 6-month claim data), although it is extremely important for the manufacturers to make better prediction of the number of warranty claims for their recently lunched products.

The objective of this paper is to develop an algorithm for forecasting the number of warranty claims of a recently launched product, referred as *target product* in the following. We assume that the manufacturer has already received quite long records of warranty claim data of similar products that the manufacturer has produced and refer such products as *reference products*. A two-phase forecasting algorithm is proposed: in Phase I, we estimate the lower and the upper bounds of warranty claim rates of the reference products; in Phase II, we build forecasting models for the recently launched products and assume that the claim rates of the products lie in the bound constraints derived from Phase I.

Although this work is motivated by the desire to forecast warranty claims, the approach can also be used in similar cases such as inventory planning and insurance claim forecasting, when the historical data of reference products are available.

The remainder of this paper is structured as follows. Section 2 details the problems in warranty claim forecasting. Section 3 proposes an algorithm for forecasting warranty claims of recently launched products. Section 4 applies the proposed algorithm to both artificially generated data and warranty claim data collected from an electronics manufacturer, and compares the performance of the proposed algorithm to the commonly used algorithm. Discussion is carried out in Section 5. The final section concludes the paper.

2 Problem description

We assume that the claim data discussed in this paper are aggregated on a monthly basis. These data can be expressed as shown in Table 1. At a given calendar month T, our objective is to predict the total number of warranty claims that might be reported in the next K months. In Table 1, the number of warranty claims of a type of product shipped to the end-users in months i and then claimed in month j is denoted as $d_{i,j}$, and the predicted number of warranty claims is denoted as $\hat{d}_{i,j}$. 'MoS' stands for the month when the

Table 1
Field warranty data

MoS	Shipments	Calendar months when claims are received.								
		1	2	3	...	T	T+1	T+2	...	T+K
1	S_1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	...	$d_{1,T}$	$\hat{d}_{1,T+1}$	$\hat{d}_{1,T+2}$	...	$\hat{d}_{1,T+K}$
2	S_2		$d_{2,2}$	$d_{2,3}$	...	$d_{2,T}$	$\hat{d}_{2,T+1}$	$\hat{d}_{2,T+2}$	...	$\hat{d}_{2,T+K}$
3	S_3			$d_{3,3}$	...	$d_{3,T}$	$\hat{d}_{3,T+1}$	$\hat{d}_{3,T+2}$	...	$\hat{d}_{3,T+K}$
$\vdots$	$\vdots$				...			$\vdots$		
T	S_k					$d_{T,T}$	$\hat{d}_{T,T+1}$	$\hat{d}_{T,T+2}$	...	$\hat{d}_{T,T+K}$

products are shipped to the end-users.

When T is large, one can develop a forecasting model with good performance as there is a long history of warranty claim data available. Research on such a forecasting problem can be found in [1,3–6,8,10–12,15,16]. Most of those publications, however, conduct little discussion on the availability of historical claim data. For a manufacturer, one of their important concerns is how to forecast the number of warranty claims when T is small. For example, $T = 3$ or $T = 6$, which means the manufacturer has only received warranty claims of the first 3 months or the first 6 months. This usually happens for those recently launched products.

3 Algorithm development

3.1 The algorithm

Modern manufacturing is characterised by often nearly identical products due to common components and technology being used [17]. This property has been used in warranty data analysis such as early detection of reliability problems (see [18], for example). Denote $\hat{r}_k$ as the estimated warranty claim rate of the target product during its k-th month. For early detection of reliability problems, the reference value for $\hat{r}_k$ is denoted by $\hat{r}_k^U$, which can be obtained from previous experience with similar products or design specifications, as assumed in [18]. Then the early detection of reliability problems can be formulated as a test of the multiple-parameter hypothesis [18]:

$$\hat{r}_1 \leq \hat{r}_1^U, \hat{r}_2 \leq \hat{r}_2^U, \ldots, \hat{r}_{T+K} \leq \hat{r}_{T+K}^U \tag{1}$$

The above inequalities have also been used as an assumption in designing early detection algorithm (also see [19,20]). Similarly, we can assume that the other reference values for $\hat{r}_k$, denoted by $\hat{r}_k^L$, which can also be obtained from other reference products and satisfies

$$\hat{r}_1^L \leq \hat{r}_1, \hat{r}_2^L \leq \hat{r}_2, \ldots, \hat{r}_{T+K}^L \leq \hat{r}_{T+K} \tag{2}$$

Denote the estimated claim rate of the i-th reference product in month k as $\hat{r}_{i,k}$. The inequalities in (1) and (2) can be used to build models for forecasting warranty claims, based on which we can derive a new algorithm to build a model for forecasting warranty claims as following:

- Assume that the claim rates of both the reference and the target products are stable over months in service [1].
- If the observed claim rates of the target product in the first T months are larger than the claim rates $\hat{r}_k^L$ derived from a set of reference products and less than the claim rates $\hat{r}_k^U$ derived from this set reference products for $k = 1, 2, ..., T$, then we can assume that the claim rates of the model in months $\{T+1, T+2, ..., T+K\}$ lie in $(\hat{r}_k^L, \hat{r}_k^U)$ for $k = T+1, T+2, ..., T+K$.

The above algorithm can also be re-written in Table 2.

Table 2
The algorithm.

Assumptions:	There are M reference products that have been in the market for $T + K$ months. A target product has been in the market for T months, where $T > 0$.
Phase 1:	Search the warranty claim database and find M_0 reference products from the M products that are similar to the target product with respect to claim rates within the T months and that satisfies $\hat{r}_k^L \leq \hat{r}_k \leq \hat{r}_k^U$ for all of the M_0 products, where $k = 1, 2, ..., T$.
Phase 2:	Develop a warranty forecasting model based on the warranty claims received for the T months of the target product and the claim rates, r_k, derived from the model for products older than T months are subject to $\hat{r}_k^L \leq r_k \leq \hat{r}_k^U$, where $k = T+1, T+2, ..., T+K$.

3.2 Implementation of the algorithm

The non-homogeneous Poisson process (NHPP) is a stochastic process that has been widely used in the reliability engineering as well as in estimating warranty claims (see [10,11] for example).

[1] In practice, due to various reasons such as design modification, the claim rates of a product over month in service can change dramatically.

Denote the total number of warranty claims of the target product in month k as d_k and the total number of the target product on the market as N.

Then the likelihood function of the NHPP is given by:

$$\mathrm{L}_{NHPP} = \prod_{k=1}^{T} \frac{\left(N \int_{k-1}^{k} r(x)dx\right)^{d_k} e^{-\left(N \int_{k-1}^{k} r(x)dx\right)}}{d_k!} \tag{3}$$

where $r(x)$ is the intensity function of the NHPP, and $\mathrm{E}(d_k) = N \int_{t-1}^{t} r(x)dx$.

Thus the optimisation problem, based on the algorithm in Table 2, can be stated as

$$\begin{aligned} &\text{Maximise} \quad \mathrm{L}_{NHPP} \\ &\text{Subject to} \quad \hat{r}_k^L \leq \int_{k-1}^{k} r(x)dx \leq \hat{r}_k^U \ \text{ for } k = T+1, T+2, ..., T+K. \end{aligned} \tag{4}$$

The above algorithm is constrained maximum likelihood estimation (CMLE).

3.3 Discussion

The selection of reference products in Phase I in the algorithm proposed (in Table 2) is very important. For selecting such reference products, expert opinions or mathematical algorithms might be pursued. Mathematical algorithms such as the k-nearest-neighbour (K-NN) algorithm can be applied.

In this paper, we adopt the algorithm, as shown in Table 3.

One of the well-known phenomena of warranty data is *warranty data maturation*. This phenomenon witnesses that there is a tendency for the observed warranty claim frequencies to increase with time [2]. Different approaches have been suggested to tackle warranty data maturation in warranty data analysis. For example, Singpurwalla and Wilson suggested that a better approach would be the consideration of dynamic linear models, with innovation terms that account for the added uncertainties due to a maturation of the data [2]. Weighted regression modelling and weighted maximum likelihood estimation have also been suggested in [12,21]. In the present article, as the proposed algorithm (shown in 2) aims to find a region defined by the lower and upper bounds, which are not fixed values, the phenomena of warranty data maturation does not affect the efficiency of the algorithm significantly. This can be seen from the following data experiments, based on both artificially generated data and data collected from a manufacturer.

Table 3
The two-phase algorithm.

(1)	Define a cut-off value C_o, where $0 < C_o \leq 1$;
(2)	Calculate $\rho_j = \frac{\sum_{i=1}^{T}(\hat{r}_{i,j}-\hat{\mu}_j)(\hat{r}_i-\hat{\mu})}{\sqrt{\sum_{i=1}^{T}(\hat{r}_{i,j}-\hat{\mu}_j)^2 \sum_{i=1}^{T}(\hat{r}_i-\hat{\mu})^2}}$, which is the Pearson product-moment correlation coefficient, where $j = 1, 2, ..., M$, and $\hat{\mu}_j = \sum_{i=1}^{T} r_{i,j}$, and $\hat{\mu} = \sum_{i=1}^{T} r_i$; where $r_{i,j}$ is the claim rate of product j in month i and r_i is the claim rate of the target product in month i;
(3)	Assume that there are M_0 reference products satisfying $\rho_j > C_o$ and the M_0 products have claim rates $\{\hat{r}_{i_1,k}, \hat{r}_{i_2,k}, ..., \hat{r}_{i_{M_0},k}\}$, for $k = 1, 2, ..., T+K$;
(4)	Calculate $\hat{\mu}'_k = \frac{1}{M_0}\sum_{j=1}^{M_0} \hat{r}_{i_j,k}$ and $\hat{\sigma}_k = \frac{1}{M_0-1}\sum_{j=1}^{M_0}(\hat{r}_{i_j,k} - \hat{\mu}'_k)^2$, where $k = 1, 2...T + K$;
(5)	Find $\hat{r}^L_k$ and $\hat{r}^U_k$ so that $\hat{r}^L_k \leq \hat{r}_k \leq \hat{r}^U_k$, where $\hat{r}^L_k$ and $\hat{r}^U_k$ are the least lower bound and the greatest upper bound chosen from $\{\hat{\mu}'_k - 5\hat{\sigma}_k, \hat{\mu}'_k - 4\hat{\sigma}_k, ..., \hat{\mu}'_k + 4\hat{\sigma}_k, \hat{\mu}'_k + 5\hat{\sigma}_k\}$ for $v = -5, -4, -3, ..., 3, 4, 5$;
(6)	Build a model based on Eq. (5).

4 Data experiments

In this section, we evaluate the proposed algorithm shown in Table 3, based on artificially generated data and data collected from an electronics manufacturer, respectively.

We set T=6, 9, and 12, respectively, as the number of months that the recently lunched products have been in the market and K=3, 6, 9, 12, 15, 18, 21, and 24 as prediction horizons, respectively.

For both the artificially generated dataset and the field data, and for given M products, we choose one of the M products as the target product and the rest $M - 1$ products as the candidate reference products for selection. We build models based on the T month data and test the models on the claim data within interval $\{T+1, T+2, ..., T+k\}$. The prediction performance of a model is measured with the following normalised rooted mean squared error (NRMSE):

$$\text{NRMSE} = \sqrt{\frac{\sum_{k=T+1}^{T+K}(d_k - \hat{d}_k)^2}{\sum_{k=T+1}^{T+K} d_k^2}} \tag{5}$$

The experiments are performed for different cut-off values of the Pearson correlation coefficient, C_o=0.6, 0.7, and 0.8, respectively.

4.1 Simulation study

The number of warranty claims received in the k-th month is sampled from a Poisson distribution with mean $\mu_k = N \int_{k-1}^{k} r(x)dx$ for $k = 1, 2, ..., 36$, where $r(x)$ is the hazard rate function of the inverse Weibull distribution, that is,

$$r(x) = \beta\alpha^{\beta}x^{-(\beta+1)} \exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\left\{1 - \exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\right\}^{-1} \tag{6}$$

where the parameters α and β in $r(x)$ are randomly selected from uniform distributions, respectively. We have generated data based on the inverse Weibull distributions with their hazard rates $r(x)$, whose scale parameter α are randomly selected from $[100, 500]$ and whose shape parameter is randomly selected from $[0.2, 0.8]$. The data generated with those different inverse Weibull distributions represent warranty claim data of different products.

We have generated 30 (or $M = 30$) products using the parameters of $r(x)$ as shown in Table 4, and set $N = 20000$ for all products.

Table 4
Parameters of the inverse Weibull distributions.

α	466.23	100.97	329.01	309.33	346.65	395.15	122.27	374.71	324.40	366.03
β	0.390	0.534	0.442	0.435	0.344	0.283	0.394	0.300	0.475	0.460
α	112.28	337.42	206.94	448.50	401.07	390.48	427.22	488.84	120.49	233.01
β	0.465	0.381	0.378	0.399	0.320	0.305	0.334	0.447	0.389	0.363
α	151.03	465.64	464.42	113.19	457.55	495.59	286.92	217.95	308.77	481.33
β	0.520	0.417	0.271	0.374	0.298	0.279	0.422	0.364	0.379	0.309

Table 5 shows the NRMSE values derived from the forecasting models with the maximum likelihood estimation on the artificially generated dataset, where we use Eq. (3) to build the forecasting models. Those values in Table 5 are the average values over the 30 simulated products. It is clear from the table that as T increases, the prediction accuracy improves.

Table 5
Average NRMSE values from the forecasting models based on the maximum likelihood estimation on the artificially generated dataset.

	K = 3	K = 6	K = 9	K = 12	K = 15	K = 18	K = 21	K = 24
T = 6	0.170	0.220	0.207	0.219	0.228	0.238	0.233	0.234
T = 9	0.139	0.138	0.140	0.145	0.151	0.147	0.147	0.150
T = 12	0.131	0.130	0.134	0.137	0.136	0.136	0.137	0.140

Table 6 shows the NRMSE values derived from the forecasting models with the constrained maximum likelihood estimation on the artificially generated dataset, where different Pearson correlation coefficients C_o are applied. We use Eq. (4) to estimate the parameters in the models. Those values in Table 6 are the average values over the 30 simulated products.

Table 6
Average NRMSE values from the forecasting models based on the constrained maximum likelihood estimation on the artificially generated dataset.

		K = 3	K = 6	K = 9	K = 12	K = 15	K = 18	K = 21	K = 24
	T = 6	0.143	0.171	0.159	0.166	0.171	0.176	0.173	0.173
$C_o = 0.60$	T = 9	0.141	0.137	0.138	0.142	0.148	0.144	0.144	0.146
	T = 12	0.126	0.126	0.131	0.134	0.133	0.132	0.133	0.136
	T = 6	0.139	0.166	0.157	0.164	0.168	0.173	0.171	0.171
$C_o = 0.70$	T = 9	0.139	0.137	0.139	0.143	0.150	0.146	0.145	0.148
	T = 12	0.128	0.128	0.134	0.136	0.135	0.134	0.135	0.139
	T = 6	0.137	0.164	0.156	0.163	0.168	0.173	0.170	0.170
$C_o = 0.80$	T = 9	0.140	0.138	0.139	0.145	0.150	0.147	0.146	0.149
	T = 12	0.129	0.128	0.134	0.136	0.135	0.135	0.136	0.139

We then use the t-test (using *paired two sample for means* in Microsoft Excel©) to compare the NRMSE values with the same combination (T, K) (for example, T=6 and K=9) in Table 5 and Table 6, and found the assumption that the NRMSE values shown in Table 5 are larger than those in Table 6 is statistically significant with a level of significance 0.95, except the case when $C_o = 0.8$ and $T = 9$.

To further investigate the performance of the models, we create another table, Table 7, to compare the averages of NRMSE values under each combination (T, K) based on Tables 5 and 6. The percentage columns represent the difference between the NRMSE averages obtained with the constrained maximum likelihood estimation and the NRMSE averages obtained with the maximum likelihood estimation. The negative values show that the proposed algorithm, i.e., the NHPP with a constrained maximum likelihood estimation, outperforms the commonly used NHPP, which is especially the case for $T = 6$. It also shows that the model performance of the proposed algorithm becomes similar to that of commonly used NHPP for larger T, as expected.

Table 7
Comparing the average NRMSE values over K's, based on Table 5 and Table 6.

	ML	CML					
		$C_o = 0.60$		$C_o = 0.70$		$C_o = 0.80$	
	average	average	percentage	average	percentage	average	percentage
T = 6	0.219	0.166	-23.9%	0.164	-25.1%	0.163	-25.7%
T = 9	0.145	0.143	-1.4%	0.143	-0.80%	0.144	-0.18%
T = 12	0.135	0.131	-2.7%	0.134	-1.2%	0.134	-0.88%

4.2 Validation of the algorithm on warranty claims of an electronic manufacturer

The warranty claim data used in this paper are provided by an electronics manufacturer. Its name will not be disclosed here for the confidentiality reason. The products are internet networking equipment. The original dataset includes three variables: monthly sales amounts, monthly warranty claims, and months in service (MIS).

We present a figure, see Figure 1, showing the warranty claim rates of the 9 products (or $M = 9$). The warranty claims data have the following properties:

- The warranty claims are aggregated on a monthly basis.
- The warranty policy of the products is a long-term warranty policy. It expires after the technological obsolescence is announced.
- The products are repairable. Once a claim is received, the manufacturer will immediately send out an identical product with the same age to the customer. The received (or claimed) product will be tested and/or repaired.

In order to use the NHPP model, we further assume that the repair is minimal repair.

Figure 2 shows the bounds derived in Phase I, after the first five steps in the algorithm shown in Table 3 are conducted.

Table 8 shows the NRMSE values derived from the maximum likelihood method, (i.e., with the Eq. (3)).

Table 8
Prediction NRMSE estimated using constrained maximum likelihood method on the case study dataset as an average over 9 tests.

	K = 3	K = 6	K = 9	K = 12	K = 15	K = 18	K = 21	K = 24
T = 6	0.477	0.728	0.790	0.907	0.946	1.011	1.073	1.126
T = 9	0.917	0.885	1.052	1.036	1.113	1.185	1.251	1.171
T = 12	0.637	0.774	0.760	0.816	0.867	0.922	0.865	0.884

Table 9 shows the NRMSE values derived from the constrained maximum likelihood method, (i.e., with the Eq. (4)).

We have used the t-test (using *paired two sample for means* in Microsoft Excel©) to compare the NRMSE values with the same combination (T, K) (for example, T=6 and K=9) in Table 8 and Table 9, and found the assumption that the NRMSE values shown in Table 5 are larger than those in Table 6 is statistically significant with a level of significance: 0.05.

To further investigate the performance of the models, we create another table,

Table 9
Prediction NRMSE estimated using constrained maximum likelihood on the case study dataset as an average over 9 tests.

		K = 3	K = 6	K = 9	K = 12	K = 15	K = 18	K = 21	K = 24
	T = 6	0.356	0.510	0.537	0.615	0.625	0.665	0.705	0.741
$C_o = 0.60$	T = 9	0.776	0.715	0.849	0.822	0.877	0.932	0.981	0.888
	T = 12	0.451	0.606	0.599	0.637	0.677	0.719	0.652	0.669
	T = 6	0.356	0.510	0.537	0.615	0.625	0.665	0.705	0.741
$C_o = 0.70$	T = 9	0.818	0.757	0.904	0.882	0.946	1.004	1.058	0.971
	T = 12	0.458	0.628	0.628	0.674	0.715	0.760	0.695	0.714
	T = 6	0.329	0.520	0.569	0.657	0.676	0.723	0.768	0.809
$C_o = 0.80$	T = 9	0.863	0.762	0.907	0.878	0.937	0.993	1.043	0.951
	T = 12	0.632	0.769	0.757	0.813	0.863	0.918	0.860	0.880

Table 10, to compare the averages of NRMSE values under each combination (T, K) based in Table 8 and Table 9. The percentage columns represent the difference between the NRMSE averages obtained using the constrained maximum likelihood estimation and the NRMSE averages obtained using the maximum likelihood estimation, respectively. The negative values show that the proposed algorithm, i.e., the NHPP with constrained maximum likelihood estimation, outperforms the commonly used NHPP.

Table 10
Comparing the average NRMSE values over K's, based on Table 8 and Table 9.

	ML	CML					
		$C_o = 0.60$		$C_o = 0.70$		$C_o = 0.80$	
	average	average	percentage	average	percentage	average	percentage
T = 6	0.882	0.594	-32.66%	0.594	-32.66%	0.631	-28.44%
T = 9	1.076	0.855	-20.56%	0.918	-14.75%	0.917	-14.81%
T = 12	0.815	0.626	-23.22%	0.659	-19.18%	0.812	-0.46%

5 Conclusions

This paper developed an algorithm for forecasting warranty claims for recently launched products, i.e., target products. We develop a constrained maximum likelihood estimation, where the objective function is the likelihood function based on warranty claims observed from the target products and the constraints are the claim rates estimated from the reference products.

The data experiments show that the proposed algorithm outperforms the commonly used approach, especially for the case when the history of the warranty claims of the target products is short.

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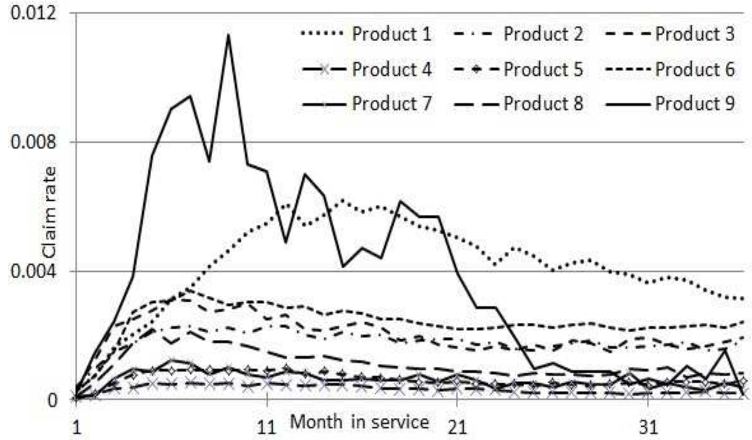

Fig. 1. Warranty claim rates of the 9 products in the case study.

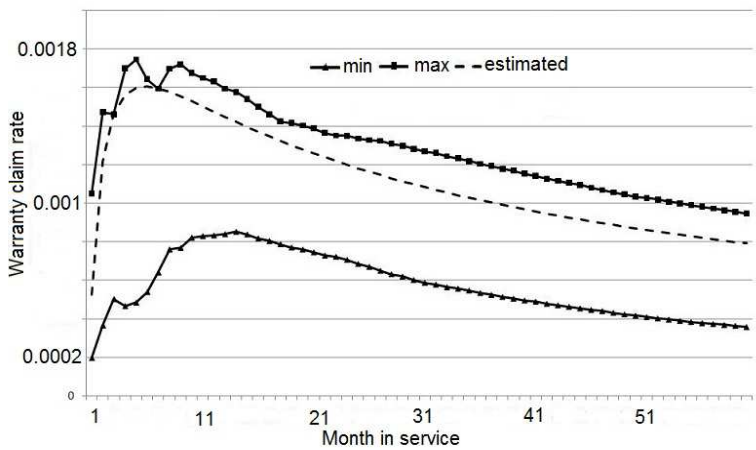

Fig. 2. Estimated warranty claim rates and the bounds for one of the products in the case study.