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Relativistic analysis of the pairing symmetry of the noncentrosymmetric superconductor LaNiC$_2$

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(Received 27 May 2010; revised manuscript received 18 October 2010; published 16 November 2010)

We present a relativistic symmetry analysis of the allowed pairing states in the noncentrosymmetric superconductor LaNiC$_2$. The case of zero spin-orbit coupling (SOC) is discussed first and then the evolution of the symmetry-allowed superconducting instabilities as SOC is adiabatically turned on is described. In addition to mixing singlet with triplet pairing, SOC splits some triplet pairing states with degenerate order-parameter spaces into nondegenerate pairing states with different critical temperatures. We address the breaking of time-reversal symmetry detected in recent muon spin-relaxation experiments and show that it is only compatible with such nonunitary triplet pairing states. In particular, an alternative scenario featuring conventional singlet pairing with a small admixture of triplet pairing is shown to be incompatible with the experimental data.

DOI: 10.1103/PhysRevB.82.174511 PACS number(s): 74.20.Rp, 71.70.Ej

I. INTRODUCTION

Noncentrosymmetric superconductors have been a subject of considerable interest since the discovery of superconductivity in the heavy-fermion material CePt$_3$Si.1 In particular, it is the unique property of such noncentrosymmetric superconductors that in the presence of spin-orbit coupling (SOC) both spin singlet and spin triplet Cooper pairs can, and must, coexist within a single material. This is quite general as while in the complete absence of SOC the two kinds of pairing are distinguished by their different behavior under rotations in spin space, once SOC is finite then spin and space rotations cannot be separated and it is only the parity of the Cooper pair wave function under spatial inversion, $P$, which separates spin singlet (even) from spin triplet (odd) states.2–6 In a noncentrosymmetric superconductor there is no lattice center of inversion and so the parity operator, $P$, is not a well-defined symmetry of the crystal, leading to mixing of singlet and triplet pairing states within a single material. An interesting analogy can be made with particle physics where the mixing of neutrino flavors is induced by violation of CP symmetry.7 The implication is that in noncentrosymmetric superconductors the order parameter is always unconventional. On the other hand the experimental situation is quite complex as some noncentrosymmetric superconductors such as CePt$_3$Si are additionally strongly correlated while the superconducting state of others such as Li$_3$Pd$_3$B, BaPtSi$_3$, or Re$_3$W appears to feature pure singlet pairing.3–10

An important recent development has been the observation, through zero-field muon spin resonance ($\mu$SR), of time-reversal symmetry (TRS) breaking at the superconducting instability of LaNiC$_2$.11 Superconductivity in this intermetallic compound was discovered in the mid 1990s with critical temperature $T_c=2.7$ K.12 There was some discussion of whether it was a type-II or a dirty type-I superconductor14 and the possibility that the symmetry of the superconducting order parameter was unconventional was debated.13,14 At the time, however, the lack of inversion symmetry was largely overlooked. In contrast, very recently there has been a surge of experimental11,15,16 and theoretical11,17–19 work on this system. Some of this has been motivated by the results in Ref. 11 which constitute very strong and direct evidence of unconventional pairing. In addition to this dramatic dependence of $T_c$ on Cu, Y, and Th doping have been identified.15,16,20

Two broad and mutually exclusive scenarios have been proposed to describe the breaking of TRS in the superconducting state of LaNiC$_2$.11,19 In the first scenario, which is based on group-theoretical considerations,11 the superconducting order parameter is intrinsically unconventional: a nonunitary triplet pairing state. In the second scenario, based on first-principles calculations,19 LaNiC$_2$ is essentially a conventional superconductor but a small amount of triplet pairing is induced by SOC, as described above and is responsible for the observed breaking of TRS. Unfortunately both the group-theoretical analysis of Ref. 11 and the first-principles calculations of Ref. 19 ignore relativistic effects. It is therefore unclear whether any of the eight superconducting instabilities that are allowed by symmetry and that preserve TRS (Ref. 11) acquire a TRS breaking component when SOC is adiabatically turned on. More specifically it is not known whether the conventional superconducting state assumed in Ref. 19, which does not break TRS, can acquire the necessary TRS breaking component in this way. Indeed it is well known22 that TRS breaking requires a superconducting order parameter with degeneracy. However no such degeneracy should occur in an orthorhombic crystal with finite SOC. In the present work we address this question directly by extending the previous symmetry analysis of Ref. 11 to include the effect of SOC. The more general analysis that we present here allows us to conclude that the observation of time-reversal symmetry breaking at $T_c$ is not compatible with a conventional mechanism of the type proposed in Ref. 19.

To address the pairing symmetry in the LaNiC$_2$ crystal structure we first consider, following the original analysis, the possible pairing states if spin-orbit interaction is negligible. We then study how these states evolve when perturbed by SOC. In particular, we note in this paper that simply a combination of $s$-wave pairing and noncentrosym-
metric crystal structure does not automatically lead to time-
reversal symmetry breaking at $T_c$. It turns out that the low
symmetry of the orthorhombic Amm2 structure leads to only
a small number of time-reversal symmetry-breaking states in
the absence of SOC, all of which have degeneracy which is
lifted when spin-orbit interaction is finite. Therefore the ob-
ervation of time-reversal symmetry breaking at $T_c$ provides
a very strong constraint on the pairing state and is not natu-
really consistent with the conventional electron-phonon pair-
ing mechanism or $s$-wave pairing. Instead, the observation is
only compatible with SOC being small and with the system
entering a nonunitary triplet pairing state at $T_c$.

Our arguments are based on group theory and in that spirit
the present analysis of the pairing symmetry in LaNiC$_2$ does
not rely on any specific assumptions about the origin of the
pairing interaction, the band structure or the strength of SOC.
The method is very well established and has been very suc-
cessful in the past for many other superconductors with a
center of inversion, e.g., the cuprates. More recently similar
methods have been applied to noncentrosymmetric
superconductors. We will nevertheless describe some of the
main arguments in considerable detail to highlight the
issue of TRS breaking, both in the presence and absence of
SOC, as well as the features specific to the point symmetry
of LaNiC$_2$.

II. SYMMETRY ANALYSIS IN THE ABSENCE
OF SPIN-ORBIT COUPLING

The possible symmetries of the superconducting in-
stability in LaNiC$_2$ assuming that SOC can be neglected were
enumerated in Ref. 11. In this section we give the details of
the derivation emphasizing the similarities and differences
with the case where there is a center of inversion. In the
absence of spin-orbit coupling, the point group $G$ is

$$G = G_c \times SO(3),$$

where $\times$ represents the direct product, $G_c$ is the point group
of the crystal structure and $SO(3)$ represents all spin rota-
tions. The irreducible representations therefore have the form
$\Gamma = \Gamma_c \times \Gamma_s$, where $\Gamma_c$ and $\Gamma_s$ are irreducible represen-
tations of $G_c$ and $SO(3)$, respectively (in principle, the full space
group of the crystal must be taken into account; however, we
assume that the translational symmetries are the same above
and below $T_c$, so it is enough to refer to the point group).
A basis of $\Gamma$ is given by the functions $\hat{1}_{\mu}(k) = \Gamma_{\mu}(k)\mathbf{1}_n$, where
$\{\Gamma_{\mu}(k)\}_{\mu=1, \ldots, d_{\Gamma}}$ forms a basis of $\Gamma_c$ and $\{\mathbf{1}_n\}_{n=1, \ldots, d_{\Gamma_s}}$ forms a
basis of $\Gamma_s$. The dimensionality of $\Gamma$ is $d_{\Gamma} = d_{\Gamma_s}d_{\Gamma_s}$. The gap
function just below $T_c$ is thus

$$\hat{\Delta}(k) = \sum_{\mu=1}^{d_{\Gamma}} \sum_{n=1}^{d_{\Gamma_s}} \eta_{\mu n} \Gamma_{\mu}(k)\mathbf{1}_n.$$ 

The spin rotation group $SO(3)$ is the same for all crystals.
As is well known it has two irreducible representations (ir-
reps). The first of these is the singlet representation, of
dimension 1. This corresponds to order parameters of the form

$$\hat{\Delta}(k) = \sum_{\mu=1}^{d_{\Gamma}} \eta_{\mu \mu} \Gamma_{\mu}(k)\mathbf{1}_n$$

meaning that we must have $\Gamma_{\mu}(k) = \Gamma_{\mu}(-k)$. Thus for singlet
order parameters only the first term in

$$\hat{\Delta}(k) = \Delta(k) = \Delta(k) i\hat{\sigma}_y + [\mathbf{d}(k) \cdot (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)]\hat{\sigma}_y$$

is finite. The second irrep of $SO(3)$ is the triplet representa-
tion, of dimension 3. For it we thus have

$$\hat{\Delta}(k) = \sum_{n=1}^{d_{\Gamma_s}} \eta_{mn} \Gamma_{\mu}(k)\mathbf{1}_n,$$

where

$$\Gamma_\mu(k) = \sum_{n=1}^{d_{\Gamma_s}} \eta_{mn} \Gamma_{\mu n}(k).$$

Moreover we have

$$\Gamma_{\mu n}(k) = (\Gamma_{\mu n}(k))^T$$

whereby the $G_c$ basis functions must be odd, $\Gamma_{\mu n}(k) = -\Gamma_{\mu n}(-k)$, meaning that for triplet pairing in $\hat{\Delta}(k)$ has only the second term.

The above results are very well known from the group-
theory analysis of nonsymmetric superconductors. They
are also valid in the centrosymmetric case as long as SOC can be neglected. In particular, the pairing symmetry
must be purely of the singlet or triplet type in the limit in
which SOC does not play a role. The only difference with the
case of centrosymmetric superconductors is that in a noncen-
trosymmetric superconductor the irreps of the crystal point
group do not have distinct symmetries under inversion, so
each of them is compatible with both singlet and triplet pairing.
Thus in LaNiC$_2$, where $G_c = C_{2v}$, each of the four irreps
$A_1$, $A_2$, $B_1$, and $B_2$ [Table I in Ref. 11] is compatible with
singlet and triplet superconducting instabilities. Since in this
case all four irreps of $G_c$ are one-dimensional, this leads to
a total of 12 possible instabilities: four in the singlet channel
and eight in the triplet channel (see Ref. 11 for details). The
possible symmetries of the gap function are reproduced in
Table I here for completeness. Note that the nonunitary
triplet pairing instabilities $A_1(b), A_2(b), B_1(b),$ and $B_2(b)$ are the only ones that break TRS, leading to the conclusion that
the superconducting state just below $T_c$ features nonunitary
triplet pairing. As noted in that reference one of these four
forms of the gap function has the same point-group sym-
metry as the crystal, which would not have been possible for
triplet pairing in a centrosymmetric superconductor. The
other three break additional symmetries. In the following

<table>
<thead>
<tr>
<th>Irrep of $SO(3) \times C_{2v}$</th>
<th>$\Delta_0(k)$</th>
<th>$\mathbf{d}(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$XY$</td>
<td>0</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$XZ$</td>
<td>0</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$YZ$</td>
<td>0</td>
</tr>
<tr>
<td>$A_1(a)$</td>
<td>0</td>
<td>(0,0,1)Z</td>
</tr>
<tr>
<td>$A_2(a)$</td>
<td>0</td>
<td>(0,0,1)XYZ</td>
</tr>
<tr>
<td>$B_1(a)$</td>
<td>0</td>
<td>(0,0,1)X</td>
</tr>
<tr>
<td>$B_2(a)$</td>
<td>0</td>
<td>(1,0,0)Z</td>
</tr>
<tr>
<td>$A_1(b)$</td>
<td>0</td>
<td>(1,0,0XYZ)</td>
</tr>
<tr>
<td>$A_2(b)$</td>
<td>0</td>
<td>(1,0,0X)</td>
</tr>
<tr>
<td>$B_1(b)$</td>
<td>0</td>
<td>(1,1,0)XYZ</td>
</tr>
<tr>
<td>$B_2(b)$</td>
<td>0</td>
<td>(1,1,0)Y</td>
</tr>
</tbody>
</table>

TABLE I. Possible symmetries of the gap function of LaNiC$_2$ just below $T_c$ in the case where SOC can be neglected, written in terms of $\Delta_0(k)$ and $\mathbf{d}(k)$ in Eq. (2). Each of the functions $X, Y,$ and $Z$ depend on the wave vector $k$ and they have the same symmetries under the operations of the point group $C_{2v}$ as its three components $k_x, k_y,$ and $k_z$, respectively.
section we analyze how this conclusion is affected by the inclusion of SOC in the analysis.

III. SYMMETRY ANALYSIS IN THE PRESENCE OF SPIN-ORBIT COUPLING

Now suppose that spin-orbit coupling is strong enough that it cannot be neglected. Then, as in the case of centroindie superconductors, \( G = G_{G, j} \), which is the “double group” obtained by appending to each rotation carried out on the coordinates in \( G \) an equivalent operation carried out on the spins. Take, for example, the reflection through the \( x \)-\( z \) plane contained in the point group of the \( \text{LaNiC}_2 \) crystal structure, \( C_{2v} \). This is \( \sigma_{x z} = i C_{z} \), where \( I \) represents inversion through the central point and \( C_{z} \) a rotation by \( 180^\circ \) around the \( y \) axis. Then \( G_{G, j} \) contains the similar operation, \( \sigma_{x z, j} \), involving this reflection as well as a \( C_{z} \) rotation carried out on the spins (i.e., a rotation of the \( d \) vector). The gap function just below \( T_c \) is now \( \hat{\Delta}(k) = \sum_{j} \eta_j \hat{\Gamma}_j(k) \), where \( \hat{\Gamma}_j(k) \) is the \( j \)th basis function of the irrept \( \Gamma \) of \( G_{G, j} \). In general, unlike the case of vanishing SOC, the gap function is not of the singlet or triplet forms. Note, however, that such mixture of the singlet and triplet channels occurs only when both of the following conditions are met: (i) there is no center of inversion and (ii) SOC cannot be neglected. As has been extensively remarked,\(^{26-30}\) this makes noncentrosymmetric superconductors special in that SOC has a more dramatic effect on the pairing symmetry than it has in centrosymmetric superconductors.\(^{23,27} \)

On the other hand that is quite different from saying that SOC has to be strong in these systems. Indeed, as we will see shortly in the case of \( \text{LaNiC}_2 \) it is difficult to reconcile the observation of TRS breaking\(^{11} \) with SOC being strong.

Through SOC, spin rotations cease to be independent degrees of freedom. Thus unlike the case of zero SOC the irreps of \( G_{G, j} \) are in one-to-one correspondence to those of \( G_c \). For \( \text{LaNiC}_2 \) this leads to a dramatic reduction in the number of symmetry-allowed superconducting instabilities of the normal state from 12 when SOC can be neglected (see above) to only four, corresponding to the four irreps of the point group of the crystal structure. The basis functions depend both on \( k \) and the spin indices (i.e., they are matrices), just like the basis functions of the irreps of \( G_c \times SO(3) \). Constructing the four symmetry operations \( E_c, C_{2v}, \sigma_{x z, j} \), and \( \sigma_{y z, j} \) in the way described above one can find a set of basis functions that is compatible with the group’s character table [Table I in Ref. 11]. One such set is given in Table II. The \( A, B, C, \) and \( D \) coefficients should be determined by a microscopic theory but should be real. Note that, as a direct result of all the irreps of \( G_c \) being one-dimensional [Table I in Ref. 11], all the possible order parameters just below \( T_c \) are one-dimensional, too. Since a one-dimensional order parameter cannot break TRS (Ref. 22) we are led to the inescapable conclusion that the superconducting instability in \( \text{LaNiC}_2 \) can only break TRS if SOC is negligible. In view of the experimental observation of TRS breaking,\(^{11} \) this suggests that the effect of SOC on the superconductivity must be small and confirms our original conclusion,\(^{11} \) reached on the basis of a nonrelativistic analysis, of nonunitary triplet pairing.

Note that the case of the orthorhomic-symmetry group \( C_{2v} \) appropriate for \( \text{LaNiC}_2 \) is quite different from the tetragonal \( \Gamma_{4v} \) appropriate to \( \text{CePt}_3\text{Si} \).\(^{24-26} \) For \( \Gamma_{4v} \) one of the irreducible representations is two-dimensional so time-reversal symmetry breaking is allowed even in the presence of strong SOC. The point-group studied here is also somewhat different from the monoclinic \( C_{2} \), studied by Sergienko and Curnoe.\(^{24} \) In this case there is only one twofold rotation axis and hence only two irreducible representations, \( A_1 \) and \( A_2 \), both one-dimensional. Nevertheless the general pattern of possible symmetry breakings for \( C_{2v} \) is similar to those given in Tables I and II. Under \( C_{2v} \) the \( A_1 \) representation is equivalent to both \( A_1 \) and \( A_2 \) of \( C_{2v} \), while the \( A_2 \) representation of \( C_{2v} \) is equivalent to \( B_1 \) and \( B_2 \) under \( C_{2v} \).

IV. SPIN-ORBIT COUPLING-INDUCED SPLITTING OF THE SUPERCONDUCTING INSTABILITY

Our main conclusion so far is that the observation of TRS symmetry breaking implies that SOC must be very weak, for no TRS breaking superconducting instability of the normal state is compatible with the crystal’s symmetry in the presence of SOC. One the other hand, a small amount of SOC must be present in any crystal, which raises the question of how the results of Secs. II and III can be reconciled. To clarify this we consider the evolution of the instability as a small amount of SOC is adiabatically turned on.

Each of the symmetry-allowed superconducting instabilities listed in Table I will evolve into one of those listed in Table II, as shown in Fig. 1. To ascertain the relationships depicted in the figure, we must express the gap function given in Table I as a linear combination of those in Table II. Such linear combinations are unique. In particular, the \( k \) dependences of the gap function just below the singlet superconducting instabilities are given by

\[
\hat{\Gamma}_A(k) = \hat{\Gamma}_A(k)_{\|A,B,C,D=1,0,0,0^+} \\
\hat{\Gamma}_B(k) = \hat{\Gamma}_B(k)_{\|A,B,C,D=1,0,0,0^+} \\
\hat{\Gamma}_C(k) = \hat{\Gamma}_C(k)_{\|A,B,C,D=1,0,0,0^+}
\]

Table II. Possible symmetries of the gap function of \( \text{LaNiC}_2 \) just below \( T_c \) in the case where SOC cannot be neglected. \( A, B, C, \) and \( D \) denote four \( k \)-independent quantities with the same phase. All other notations as in Table I.

<table>
<thead>
<tr>
<th>Irrep of ( C_{2v,j} )</th>
<th>( \Delta_0(k) )</th>
<th>( d(k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>( A )</td>
<td>( (BY,CX,DXYZ) )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( AXY )</td>
<td>( (BX,CY,DZ) )</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>( AZY )</td>
<td>( (BXYZ,CZ,DY) )</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>( AVZ )</td>
<td>( (BZ,CXYZ,DX) )</td>
</tr>
</tbody>
</table>
Moreover adiabatic continuity in the limit of vanishing SOC
situation is somewhat more complicated. Since they break
Finally, for the four nonunitary triplet pairing instabilities the
situations is arbitrary. The relative temperatures of the different
instabilities in this diagram are
Thus the $A_1$, $A_2$, $B_1$, and $B_2$ instabilities evolve into
instabilities with $A_1$, $A_2$, $B_1$, and $B_2$ symmetries, respectively.
Moreover adiabatic continuity in the limit of vanishing SOC
places constraints on the coefficients $A$, $B$, $C$, and $D$ in Table II:
in order that the coefficients $A$, $B$, and $C$ vanish in the
limit of zero SOC, it is necessary for them to be small, compared
to $A$, when SOC is weak but finite. By this mechanism
a small triplet component [i.e., a finite $\delta(d(k))]$ could be induced
in an otherwise singlet superconductor by the action of
SOC alone. Note, however, that such triplet component does
not break TRS. This is at variance with the claim made in
Ref. 19, as we discuss in detail in Sec. V.

Similarly, the $k$ dependences of the gap function just below
the four unitary triplet pairing instabilities are also in
one-to-one correspondence with those of the relativistically
allowed ones
Finally, for the four nonunitary triplet pairing instabilities the
situation is somewhat more complicated. Since they break
TRS, they cannot evolve smoothly into one of the four
symmetry-allowed instabilities as SOC is turned on, as all of
them preserve TRS. Indeed the gap matrix just below $T_c$ is a
linear combination of two of the forms allowed in the
presence of SOC

$$\hat{\Gamma}_{B_2}(k) = \hat{\Gamma}_{B_2}(k)_{A,B,C,D=1,0,0,0} \quad (6)$$

$$\hat{\Gamma}_{A_1(a)}(k) = \hat{\Gamma}_{A_1(a)}(k)_{A,B,C,D=0,0,0,1} \quad (7)$$

$$\hat{\Gamma}_{A_2(a)}(k) = \hat{\Gamma}_{A_2(a)}(k)_{A,B,C,D=0,0,0,1} \quad (8)$$

$$\hat{\Gamma}_{B_1(a)}(k) = \hat{\Gamma}_{B_1(a)}(k)_{A,B,C,D=0,0,0,1} \quad (9)$$

$$\hat{\Gamma}_{B_2(a)}(k) = \hat{\Gamma}_{B_2(a)}(k)_{A,B,C,D=0,0,0,1} \quad (10)$$

This implies that, unlike the singlet and unitary triplet instabilities
the nonunitary triplet instabilities split under the influence of SOC:
as SOC is increased the critical temperature $T_c$ splits into two transitions, one in which the order parameter takes one form and a second one where another component develops. The first transition does not break TRS but the second one does (it wouldn’t if the system went into that state straight from the normal state; TRS breaking is due to the presence of the other component of the order parameter and their relative phase, which is fixed by the requirement that the correct form is recovered in the limit of zero SOC). In the limit of weak SOC, the two transitions happen so close that they are indistinguishable from a single transition going straight into the state with broken TRS.

V. DISCUSSION

Figure 1 shows the evolution of the superconducting instabilities allowed by symmetry in the absence of SOC as the latter is adiabatically turned on. We can pose the opposite question, which is: in the presence of strong SOC, how is a general pairing state decomposed into the components that would be allowed in its absence? This is shown in Fig. 2. We note that in general the pairing states allowed in the presence of SOC contain singlet, unitary and nonunitary triplet components. Interestingly, the nonunitary states, which are the
only ones that can break TRS, are always shared between
two different strong SOC pairing states. On the other hand

$\hat{\Gamma}_{A_1(b)}(k) = \hat{\Gamma}_{B_1(b)}(k)_{A,B,C,D=0,1,0,0} + i\hat{\Gamma}_{B_1(b)}(k)_{A,B,C,D=0,0,0,1}$

$\hat{\Gamma}_{A_2(b)}(k) = \hat{\Gamma}_{B_1(b)}(k)_{A,B,C,D=0,1,0,0} + i\hat{\Gamma}_{B_1(b)}(k)_{A,B,C,D=0,0,0,1}$

$\hat{\Gamma}_{B_1(b)}(k) = \hat{\Gamma}_{A_1(b)}(k)_{A,B,C,D=0,1,0,0} + i\hat{\Gamma}_{A_1(b)}(k)_{A,B,C,D=0,0,0,1}$

$\hat{\Gamma}_{B_2(b)}(k) = \hat{\Gamma}_{A_1(b)}(k)_{A,B,C,D=0,1,0,0} + i\hat{\Gamma}_{A_1(b)}(k)_{A,B,C,D=0,0,0,1}$

FIG. 1. Evolution of all the superconducting instabilities of the
normal state of LaNiC$_3$ allowed by symmetry in the absence of
SOC as the latter is adiabatically turned on: (a) singlet pairing
instabilities; (b) unitary triplet pairing instabilities; and (c) nonunitary
triplet pairing instabilities. The relative temperatures of the different
instabilities in this diagram are arbitrary.

FIG. 2. (Color online) Venn-Euler diagram showing how the
four pairing states allowed just below $T_c$ in the presence of spin-
orbit coupling decompose into the 12 singlet, nonunitary triplet, and
unitary triplet states allowed in its absence.
the singlet s-wave state never contributes to a TRS breaking instability. Also interestingly, as shown in Fig. 2, the singlet $^1A_1$ state does mix with several triplet states, including part of the nonunitary triplet pairings $^3B_1(2b)$ and $^3B_2(2b)$. Nevertheless, and somewhat counterintuitively, none of these combinations break TRS.

In the light of the above analysis let us now consider possible pairing states in LaNiC$_2$. The authors of Ref. 19 have argued that the normal state of LaNiC$_2$ is weakly correlated and that the superconducting instability is of the conventional s-wave type, resulting from phonon-mediated pairing of electrons. The justification provided for these assumptions is that a value of $T_c$ very close to that encountered in the experiments follows from them. To explain the observation of TRS breaking, a small triplet component induced by SOC is invoked. Indeed an order parameter with $^1A_1$ symmetry would develop a small triplet component as a result of SOC, as shown in Eq. (3) and Fig. 2. Unfortunately, however, such triplet component is not TRS breaking: only the nonunitary triplet pairing instabilities can break TRS. These instabilities, on the other hand, only have a small s-wave component, which vanishes completely as SOC is turned off. Our results imply that only nonunitary triplet pairing is compatible with the observation of TRS breaking.

A second consequence of our results, as shown in Fig. 1, is that the superconducting instability must be split by SOC. Since this only happens for the nonunitary triplet pairing instabilities, the observation of a split transition would be a direct consequence of TRS breaking and confirm the nonunitary triplet pairing in this system. On the other hand, given that it has not been detected in any experiment to date, the splitting must be quite small. Its observation may require the availability of single crystals, where any splitting may be more easily observed.

An outstanding issue is the quantitative estimation of the size of SOC in LaNiC$_2$. The band splitting has been calculated perturbatively using as the starting point a band structure obtained in the local-density approximation (LDA). An average band splitting of $\sim 3.1$ mRy, about half the value of that obtained by a similar method in the noncentrosymmetric heavy fermion superconductor CePt$_3$Si, was found. Given that the critical temperature of LaNiC$_2$ is about three times higher than that of CePt$_3$Si this suggests that the possible role played by SOC in LaNiC$_2$ is smaller. That said, even in this case the obtained splitting is an average and for some parts of the Fermi surface it can be either larger or smaller than that value. The importance of SOC thus depends on a number of details that are as yet unknown, such as the exact functional form of the superconducting order parameter. In any case the average value is much larger than the superconducting gap and of the same order of magnitude as the Debye energy. Yet as we have shown above if SOC had a strong effect on the superconducting instability the latter would not break TRS, which is at variance with the experimental data. We note that LDA-based estimates of SOC have been called into question in the case of the heavy fermion noncentrosymmetric superconductor CePt$_3$Si (Ref. 31) where de Haas-van Alphen oscillations have failed to detect the predicted band splitting.

All discussions so far of the implications of the observation of TRS breaking in LaNiC$_2$, including the one presented here, assume that this is a bulk phenomenon. However albeit very pure, the samples on which this was observed were polycrystalline. A distinct possibility is that the observations could correspond to a breaking of TRS at the boundaries between crystallites. On such surfaces the crystal symmetry is broken and the list of symmetry-allowed superconducting instabilities is altered. On the other hand in the experiment described in Ref. 11 muons were deposited uniformly throughout the bulk of the sample. Any magnetic fields occurring only at the boundaries between crystallites would have been screened over distances of the order of the penetration depth, $\lambda$. In order to discard completely this possibility it would therefore be required to know this number, which can be obtained for example in a transverse-field $\mu$SR experiment.

VI. CONCLUSION

In conclusion, we have studied, on the basis of grouptheoretical considerations, the effect of SOC of arbitrary strength on the superconducting instability of the noncentrosymmetric intermetallic compound LaNiC$_2$. We have paid particular attention to the issue of TRS breaking. While in the absence of SOC there are 12 possible superconducting instabilities, of which four break TRS, when SOC is taken into account there are only four superconducting instabilities of the normal state and none of them break TRS. To reconcile this result with the experimental observation of TRS breaking on entering the superconducting state we have studied the evolution of the superconducting instability as a small amount of SOC is adiabatically turned on. We have found that each of the eight TRS preserving singlet and unitary triplet instabilities evolve smoothly into one of the four that are allowed in the presence of SOC and we have obtained the form these must take when SOC is small but finite. In particular, our analysis shows a small triplet component developing on top of an s-wave order parameter. However, this mechanism is found not to lead to TRS breaking. A similar analysis for the case of the four nonunitary triplet pairing instabilities reveals that each of them splits into two distinct transitions: an instability of the normal state where superconductivity emerges without the breaking of TRS, followed by a second superconducting instability where the order parameter acquires an additional component and TRS is broken. We thus conclude that the superconducting instability must be of the nonunitary triplet type and that SOC must be comparatively small in this system so as to make the first and second transitions indiscernible. A distinct prediction of this analysis is a splitting of the superconducting transition that could be observed in single crystals and enhanced by the application of pressure. Since only the nonunitary triplet pairing instabilities are split in this way, its observation would be a direct consequence of the broken TRS and constitute definitive proof of nonunitary triplet pairing in this system.

Nonunitary triplet pairing is believed to be realized in the ferromagnetic superconductors. In contrast, the normal state of LaNiC$_2$ just above $T_c$ is paramagnetic. This material therefore constitutes an example of an “intrinsically” nonuni-
tary triplet superconductor where the pairing of electrons with only one value of the spin does not result from a pre-existing exchange splitting. Elucidating the mechanism by which this comes about and the possible role that the lack of inversion symmetry may play in it, is an outstanding challenge.

14 V. P. Mineev (private communication).
19 V. P. Mineev (private communication).
33 V. P. Mineev (private communication).

ACKNOWLEDGMENTS

The authors thank Vladimir P. Mineev for a stimulating discussion. J.Q. gratefully acknowledges funding from STFC in association with St. Catherine’s College, Oxford and from HEFCE through the South-East Physics network (SEPnet).