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Working Paper Series

Modifications to the g-index to Improve its Discriminatory Power

Wenbin Liu
Kent Business School

Fang Xu
Kent Business School

John Mingers
Kent Business School

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Modifications to the g-index to Improve its Discriminatory Power

Wenbin Liu¹

Fang Xu²

And John Mingers³

^{1,2,3}. Kent Business School, University of Kent, Canterbury, CT2 2PE, UK

Phone:

+44(0)1227 824008

Fax:

+44(0)1227 761187

E-mail:

w.b.liu@kent.ac.uk

j.mingers@kent.ac.uk

fx7@kent.ac.uk

Modifications to the g-index to improve its Discriminatory Power

Abstract

In 2006, Egghe introduced the g-index which provides a more comprehensive measure of scientific contribution. In this paper we first show that in many cases significant citation counts are wasted in computing the g-index and being an integer it often lacks discriminatory power. In this paper we introduce several indices using different extension approaches. Empirical studies are provided to compare these indices in some bibliometrics rankings.

Keywords: bibliometrics, citations, g-index, research evaluation

Introduction

Measuring the impact of a researcher's output is of increasing importance and has led to much work on different ways of measuring the number of citations that a paper receives (Burrell, 2003; Glänzel, 1997; Van Raan, 2003). Obvious measures are the total number of citations or the mean number of citations per paper but these have several shortcomings, and a new measure, the h-index, was suggested (Hirsch, 2005). This in turn created much interest (Glanzel, 2006; Meho et al., 2008; Mingers, 2008; Van Raan, 2005) and its strengths and weaknesses have been debated (Cronin et al., 2006; Bornmann et al., 2005).

The h-index is, defined as “a scientist has index h if h of his/her N papers have at least h citations each and the other $(N-h)$ papers have no more than h citations each” (Hirsch, 2005). This is clear and easy to understand but has the limitation that it excludes from consideration all the citations in excess of h that the top h papers have acquired. To overcome this, Egghe (Egghe, 2006) introduced the g-index to evaluate researchers' scientific contribution through measuring the global performance of a set of publications. Ever since the g-index was proposed, it has received much attention (Egghe, 2008; Perez-Enriquez et al., 2006; Rinker et al., 2007; Tol, 2008). Suppose all a researcher's publications are arranged in descending order of individual citations, the focus of the g-index is based on the comparison of the total citations of the first n papers and square of the n^{th} paper's rank (i.e. n^2).

If $s(n)$ is the sum of the citations of the first n papers, the g-index is defined as

$$g=n^*$$

$$\text{where } s(n^*) \geq n^{*2} \text{ and } s(n^* + 1) < (n^* + 1)^2 \quad (1)$$

The g-index utilizes information about both the total citations of the top g papers and the number of publications while accumulative citations are not taken into consideration in the h-index. In other words, higher ($>h$) citations of the first h papers are not utilized at all in the h-index. This g-index is claimed to give a better measurement of researchers' scientific contribution.

However, the g-index and the h-index are limited in their discriminatory power because they are defined to be integers. This means that several researchers may receive the same g-index although they have significantly different numbers of citations. Or, put another way, many of their citations are essentially wasted as they are not reflected in the g-index (Adler *et al.*, 2008). In this paper we are going to explore the wasted citations in the g-index and propose several indices through different extension approaches.

Illustrations of Wasted Citations

Firstly from the definition, it is clear that any increments in the total citations of the first g papers will not change the g-index as long as the increased citations are less than $(g + 1)^2 - g^2$, (i.e., $2g+1$). In other words this part of contribution is not counted. To discuss the influence of these wasted citations, we calculated its proportion (p) in the total citations of the first g papers (i.e. $s(g)$). Although this proportion varies with the value of the g-index, we can estimate it through the following formula:

$$p = \frac{2g+1}{s(g)}$$

As $g \leq \frac{s(g)}{g} < (g+1)$

then $\frac{2g+1}{g^2} \geq p > \frac{2g+1}{g(g+1)} \quad (2)$

Then we examine the percentage of wasted citations in more details. For example when $g=5$, $0.44 \geq p > 0.367$, and if $g=15$, $0.138 \geq p > 0.129$ and so on. It is thus clear that this part often

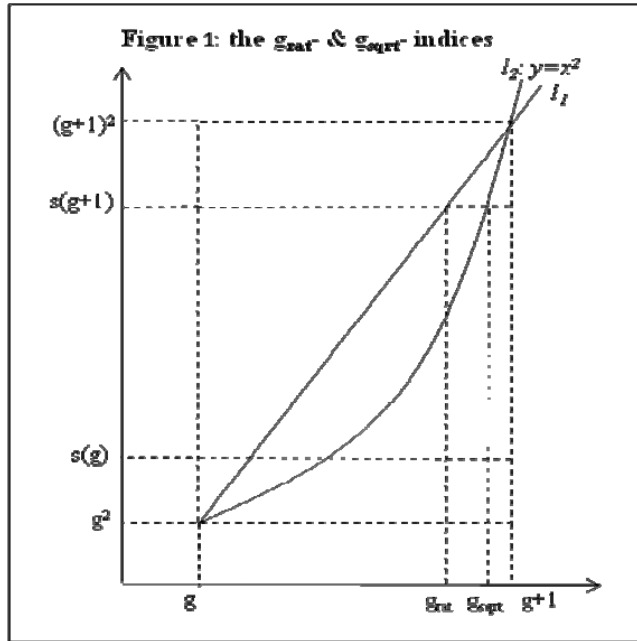
represents quite a significant contribution towards the accumulated citations. We find that the smaller g-index is the higher impact wasted citations could have. Therefore, wasted citations appear important for young researchers due to their short career length; or for fixed period evaluation (like five to eight years for the new UK Research Excellence Framework (REF)), where many researchers' g-indices are below 20, see empirical study.

Secondly, in light of the ability to next g-index (i.e., $g+1$), we see that as far as the citations of the $(g+1)^{th}$ paper (i.e. $c(g+1)$) is less than $(g+1)^2 - (s(g) - g^2)$ (i.e. the extra citations which excludes the part generated from $s(g)$), the g-index still keeps the same. Hence, this part of citations is wasted as well.

Consequently, the discrimination power of the g-index is reduced so that there are more researchers end up having the same rank (see empirical study). That is why we would like to overcome this problem by reusing the wasted citations. It is a problem that how to consider the wasted citations in an appropriate way. In this paper we explain two different approaches. Each one will lead to new indices. In Case 1, the above two parts of wasted citations are summed together (i.e. $s(g+1)$) while in Case 2, more emphasize is put on $s(g)$ than $c(g+1)$. Let us note that citations are not uniformly accounted in the original g-index – the citations after $s(g)$ are not even counted. Then empirical studies are presented to compare these indices. All these modifications are just the g-index if they happen to be integers.

Case 1: Modifications using $s(g+1)$ for discriminating research performance

In this case, we simply sum the two parts of wasted citations together, (giving $s(g+1)$) so that they are equally weighted. Here, we introduce two different modifications of the g-index to account $s(g+1)$, as shown in Figure 1.



- **Modification 1**

In literature, Tol (Tol, 2008) suggested a straight line interpolation (see line l_1 in Figure 1) connecting two points (g, g^2) and $(g+1, (g+1)^2)$. An index called the g_{rat} -index was generated where line l_1 meets line $y=s(g+1)$. In this extension, we can see that the wasted citations in $s(g)$ and $c(g+1)$ are reflected by the values of $s(g+1)$. In order to obtain the explicit formula of this index, one just firstly finds the equation of line l_1 and then the abscissa of the intersection generated with equation $y=s(g+1)$. In Tol's paper, the formula is shown as below:

$$g_{rat} = g + 1 - \frac{(g + 1)^2 - s(g + 1)}{2g + 1} \quad (3)$$

The idea of this extension approach is to apply a straight line to interpolate two critical points. However, if we recall that the original g -index is obtained by approximately solving the equation: $s(n) = n^2$ it should be more natural to use the curve $y=x^2$ for the interpolation. And then we will have the second modification.

- **Modification 2**

In this modification, the curve $y=x^2$ (i.e. line l_2 in Figure 1) is used to interpolate the two points (g, g^2) and $(g+1, (g+1)^2)$. The abscissa of intersection, where the lines l_2 and l_1 meet gives a new index. We call it the g_{sqrt} -index. And the formula is easily found to be :

$$g_{sqr} = \sqrt{s(g+1)} \quad (4)$$

With these two indices, we would like to have an empirical comparison. We have collected the relevant bibliometric information for researchers in Kent Business School at the University of Kent. With regard to types of publications, only journal articles and conference papers are considered. The other types such as books, book sections and so on are excluded. The citation counts are manually collected from Google Scholar in July, 2008. By applying formula (3) & (4), we have the following results for the individual researcher shown in Table 1.

Table 1: Ranking of KBS Researchers using the g-, g_{rat}- and g_{sqr}-indices

Researchers	g-index	Rank 1	g _{rat} - index	Rank 2	g _{sqr} - index	Rank 3
Researcher 1	47	1	47.316	1	47.318	1
Researcher 2	23	2	23.447	2	23.452	2
Researcher 3	16	3	16.970	3	16.971	3
Researcher 4	16	3	16.485	4	16.492	4
Researcher 5	15	5	15.774	5	15.780	5
Researcher 6	14	6	14.517	6	14.526	6
Researcher 7	13	7	13.222	7	13.229	7
Researcher 8	12	8	12.880	8	12.884	8
Researcher 9	12	8	12.840	9	12.845	9
Researcher 10	12	8	12.840	9	12.845	9
Researcher 11	11	11	11.478	11	11.489	11
Researcher 12	10	12	10.286	12	10.296	12
Researcher 13	9	13	9.790	13	9.798	13
Researcher 14	9	13	9.263	14	9.274	14
Researcher 15	9	13	9.000	15	9.000	15
Researcher 16	8	16	8.824	16	8.832	16
Researcher 17	8	16	8.824	16	8.832	16
Researcher 18	8	16	8.235	18	8.246	18
Researcher 19	8	16	8.177	19	8.185	19
Researcher 20	7	20	7.867	20	7.874	20
Researcher 21	7	20	7.400	21	7.416	21
Researcher 22	7	20	7.333	22	7.349	22
Researcher 23	6	23	6.615	23	6.633	23
Researcher 24	6	23	6.000	24	6.000	24
Researcher 25	5	25	5.909	25	5.916	25

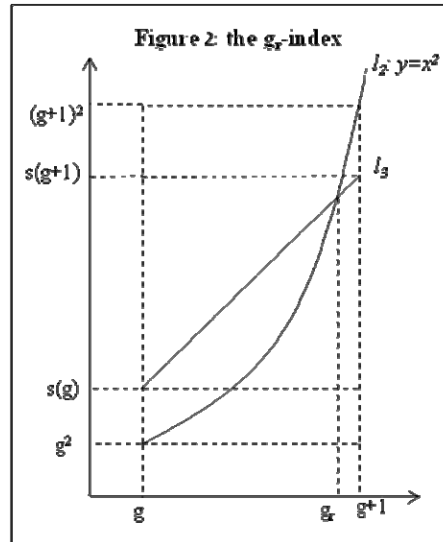
Researcher 26	5	25	5.818	26	5.831	26
Researcher 27	5	25	5.727	27	5.745	27
Researcher 28	5	25	5.182	28	5.196	28
Researcher 29	4	29	4.556	29	4.583	29
Researcher 30	4	29	4.556	29	4.583	29
Researcher 31	4	29	4.333	31	4.359	31
Researcher 32	3	32	3.857	32	3.873	32
Researcher 33	3	32	3.571	33	3.606	33
Researcher 34	3	32	3.429	34	3.464	34
Researcher 35	3	32	3.286	35	3.317	35
Researcher 36	2	36	2.600	36	2.646	36
Researcher 37	2	36	2.600	36	2.646	36
Researcher 38	2	36	2.400	38	2.450	38
Researcher 39	1	39	1.667	39	1.732	39
Researcher 40	1	40	1.000	40	1.000	40

Ranks 1, 2 and 3 represent the ranks generated according to the values of g -, g_{rat} - and g_{sqr} - indices respectively. We can see that the g -index generates a significant number of ties especially in the lower areas, such as for values of 9 and 5. Some of these ties can be resolved by using the g_{rat} - and g_{sqr} -indices. Clearly these two modifications will produce the **same** rank. Furthermore there are still some ties for the two indices.

At this point, one may ask whether we can further distinguish these ties. As discussed before, wasted citations in $s(g)$ and $c(g+1)$ are regarded as equally important in Case 1. However, one can differentiate the importance of the two parts of citations,

Case 2: Modifications using $s(n)$ and $s(g+1)$ for discriminating research performance

In this case, we reasonably believe that $s(g)$ has more contributions compared with $c(g+1)$ in evaluation of scientific performance. Using this idea, we are able to find new indices through a different approach, as shown in Figure 2.



- **Modification 3**

Guns (Guns *et al.*, 2009) has suggested a modification. Basically, he used a straight line (i.e. l_3) to interpolate the two points $(g, s(g))$ and $(g+1, s(g+1))$. Then the intersection generated by line $l_2: y=x^2$ and line l_3 results a new index called the g_r -index, which can reflect $s(g)$ and $s(g+1)$. One just needs to find out the equation of line l_3 to arrive the formula as follows:

$$g_r = \frac{1}{2}c(g+1) + \sqrt{s(g) + \frac{1}{4}c(g+1)^2 - gc(g+1)} \quad (5)$$

In Appendix A, we show that for a fixed $s(g+1)$ this index is increasing with respect to $s(g)$ (it decreases with respect to $c(g+1)$). This suggests that when $s(g+1)=s(g)+c(g+1)$ is fixed, the index puts more emphasize on $s(g)$. Thus this will further increase its discrimination power as to be seen in our empirical test.

- **Modification 4**

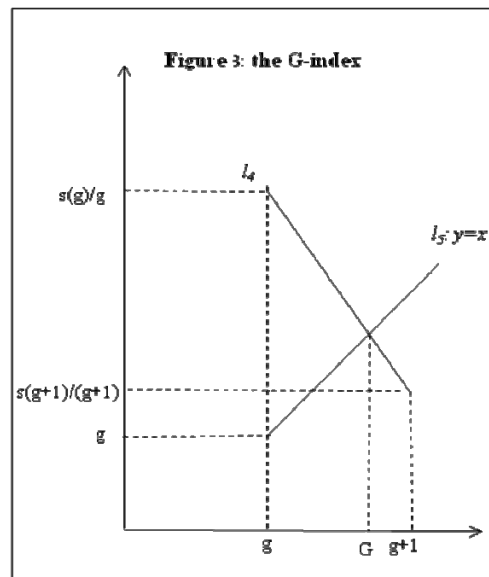
Here we introduce another approach which regards the g -index from a different perspective. Back to the definition, the g -index can be regarded in the following way:

$$g = n^*$$

$$\text{where } \frac{s(n^*)}{n^*} \geq n^* \text{ and } \frac{s(n^*+1)}{n^*+1} < (n^*+1) \quad (6)$$

This can also be seen as the point at which the top g papers have mean citations per paper of at least g . Thus the g -index is obtained by approximately solving the equation: $\frac{s(n^*)}{n^*} = n^*$

Thus we can use the average citation curve in the above interpolation as follows. Then we are able to introduce a new modification on the base of average citations, as shown in Figure 3.



A line segment (i.e. line l_4) is extended to reflect the wasted citations, which interpolates two points $(g, \frac{s(g)}{g})$ and $(g+1, \frac{s(g+1)}{g+1})$ when $\frac{s(g)}{g} \neq g$ as shown in Figure 3. Then the G-index is defined as the y or x coordinate of the intersection point where line l_4 meets line $l_3: y=x$. It is just the g-index if it is an integer.

Using this modification, the wasted citations and their variations can be reflected by the values of the G-index. It is clear that one does not necessarily use the straight line segment to achieve these purposes, but it seems to be the simplest to work. The calculation of the G-index is shown in Appendix B. The final formula is as below.

$$G = \frac{(g+1)^2 s(g) - g^2 s(g+1)}{g(g+1) - gs(g+1) + (g+1)s(g)} \quad (7)$$

This index is found to be a fraction between $(g, g+1)$. We are able to prove that this index is also increasing with respect to $s(g)$ when $s(g+1) = s(g) + c(g+1)$ is fixed, see Appendix C.

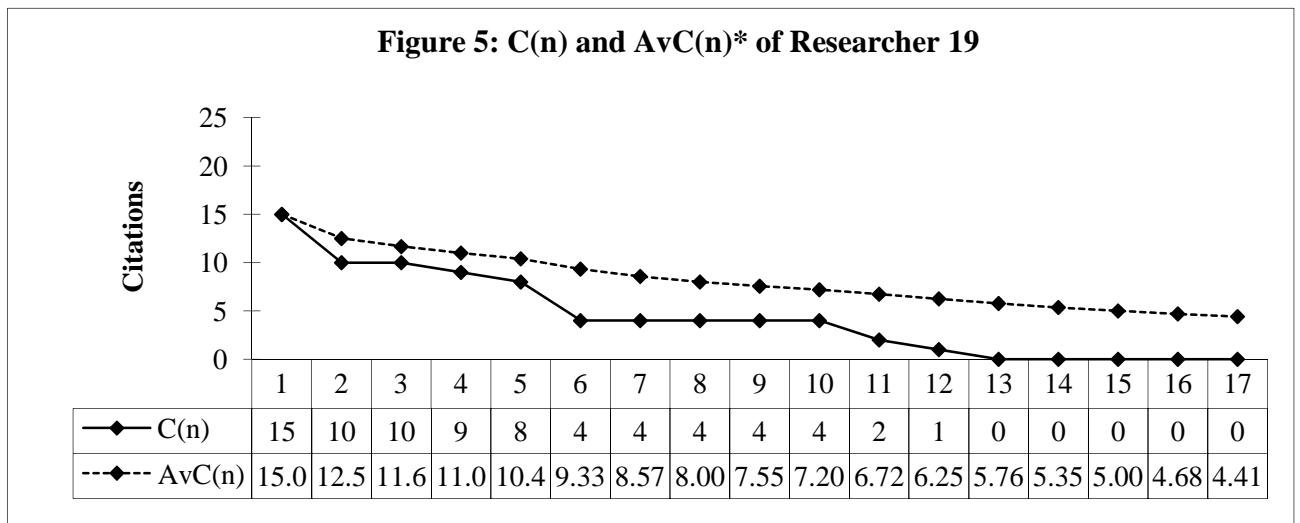
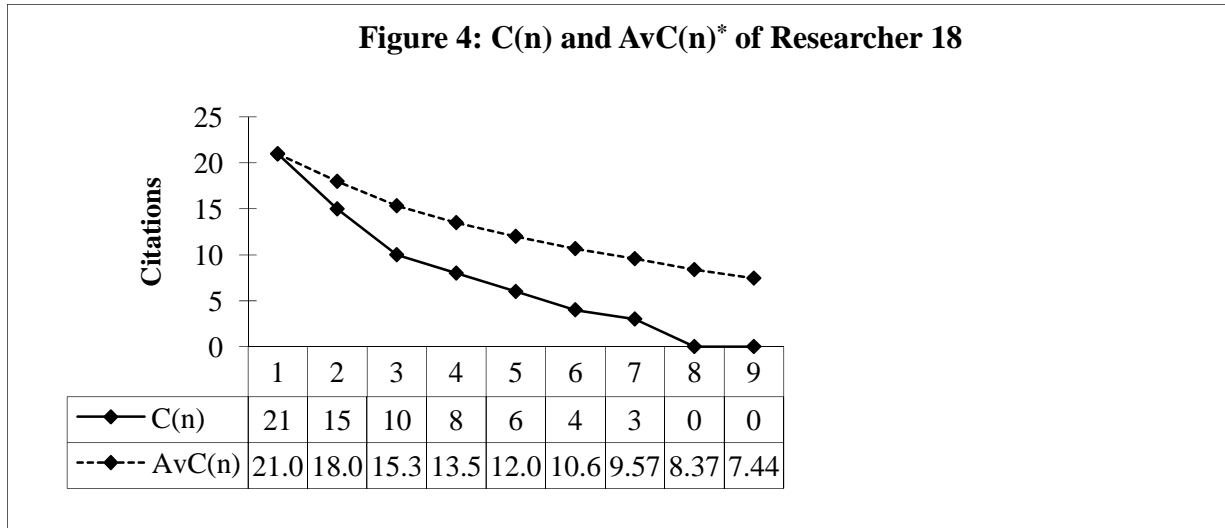
In the follows, we will examine the discriminatory power of these four indices with the same data studied before. The results are shown in Table 2.

Table 2: Ranking of KBS researchers using five indices

Researchers	g-index	Rank 1	g _{rat} -index	Rank 2	g _{sqr} t-index	Rank 3	g _r -index	Rank 4	G-index	Rank 5
researcher 1	47	1	47.316	1	47.318	1	47.288	1	47.290	1
researcher 2	23	2	23.447	2	23.452	2	23.322	2	23.325	2
researcher 3	16	3	16.970	3	16.971	3	16.963	3	16.964	3
researcher 4	16	3	16.485	4	16.492	4	16.460	4	16.467	4
researcher 5	15	5	15.774	5	15.780	5	15.780	5	15.785	5
researcher 6	14	6	14.517	6	14.526	6	14.526	6	14.534	6
researcher 7	13	7	13.222	7	13.229	7	13.048	7	13.049	7
researcher 8	12	8	12.880	8	12.884	8	12.884	8	12.888	8
researcher 9	12	8	12.840	9	12.845	9	12.845	9	12.850	9
researcher 10	12	8	12.840	9	12.845	9	12.817	10	12.822	10
researcher 11	11	8	11.478	11	11.489	11	11.489	11	11.500	11
researcher 12	10	12	10.286	12	10.296	12	10.296	12	10.306	12
researcher 13	9	13	9.789	13	9.798	13	9.761	13	9.769	13
researcher 14	9	13	9.263	14	9.274	14	9.232	14	9.241	14
researcher 15	9	13	9.000	15	9.000	15	9.000	15	9.000	15
researcher 16	8	16	8.824	16	8.832	16	8.832	16	8.840	16
researcher 17	8	16	8.824	16	8.832	16	8.797	17	8.805	17
researcher 18	8	16	8.176	19	8.185	19	8.185	18	8.194	18
researcher 19	8	16	8.235	18	8.246	18	8.000	19	8.000	19
researcher 20	7	20	7.867	20	7.874	20	7.831	20	7.837	20
researcher 21	7	20	7.400	21	7.416	21	7.325	21	7.337	21
researcher 22	7	20	7.333	22	7.348	22	7.179	22	7.186	22
researcher 23	6	23	6.615	23	6.633	23	6.472	23	6.483	23
researcher 24	6	23	6.000	24	6.000	24	6.000	24	6.000	24
researcher 25	5	25	5.909	25	5.916	25	5.887	25	5.894	25
researcher 26	5	25	5.818	26	5.831	26	5.772	26	5.783	26
researcher 27	5	25	5.727	27	5.745	27	5.690	27	5.706	27
researcher 28	5	25	5.182	28	5.196	28	5.000	28	5.000	28
researcher 29	4	29	4.556	29	4.583	29	4.464	29	4.484	29
researcher 30	4	29	4.556	29	4.583	29	4.372	30	4.385	30
researcher 31	4	29	4.333	31	4.359	31	4.000	31	4.000	31
researcher 32	3	32	3.857	32	3.873	32	3.873	32	3.889	32
researcher 33	3	32	3.571	33	3.606	33	3.449	33	3.471	33
researcher 34	3	32	3.429	34	3.464	34	3.236	34	3.250	34
researcher 35	3	32	3.286	35	3.317	35	3.193	35	3.211	35
researcher 36	2	36	2.600	36	2.646	36	2.646	36	2.692	36
researcher 37	2	36	2.600	36	2.646	36	2.414	37	2.429	37
researcher 38	2	36	2.400	38	2.449	38	2.000	38	2.000	38
researcher 39	1	39	1.667	39	1.732	39	1.618	39	1.667	39
researcher 40	1	40	1.000	40	1.000	40	1.000	40	1.000	40

Ranks 4 and 5 are generated using the values of the g_r-index and G-index respectively. By examining the empirical results, we see that the two latter indices produced the same rank and can further distinguish the ties as expected.

It is interesting to notice that there is a conflict between ranks for researchers 18 and 19 when using the two groups of indices. The two figures below, Figure 4 and Figure 5 show the citation profiles of researchers 18 and 19 respectively, which can help us to understand the difference of the citation characteristics of the researchers.



*: AvC(n) represents the average citations of the first n^{th} papers.

We see that both of the two researchers have a g-idnex of 8. And for researcher 18, citations of the top $g+1$ papers (i.e. $s(g+1)$) are 67, which are slightly less than those of researcher 19 (68). Therefore researcher 18 has the lower scores in the first group indices than researcher 19 due to the smaller value of $s(g+1)$. However if we have a closer look at their citations, and break $s(g+1)$ down to $s(g)$ and $c(g+1)$, then it is clear that the total citations of the first g papers ($s(g)$) of researcher 18 is 67 (that is $c(g+1)=0$) while they are only 64 for researcher

19 (that is $c(g+1)=4$). As $s(g+1)$ of the two researchers are almost the same, researcher 18 has a higher rank using the second group of indices.

It can be clearly seen that the citation profile of researcher 19 has a thinner body and a much longer tail. It is also clear that researcher 18 has a much higher average citations (7.44) than researcher 19 (4.41). Thus, it seems reasonable to claim that researcher 18 should have a higher research quality rank than researcher 19. And this information is reflected by the second group of indices.

Conclusions

Evaluating research quality is an important but quite difficult issue in scientific communication. Particular attention must be paid to appropriate bibliometrics methods. The most well-known index is the h-index which is easy to calculate and to understand. However, a disadvantage is that it loses some valuable information about scientific contribution and so the g-index was proposed. Although the g-index can provide a more comprehensive measure of scientific contribution, we believe that some important information is still wasted and it may not discriminate sufficiently well between researchers. We have proposed amendments to the g-index which includes all the relevant citations for a researcher and generates a continuous rather than a discrete index thus reducing the problem of tied ranks. Theoretically, the calculations of these indices were explained in details, and practically, the results of the comparison study show the discriminatory power and the differences.

Appendix A: Monotonicity of the g_r -index with respect to $s(g)$

We show the monotonicity by directly computing the first derivation.

$$\begin{aligned}
 Q g_r &= \frac{1}{2}c(g+1) + \sqrt{s(g) + \frac{1}{4}c(g+1)^2 - gc(g+1)} \\
 \therefore g_r &= \frac{1}{2}(s(g+1) - s(g)) + \sqrt{s(g) + \frac{1}{4}(s(g+1) - s(g))^2 - g(s(g+1) - s(g))} \\
 \therefore \frac{\partial(g_r)}{\partial(s(g))} &= -\frac{1}{2} + \frac{1}{2}(s(g) + \frac{1}{4}(s(g+1) - s(g))^2 - g(s(g+1) - s(g)))^{-\frac{1}{2}}(\frac{1}{2}s(g) - \frac{1}{2}s(g+1) + g + 1)
 \end{aligned}$$

Now we only need to prove that the following formula is larger than 1.

$$\frac{(\frac{1}{2}s(g) - \frac{1}{2}s(g+1) + g + 1)}{\sqrt{s(g) + \frac{1}{4}(s(g+1) - s(g))^2 - g(s(g+1) - s(g))}} > 1$$

We divide the proof into two parts. One is to find out the sign of the numerator.

$$\begin{aligned}
 Q(g+1)^2 &> s(g+1) = c(1) + c(2) + \dots + c(g+1) \geq (g+1)c(g+1) \\
 \therefore g+1 &> c(g+1) = s(g+1) - s(g) \\
 \therefore g+1 + \frac{1}{2}s(g) - \frac{1}{2}s(g+1) &> 0
 \end{aligned}$$

And the other is to find out whether this inequality holds.

$$\begin{aligned}
 Q(g+1)^2 &> s(g+1) \\
 \therefore (g+1)^2 + gs(g) - gs(g+1) + s(g) + (\frac{1}{2}s(g) - \frac{1}{2}s(g+1))^2 &> s(g+1) + gs(g) - gs(g+1) + s(g) + (\frac{1}{2}s(g) - \frac{1}{2}s(g+1))^2 \\
 \therefore (\frac{1}{2}s(g) - \frac{1}{2}s(g+1) + g + 1)^2 &> gs(g) - gs(g+1) + s(g) + \frac{1}{4}(s(g) - s(g+1))^2 \\
 \therefore \frac{\frac{1}{2}s(g) - \frac{1}{2}s(g+1) + g + 1}{\sqrt{s(g) + \frac{1}{4}(s(g) - s(g+1))^2 - g(s(g+1) - gs(g))}} &> 1
 \end{aligned}$$

With the above proof, we see that the first derivation is positive and therefore, the g_r -index is increasing with respect to $s(g)$ when g and $s(g+1)$ are fixed.

Appendix B: Calculations of the G-index

Using Figure 3, we can easily obtain the explicit formula for this index:

- When $\frac{s(g)}{g} = g$, and then the G-index is defined just as the g-index.
- When $\frac{s(g)}{g} > g$ and $\frac{s(g+1)}{g+1} < g+1$, we firstly find the equation for line l_4 , which is

$$y = x\left(\frac{s(g+1)}{g+1} - \frac{s(g)}{g}\right) + (g+1)\frac{s(g)}{g} - g\frac{s(g+1)}{g+1}$$

As it satisfies $y=x$, the abscissa of intersection (i.e. the G-index) is

$$G = \frac{(g+1)\frac{s(g)}{g} - g\frac{s(g+1)}{g+1}}{1 - \frac{s(g+1)}{g+1} + \frac{s(g)}{g}} = \frac{(g+1)^2 s(g) - g^2 s(g+1)}{g(g+1) - gs(g+1) + (g+1)s(g)}$$

That is the formula for the G-index.

Appendix C: Monotonicity of the G-index with respect to $s(g)$

We examine this problem similarly as in Appendix A. It is clear that

$$G = 1 - \frac{\frac{g^2}{(g+1)^2} s(g+1) + g - \frac{g}{g+1} s(g+1)}{g - \frac{g}{g+1} s(g+1) + s(g)}$$

Therefore, we only need to show that the numerator in the above formula is positive.

$$\begin{aligned} & Q(g+1)^2 > s(g+1) > 0 \\ & \therefore 1 > \frac{s(g+1)}{(g+1)^2} > 0 \\ & \therefore g > \frac{g}{(g+1)^2} s(g+1) \\ & \therefore \frac{g^2 - g^2 - g}{(g+1)^2} s(g+1) + g > 0 \\ & \therefore \frac{s(g+1)}{(g+1)^2} (g^2 - g(g+1)) + g > 0 \\ & \therefore \frac{g^2}{(g+1)^2} s(g+1) + g - \frac{g}{g+1} s(g+1) > 0 \end{aligned}$$

As the numerator is positive, then we finish the proof.

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University of Kent