ABSTRACT. The technique of linear programming (LP) is illustrated by developing a harvest schedule for an industrial forest enterprise operating in southwestern Pennsylvania. The objective was to maximize net present value of the harvest over a five-year planning period. The effect of changes in timber value and growth rates on the optimal schedule was determined. Sensitivity analysis provided additional information the manager could use to make decisions. In order to successfully apply LP, the forester must be able to define the management objective of the harvest schedule and the resource and managerial constraints that will influence its attainment. Data used in the model have to be available and reliable. Many forest enterprises should be in the position to adopt LP since commercial programs for microcomputers are now available for which a high level of computing expertise is not required.

Linear programming (LP) is a mathematical technique used to allocate limited resources optimally among competing activities to satisfy a given objective. The technique has been widely used in industrial applications to improve resource efficiency. In forestry LP has been applied successfully to maximize allowable timber yields (Leak 1964), timber utilization in sawmills (Fasinski 1979), and management of wildlife habitat (Mesaly and Horwitz 1981). Forplan is a large LP model, developed by the USDA Forest Service to allocate forest land to general management activities and to schedule treatments and resulting products (Kenyon 1984). A comprehensive review of the theory and application of linear programming in natural resource management is given by Dykstra (1966).

Harvest scheduling is well suited to an LP approach because many management problems in forestry, it deals with the optimization of certain measures of economic output (e.g., maximizing profit or minimizing cost) while having to satisfy management restrictions such as allowable cut and mill requirements. Linear pro-

LITERATURE CITED

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AN APPLICATION OF LINEAR PROGRAMMING FOR SHORT-TERM HARVEST SCHEDULING


148 NAF 11/88

145

145
### Table 1. Total stand volume by species.

<table>
<thead>
<tr>
<th>Species</th>
<th>Value (Volume)</th>
<th>% of Total Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aspen</td>
<td>32,000</td>
<td>32%</td>
</tr>
<tr>
<td>Balsam Poplar</td>
<td>24,000</td>
<td>24%</td>
</tr>
<tr>
<td>Birch</td>
<td>18,000</td>
<td>18%</td>
</tr>
<tr>
<td>Black Cherry</td>
<td>12,000</td>
<td>12%</td>
</tr>
<tr>
<td>Red Oak</td>
<td>9,000</td>
<td>9%</td>
</tr>
</tbody>
</table>

### Effect of Changes in Growth Rate on the Optimum Schedule

The results of Run 2 indicate that, due to the increase in black cherry prices and decline in the price of red oak, stands with a relatively high proportion of black cherry and little red oak were favored. (In this context, "favorable" means that the stands were selected for harvesting during the planning period.) For example, stand 15, which has no oak and a considerable amount of black cherry (Table 1) was completely harvested. In the same way, the stand was not only harvested at a rate of 3% per year, but also harvested later in the planning period than would be optimal. Unfortunately, the cost of the nonoptimal scheduling cannot be directly determined from the LP solution.

### Sensitivity Analysis

Sensitivity analysis shows us how much the optimal solution will vary with changes in the LP parameters. The % change in objective function value for any of the remaining stands is an extremely important aspect of any LP solution. The following summary describes the information provided by sensitivity analysis and its possible implications for management.

1. The dual price indicates how much the optimal value of the objective function will increase in this problem following an increase of one unit of any given LP parameter.

### Appendix

**FORMULATION OF HARVEST SCHEDULING PROBLEM**

Maximize net present value of harvested acres: $\sum_{j=1}^{n} p_j x_j$

Subject to: Constraints not shown in Table 1.

**RESULTS**

The Optimum Schedule

In Run 1 (Fig. 1), the LP model selected stands for harvest which would maximize the net present value of harvested acres. The objective function value for black cherry and red oak, and which were subjected to the lower of the two depletion rates (Table 1). For example, black cherry, red oak, and ash comprise almost 50% of the total timber volume of stands 6 and 14 and 15, all harvested in year 1. By comparison, less than 20% of timber volume in stands 7 and 8 was ascribed to these valuable species and neither stand was selected for harvest. The depletion rate also influences stand selection. Stands 3 and 10 both had a relatively high proportion of good timber but were excluded from the schedule due to the application of the higher depletion rate ($\Delta = 0.125$).

### DISCUSSION

In this study, an LP model is selected that has the maximum present value within the given constraints. However, the cost and present value of future timber growth rate had a strong influence on the stands selected for harvest. The model could, therefore, be used for two ways as a method of determining an optimal harvest schedule, and for a way of predicting the impact of price trends and growth rate on harvesting operations and profitability.

### Overall

The LP model appears to be an extremely useful tool for forest management applications. The LP model does not require a high degree of computer expertise, and as more commercial LP software for microcomputers becomes available, its use is likely to spread among smaller forest enterprises.

### APPENDIX

**DEPARTMENT OF FORESTRY AND RANGE RESOURCES**

### FORMULATION OF HARVEST SCHEDULING PROBLEM

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