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Estimating mean willingness to pay from dichotomous choice contingent valuation studies

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Summary. Methods for estimating mean willingness to pay for some environmental goal are reviewed. Logistic regression analysis of data from dichotomous choice contingent valuation studies often models the willingness-to-pay curve poorly. We develop solutions to this problem. We also show how to model responses as a function of several covariates, and how to model the case in which a proportion of respondents is not willing to pay anything. Analytic and bootstrap methods for quantifying precision are developed. We illustrate the methods by using an example in which biodiversity losses due to acid rain deposition in Scotland are valued.

Keywords: Acid rain; Contingent valuation studies; Dichotomous choice; Logistic regression; Mean willingness to pay

1. Introduction

Economists are increasingly interested in a range of approaches to the valuation of the non-use benefits of environmental programmes. Of these, the contingent valuation method using discrete choice (take it or leave it) questions is considered to be the most promising approach to the measurement of the relevant welfare measures such as willingness to pay (WTP) (National Oceanic and Atmospheric Administration, 1993).

In a discrete choice contingent valuation study, a questionnaire is sent to a representative sample of the population of interest. Each individual in the sample is offered a bid, and the respondents must state whether they would be willing to pay that sum to achieve a specified level of environmental improvement. Although each respondent is offered a single bid level, different respondents are offered different bid levels, according to some design. Hanemann (1984) showed how the analysis of such binary response data could be integrated into economic theory by using the random utility model.

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Although mean WTP is the most relevant welfare measure with regard to cost–benefit analysis (Johansson *et al.*, 1989), the median is often the preferred money measure (Hanemann, 1984). This is because the mean is sensitive to outliers in the data and the WTP curve may be poorly modelled by logistic regression. Median WTP is a measure of the amount 50% of the population would be willing to pay but is not a valid estimate of the average amount people are willing to pay (unless the probability density function of the maximum that a person is willing to pay is symmetric about the mean). In this paper, we describe measures to improve the fit of the logistic curve to discrete choice data, so that mean WTP can be more reliably estimated, and show how to model responses as a function of several covariates. We also address the case in which a proportion of respondents is not willing to pay anything.

There is a useful relationship between the WTP curve, representing the proportion of a population willing to pay at least x expressed as a function of x, and the random variable X representing the maximum amount that a respondent is willing to pay. We know that, if the respondent accepts a bid x_0, $X > x_0$. Thus the cumulative distribution function of X is 1 minus the WTP curve. Hence, given a model for the WTP curve, the corresponding probability density function of X is found by differentiating and reversing sign.

2. Methods

2.1. Logistic regression

The natural way to analyse dichotomous choice data is logistic regression. If there are no covariate data, the regression equation may be expressed as

$$E(y_j) = \frac{1}{1 + \exp\{-\beta(x_j - \mu)\}} \tag{1}$$

where $E(y_j)$ is the probability that a respondent accepts bid x_j, and β (< 0) and μ are parameters to be estimated. Each observation y is 0 (bid rejected) or 1 (accepted) and is assumed to have a binary distribution with parameter $E(y)$. Standard logistic regression software may be used to obtain estimates b and m of β and μ, and corresponding standard errors. Most software provides estimates b and b_0, where $\beta_0 = E(b_0) = -\beta\mu$, so that $m = -b_0/b$. The above parameterization is convenient, as the parameter μ is both the mean and the median WTP. The variance of its estimate m, var(m), and cov(b, m) may be estimated by using the delta method (Seber, 1982). Here var(·) and cov(·, ·) represent the estimated variance and covariance respectively. Formulae are given in equations (14) and (15) of Appendix A.

This formulation is adequate for the case that bids can be negative. For example, the specified goal may be detrimental to some individuals, and acceptance of a negative bid corresponds to an acceptable level of compensation (Johansson *et al.*, 1989). In many contingent valuation applications, it is desirable to restrict estimated WTP to a non-negative random variable. Often, negative bids are inconsistent with the hypothetical environmental change (e.g. prevention of acid rain damage). Also, it may not be appropriate to model willingness to accept and WTP by using the same mathematical functions, since they may be based on entirely different lines of reasoning (Hanemann, 1991). We now address the case where bids cannot be negative. This raises two issues: how should the analysis be modified first to acknowledge that bids must be non-negative and second to allow for a proportion of individuals who would not be willing to pay anything?

2.2. Left truncation of the logistic curve

Perhaps the simplest solution is to truncate the logistic curve to the left of $x = 0$. For the untruncated curve,

$$\Pr(X < 0) = \frac{1}{1 + \exp(-\beta\mu)} \tag{2}$$

and this might be taken as the assumed proportion ϕ_0 of respondents who are not willing to pay anything. More work is now required to estimate mean WTP. Consider first only those respondents who are willing to pay something. Differentiating the WTP curve and reversing the sign yields

$$f(x) = \frac{-\beta \exp\{-\beta(x - \mu)\}}{[1 + \exp\{-\beta(x - \mu)\}]^2}, \tag{3}$$

where $\beta < 0$. Denoting mean WTP for those who are willing to pay something by μ_1, we have

$$\mu_1 = \int_0^\infty x\, f(x)\, \mathrm{d}x. \tag{4}$$

The estimates b and m may now be substituted into equation (3) and $m_1 = \hat{\mu}_1$ obtained by numerical integration of equation (4). Again using the delta method, we obtain $\mathrm{var}(m_1)$ (Appendix A, equation (16)).

If those who are not willing to pay anything are included in the mean WTP calculation, then if μ_0 denotes this mean WTP we have $\mu_0 = (1 - \phi_0)\mu_1$. Its estimate m_0 may be obtained by substituting b and m for β and μ in the expressions for ϕ_0 and μ_1; the corresponding variance is $\mathrm{var}(m_0)$ (Appendix A, equation (17)).

2.3. *Logarithmic transformation of bid*

When negative bids are not possible, rather than left truncate a curve whose argument ranges from $-\infty$ to ∞, it seems more satisfactory to transform x so that negative values are impossible. The obvious transformation to try is $\log_e(x)$. The log-linear function, although not entirely consistent with utility theory (Hanemann, 1984), can be considered a first-order approximation for a utility difference (Bowker and Stoll, 1988). If standard logistic regression software is to be used, the logarithm of each bid is calculated; then the regression is carried out as before, with $z = \log_e(x)$ replacing x. Equation (1) now becomes

$$E(y_j) = \frac{1}{1 + \exp[-\beta\{\log(x_j) - \mu\}]} \tag{5}$$

from which

$$f(x) = \frac{-\beta \exp[-\beta\{\log(x) - \mu\}]}{x(1 + \exp[-\beta\{\log(x) - \mu\}])^2}. \tag{6}$$

Mean WTP μ_1 is again estimated from numerical integration of equation (4), with parameters β and μ replaced by estimates b and m from the logistic regression.

We could work with the density of z, say $f_z(z)$, but that would yield an estimate of the mean of the logarithm of WTP. The exponential of that mean is a biased estimate of mean WTP. By continuing to work with x and $f(x)$, we avoid this difficulty.

The estimate m_1 of μ_1 has approximate variance $\mathrm{var}(m_1)$ (Appendix A, equation (18)).

2.4. *Reciprocal bid transformation*

The logarithmic transformation might adequately handle the difficulty of fitting the lower tail of the logistic curve. However, it may create a far greater difficulty in the upper tail. Mean WTP is insensitive to poor model fits in the lower tail because this corresponds to a WTP close to 0, and large relative bias in almost zero amounts can be tolerated. This is not true of the upper tail, which is substantially lengthened by the logarithmic transform. Thus the method can give rise to absurdly high estimates of mean WTP if there are few bids that correspond to a mean probability of acceptance close to 0, or if the fit of the model is poor in the upper tail. Right truncation at some arbitrary x, corresponding for example to the largest bid in the design, can reduce the problem (Boyle *et al.*, 1988) and is readily accommodated in numerical integration routines (Duffield and Patterson, 1991). However, the estimated mean WTP will be very sensitive to the choice of truncation point. Another solution is to identify a transformation that removes the problem of the range of x in the lower tail but does not alter behaviour of the upper tail. One such transformation is $w = x - a/x$ for some a. The left truncation method described above is a limiting case of this transformation, as $a \to 0$. The value of a might be fixed arbitrarily, but it is better considered an estimate of an unknown parameter α.

We now have, instead of equation (1),

$$E(y_j) = \frac{1}{1 + \exp\{-\beta(x_j - \alpha/x_j - \mu)\}} \tag{7}$$

from which

$$f(x) = \frac{-\beta(1 + \alpha/x^2)\exp\{-\beta(x - \alpha/x - \mu)\}}{[1 + \exp\{-\beta(x - \alpha/x - \mu)\}]^2}. \tag{8}$$

We estimate mean WTP μ_1 from numerical integration of equation (4), with parameters α, β and μ replaced by estimates a, b and m obtained from the logistic regression as follows.

The linear predictor in equation (7), $-\beta(x_j - \alpha/x_j - \mu)$, may be expressed as $-\beta_0 - \beta_1 x_j - \beta_2/x_j$, where $\beta_0 = -\beta\mu$, $\beta_1 = \beta$ and $\beta_2 = -\alpha\beta$. Thus we can obtain estimates b_0, b_1 and b_2 of β_0, β_1 and β_2 respectively, together with their standard errors and covariances, from a standard logistic regression with covariates x_j and $1/x_j$. Then $b = b_1$, $a = -b_2/b$ and $m = -b_0/b$. Further, $\mathrm{var}(b) = \mathrm{var}(b_1)$, $\mathrm{var}(m)$ is given by equation (14) of Appendix A, $\mathrm{cov}(b, m)$ by equation (15) and $\mathrm{var}(a)$, $\mathrm{cov}(b, a)$ and $\mathrm{cov}(m, a)$ by equations (19), (20) and (21) respectively. The estimate m_1 of μ_1 has approximate variance $\mathrm{var}(m_1)$ given in equation (22).

2.5. *Modified model for when some respondents are willing not to pay anything*

In logistic regression, it is assumed that the upper asymptote of the logistic curve is exactly 1. In the context of estimating mean WTP, this implies that there should be a bid that everyone would accept. Because the curve is continuous, no discrete lump of probability should be attached to any particular value of x. In particular, there should not be a group of people who are willing to pay nothing, but who would reject any positive bid. This condition is unlikely to be met in contingent valuation studies. A simple modification is possible provided that respondents are first asked whether they are willing to pay anything. This modification can be used with all the methods described above. In the case of the first method (ordinary logistic regression), the correction allows for respondents who are willing to pay nothing but do not require compensation either, i.e. they are indifferent to the change. In the second method, the correction can also be applied; the apparent proportion of respondents who are willing to pay nothing under the left truncation method is an artefact of the modelling, in which the logistic curve is truncated before it attains the upper asymptote. If those respondents who are not willing to pay anything are excluded from the analysis, then in reality ϕ_0 under the left truncation method is the proportion of people who are willing to pay very little, who, owing to the poor fit of the logistic model to untransformed bid values, are estimated to be willing to pay only negative amounts.

Let π_0 be the proportion of the population willing to pay nothing, estimated by p_0, the proportion of respondents who respond negatively to the initial question of whether they are willing to pay anything. The above logistic regression analyses are now carried out using data on only those who responded positively to the initial question. If the mean WTP of those willing to pay something is μ_y with estimate m_y obtained by one of the above methods, then the mean WTP in the entire population is

$$\mu_p = \mu_y(1 - \pi_0) \tag{9}$$

with estimate

$$m_p = m_y(1 - p_0). \tag{10}$$

Assuming independence between m_y and p_0,

$$\mathrm{var}(m_p) = (1 - p_0)^2\,\mathrm{var}(m_y) + m_y^2\,\mathrm{var}(p_0) + \mathrm{var}(m_y)\,\mathrm{var}(p_0) \tag{11}$$

(Seber (1982), p. 9), where $\mathrm{var}(p_0) = p_0(1 - p_0)/n$, with n equal to the total number of respondents.

2.6. *Logistic regression with covariates*

When covariates are present, the logistic regression equation may be expressed as

$$E(y_j) = \frac{\exp\left(\beta_0 + \sum_i \beta_i x_{ij}\right)}{1 + \exp\left(\beta_0 + \sum_i \beta_i x_{ij}\right)} = \frac{1}{1 + \exp\left(-\beta_0 - \sum_i \beta_i x_{ij}\right)} \tag{12}$$

where x_{ij} is the value of covariate i for respondent j, $i \geqslant 1$ (x_{1j} is the bid offered to respondent j and is always in the model), and β_i are coefficients to be estimated, $i \geqslant 0$.

Thus equation (1) may be expressed in this form by setting x_j to x_{1j}, β to β_1 and μ to $-\beta_0/\beta_1$. Stepwise methods may be used to reduce the number of covariates, but bid level should always be retained in the model (as should the reciprocal of the bid level if the reciprocal bid transform is used). The corresponding fitted model may be expressed as

$$\hat{y} = \frac{1}{1 + \exp\left(-b_0 - \sum_i b_i x_i\right)}. \tag{13}$$

The presence of covariates complicates estimation of mean WTP of the population as a whole. As noted by Cameron (1988), most researchers have estimated mean WTP by averaging over all covariates other than bid. This yields a single logistic curve, representing probability of accepting the bid against bid value. The parameter estimates for this curve may be entered into the calculations for any of the methods listed above, to obtain a mean WTP. However, this represents the mean WTP of individuals whose covariate values correspond to the mean for the population. Even if a representative sample of the entire population of interest has been obtained, this will be a biased estimate for the mean WTP in the population, because of the non-linear relationship between the covariates and WTP. For example, the mean WTP of a group of people on average income is likely to be smaller than that for a group randomly selected from the population, which may comprise a few individuals who are willing (and able) to pay a large sum.

One solution is to take each respondent in turn, and to substitute his or her covariate values into equation (13), with the exception of the bid that each was offered. Thus, for that individual, a logistic curve is obtained which is a function of the bid alone. The mean WTP of individuals with the same covariate values as that respondent can then be estimated by any of the methods presented above. This procedure has the merit that WTP can be estimated for each respondent, and, assuming that the respondents are a random sample from the population of interest, the average across all respondents is an estimate of the mean WTP in the population. Furthermore, averaging can be carried out across subgroups, e.g. to quantify mean WTP by income band. Its disadvantage is that it assumes that just the location of the WTP curve (when plotted against $z = \log(x)$ or $w = x - a/x$ in the case of the third and fourth approaches respectively) changes when the covariates change, whereas a change in shape might occur, in which case bias should be expected.

A method that is less sensitive to such bias is the following. Evaluate $\hat{y}_j$ for each respondent j, using equation (13), and record the bid value x offered to the respondent. Evaluate the mean prediction at each bid level k: $p_k = (\Sigma\, \hat{y}_j)/n_k$, where the summation is over the n_k predictions corresponding to a bid value of x_k. (Thus n_k is the number of responses received from individuals offered a bid of x_k.) Now fit a logistic curve to the p_k, with x (suitably transformed if the log-transform is selected, and together with $1/x$ if the reciprocal transform is used) as the independent variable. This can be done using curve fitting software that allows a logistic curve with upper asymptote equal to 1 and lower asymptote equal to 0. Weights should be specified equal to $n_k/p_k(1 - p_k)$. In the absence of such software, logistic regression software might be used, by setting $n_k p_k$ values (rounded to the nearest integer) to 1 and the remaining $n_k(1 - p_k)$ values at bid value x_k to 0. If this approach is adopted, the dispersion parameter should be estimated rather than assumed to be 1, and each n_k should be reasonably large. Some grouping across bid levels may be required. In this approach, the number of values set to 1 at a given bid level may differ from the observed number of acceptances in the data, as an adjustment has been made for the covariates. Also, because an adjustment has been made for covariates, at this stage, the logistic regression is univariate, with bid (possibly transformed) as the independent variable. (In the case of the reciprocal bid transform, the curve will be bivariate.) Thus the resulting logistic curve can be analysed by any of the methods described earlier, to obtain the estimated mean WTP and its variance.

2.7. *Estimates of precision*

Where the coefficient of variation of mean WTP is small, or where the unmodified logistic method, allowing negative values for WTP, is adopted, an approximate $100(1 - 2\alpha)\%$ confidence interval for mean WTP may be obtained as

$$m_1 \pm z_\alpha \sqrt{\mathrm{var}(m_1)}$$

where z_α is the appropriate percentile from the standardized normal distribution. However, if WTP is constrained to be non-negative, a better approximation is obtained by assuming that its mean has a log-normal distribution. An approximate $100(1 - 2\alpha)\%$ confidence interval may then be found as (Burnham *et al.*, 1987)

$$(m_1/k,\ m_1 k)$$

where $k = \exp[z_\alpha \sqrt{\mathrm{var}\{\log_e(m_1)\}}]$, with

$$\mathrm{var}\{\log_e(m_1)\} = \log_e\{1 + \mathrm{var}(m_1)/m_1^2\}.$$

The variance estimates $\mathrm{var}(m_1)$ require numerical evaluation and are based on several approximations. The bootstrap (Efron, 1979) may be used either to check the adequacy of these approximations or to provide alternative variance and interval estimates. The simplest method of generating a bootstrap resample is to sample with replacement from the n respondents until the sample size is again n. Mean WTP is estimated from the resample by one of the above methods, and this estimate, together with that of any other parameter of interest, is stored. A second resample is generated, and the process is repeated. Further resamples are generated, until t estimates of each parameter of interest are available. For a given parameter, the variance of its estimate is given by the sample variance of the bootstrap estimates of that parameter. A percentile confidence interval for the parameter is obtained by ordering the t values from smallest to largest. A $100(1-2\alpha)\%$ confidence interval is then given by the rth and sth ordered values from the list, where $r = (t+1)\alpha$ and $s = (t+1)(1-\alpha)$ (Buckland, 1984). Thus if 999 resamples were generated, and a 95% confidence interval was required, the 25th smallest and 25th largest estimates would be selected.

Park *et al.* (1991) advocated use of the parametric bootstrap, in which bootstrap estimates of the parameter vector are generated from a multivariate normal distribution. This method is superior when the assumption of multivariate normality holds, but it may be less robust in practice, unless the sample size is too small for a reliable application of the nonparametric bootstrap. Duffield and Patterson (1991) advocated a bootstrap approach in which they assumed that, of the n respondents offered a given bid level, the number s that accept follows a binomial distribution with estimated probability of acceptance s/n. They generated resamples by generating a deviate from this fitted binomial distribution. This is again a parametric bootstrap, although the method is equivalent to resampling with replacement from the responses of the n respondents, which is the nonparametric bootstrap method. The nonparametric bootstrap described above includes the method of Duffield and Patterson (1991) as a special case, in which covariates are not recorded. Cooper (1994) has provided the framework for a more sophisticated application of the bootstrap to logistic regression, in which the residuals are resampled, but in a way that recognizes that their variance is a function of the mean response.

3. An example: valuing biodiversity losses due to acid rain deposition in Scotland

Contingent valuation was used to assess the mean WTP of people in Scotland to reverse the effects of acid rain through higher prices on commonly purchased consumer items (electricity, cars and central heating) caused by stiffer pollution control (Macmillan *et al.*, 1995). This was felt to be a realistic option, and fair in the sense that it reflected the 'polluter pays principle'. In an initial pilot study, 254 individuals were selected by using a systematic sample of Scottish households drawn from the telephone directory. Respondents were asked how much they were willing to pay to reverse the effects of acid rain (i.e. an open-ended format was used). The method of Cooper (1993) was then used to select bid levels (together with the sample size at each bid level) for use in the main survey, in which respondents were asked whether they were willing to pay the specified amount of the bid. In total, nearly 3000 dichotomous choice questionnaires were mailed in the main survey. Although the open-ended question of the pilot study suggested a mean bid level per household of only £75, in the main survey most respondents who were willing to pay something and were offered the highest bid of £396 accepted it. Even allowing for the 21% not willing to pay anything, this represents a strong disparity between responses to the open question of the pilot study and the closed question of the main survey, which compromises our ability to estimate mean WTP. A further 30 questionnaires were mailed with a bid level of £798, allowing improved estimation of mean WTP. The responses are summarized in Table 1. Respondents were presented with one of five potential damage scenarios. Analyses here are restricted to three of

Table 1. Summary of responses to the questionnaire to assess mean WTP for reversing the effects of acid rain in Scotland

Number not willing to pay anything	280
Number willing to pay something, but who refused the bid offered	399
Number who accepted the bid offered	638
Number who do not know	352
Total number of valid responses	1669
Number protesting about some aspect of the survey	118
Number of incomplete returns	33
Total number of questionnaires returned	1820
Number of blank questionnaires returned	110
Non-responses	790
Total number of questionnaires mailed	2720

these scenarios, corresponding to low, medium and high damage, so that subsequent sample sizes are smaller than indicated in Table 1.

The questionnaire sought information on a number of potential covariates, and the methods outlined above for when covariates other than bid level are present were used. Respondents were also asked an initial question about whether they were willing to pay anything towards reversing the effects of acid rain. This allowed us to modify estimates to take account of households which were not willing to pay anything, as outlined above. Only those willing to pay something were asked whether they would accept the bid offered or not, and the logistic regression was based on those respondents only.

Straightforward logistic regression suggested that the WTP of many respondents was negative, but because of the initial filter question it was known that all were willing to pay something. This is explained by the poor fit of the logistic curve very close to 0; many households that are willing to pay something are nevertheless not willing to pay as much as £10. The use of either log(bid level) or the reciprocal bid transformation defined earlier resolves this difficulty. However, the failure of the survey to tie down adequately the upper tail of the WTP curve renders estimation very sensitive to the choice of truncation point for WTP if a log-transform of bid level is used. Instead, we use here the reciprocal bid transformation without truncation.

The analysis carried out may be summarized as follows.

(a) Carry out stepwise logistic regression, including bid level x, its reciprocal $1/x$ and any other covariates that significantly reduce the residual deviance.
(b) Obtain the predicted probabilities (fitted values) corresponding to all respondents, by using equation (13).
(c) Calculate p_k, the mean of the n_k predictions corresponding to respondents offered bid level x_k. Fit a logistic curve to the p_k, constraining the upper asymptote to 1 and the lower asymptote to 0, to model WTP as a function of the transformed bid levels $x - a/x$. Weight the p_k by $n_k/p_k(1 - p_k)$.
(d) Estimate the mean WTP from this fitted curve. Adjust this estimate for respondents who were not willing to pay anything, by using equation (10).
(e) To estimate the precision of the estimated mean WTP, create a bootstrap data set by resampling with replacement from the list of respondents. Apply steps (a)–(d) to this resample. Repeat 999 times, and extract appropriate summaries. (For simplicity, only the model identified from stepwise logistic regression on the real data was fitted to the bootstrap resamples.)

The logistic equation from step (a) is summarized in Table 2. The questionnaire that provided the covariate information appears as an appendix in Macmillan *et al.* (1995). From the results, we see that WTP increases with income, with the number of environmental organizations that the respondent is a member of, with the projected level of damage caused by acid rain and with the level of concern about international environmental issues. WTP decreases with increasing bid level, increasing opposition to Government expenditure on environmental issues and the number of reminders sent before a response was received. Higher levels of understanding of information presented in the questionnaire and higher priority assigned to the problem of pollution relative to other social concerns (e.g. education and defence) both correlated with higher WTP.

Table 2. Summary of the analysis of deviance for the logistic regression of individual response (acceptance or rejection of the bid offered) on covariates selected by the stepwise procedure†

	Degrees of freedom	*Deviance*	*Mean deviance*
Regression	10	225.4	22.54
(bid + 1/bid	2	45.7	22.83
+ income	1	105.7	105.66
+ govt	1	24.3	24.28
+ return	1	14.8	14.78
+ understand	1	11.7	11.69
+ member	1	8.6	8.56
+ pollu	1	6.2	6.22
+ damage	1	4.5	4.49
+ abroad	1	4.0	4.05)
Residual	732	716.7	0.98
Total	742	942.1	
	Estimate	*Standard error*	*Student's t*
constant	1.079	0.738	1.46
bid	−0.00429	0.00093	−4.63
1/bid	16.3	11.0	1.48
income	0.419	0.048	8.70
govt	−0.427	0.147	−2.91
return	−0.406	0.122	−3.33
understand	−0.372	0.129	−2.88
member	0.371	0.146	2.54
pollu	−0.173	0.075	−2.31
damage	0.306	0.153	2.01
abroad	0.132	0.066	2.01

†bid, x, the bid level offered; 1/bid, $1/x$; income, annual gross household income (1 ≡ <£5000; 2 ≡ £5000–10000; 3 ≡ £10000–15000; ...; 9 ≡ >£40000); govt, attitude towards the Government's role in protecting the environment (Government should protect environment regardless of cost (1) through to Government should have no control over the environment (5)); return, the return date category (1 ≡ no reminder necessary; 2 ≡ one reminder; 3 ≡ two reminders); understand, the level of understanding of the information presented (1 ≡ 'I understand it' through to 4 ≡ 'I really do not understand most of it'); member, the number of environmental organizations or charities of which the respondent is a member (maximum 6); pollu, a priority ranking of pollution among six social issues (1 ≡ most important; 6 ≡ least important); damage, the level of future losses 'predicted' if no action is taken (1 ≡ low damage; 2 ≡ moderate damage; 3 ≡ high damage); abroad, a score reflecting the level of concern about two environmental issues not affecting the UK directly (for each issue, 1 ≡ not concerned through to 4 ≡ very concerned); abroad is the sum of the two scores.

The finding that the number of reminders is a useful predictor is not unusual. Those respondents who have strong views on the issues that are raised in the questionnaire are more likely to return it without the need for any reminder. On average, these people are likely to be willing to pay more than those who respond only after receiving reminders.

The fitted logistic curve from step (c) was

$$\hat{y}_j = \frac{1}{1+\exp\{0.003\,727(x_j - 3807/x_j - 345.9)\}}.$$

This curve is plotted in Fig. 1. Analytic and bootstrapped standard errors and correlations are compared in Table 3. For simplicity, we conditioned on the estimate $a = 3807$ obtained from step

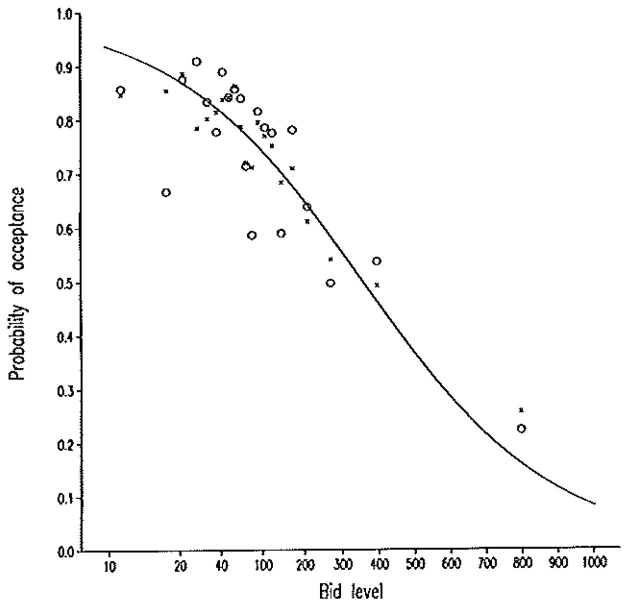

Fig. 1. Scatterplot of the proportion of respondents accepting a bid (O) and of the mean predicted probability of acceptance of a bid (×) against bid level under the reciprocal bid transformation: also shown is the fitted logistic curve (———) from which mean WTP is estimated

Table 3. Estimates of the parameters of the WTP curve†

Parameter	*Estimate*	*Standard error*		
		Analytic	*Bootstrap*	
α	3807	3101	4317	
β	−0.003727	0.000327	0.000867	
μ	345.9	20.4	42.2	
corr($\hat{\alpha}$, $\hat{\beta}$)				0.639
corr($\hat{\alpha}$, $\hat{\mu}$)				0.107
corr($\hat{\beta}$, $\hat{\mu}$)				0.593

†Also shown are analytic and bootstrap standard errors of the parameter estimates, and correlations estimated by using the bootstrap. The form of the curve is $E(y_j) = 1/[1 + \exp\{-\beta(x_j - \alpha/x_j - \mu)\}]$.

(a) when fitting the curve, although we re-estimated a at step (a) in each bootstrap resample. The number of bootstrap replications was 999, and the balanced bootstrap was used, i.e. each respondent featured in the bootstrap resamples exactly 999 times in total. The large correlation between $\hat{\alpha}$ and $\hat{\beta}$, coupled with bootstrap standard errors that are substantially larger than the analytic standard errors, suggest that precision is more reliably quantified by using the bootstrap. Table 4 provides little evidence of a systematic departure from the model, although the observed proportions fluctuate at the higher bid levels. The fluctuations exhibited by the mean predicted probabilities of acceptance in Fig. 1 are appreciably smaller than those of the observed proportions accepting the bid, suggesting that the larger differences between observed and predicted in Table 4 can be explained by differences in values of the covariates for respondents offered different bid levels.

Numerically integrating the above equation and using the bootstrap to quantify the precision, we estimate that the mean annual WTP for those willing to pay something is £425, with 95% 'percentile' confidence interval (£333, £596). Adjusting for the 21.26% of respondents who are not willing to pay anything, we estimate the mean annual WTP for the population as £335, with 95% confidence interval (£261, £475). Thus, if we assume that each respondent represents his or her household and take the number of households in Scotland to be 1.96 million, we estimate that the total annual WTP for Scotland is £656 million, with 95% confidence interval (£512, £930) million. If we make the extreme assumption that all invalid and non-responses correspond to households that are not willing to pay anything, then these estimates should be reduced by multiplying by the fraction of valid responses, $1669/2720 = 0.6136$ (from Table 1), giving a total annual WTP for Scotland of £402 million with 95% confidence interval (£314, £570) million.

In Table 5, we show the estimated mean annual WTP under various model assumptions. Whereas a logistic curve provides a very poor fit to the data at small bid levels if the bid level is untransformed (Fig. 2), the pragmatic approach of left truncating the curve at 0 to disallow

Table 4. Number of respondents offered each bid level†

Bid level x	*Number offered bid level x*	*Number accepting offer*	*Proportion accepting offer*	
			Observed	*Predicted*
11	7	6	0.857	0.927
17	6	4	0.667	0.887
21	8	7	0.875	0.868
26	11	10	0.909	0.850
31	12	10	0.833	0.836
37	9	7	0.778	0.823
42	18	16	0.889	0.813
48	19	16	0.842	0.803
55	14	12	0.857	0.793
63	25	21	0.840	0.782
71	28	20	0.714	0.773
81	29	17	0.586	0.762
92	38	31	0.816	0.750
106	42	33	0.786	0.736
122	49	38	0.776	0.721
143	56	33	0.589	0.702
170	73	57	0.781	0.677
209	80	51	0.638	0.641
270	141	70	0.496	0.583
396	69	37	0.536	0.462
798	9	2	0.222	0.159

†Also shown are the observed and predicted numbers accepting the bid and the corresponding proportions.

Table 5. Estimated mean WTP under various analysis options†

Transformation	*Truncation*	*Covariates averaged*	*Estimated mean WTP (£)*
Reciprocal	None	No	335
None	Left truncation at £0	No	308
Logarithmic	None	No	4442
Reciprocal	None	Yes	341

†Estimates are adjusted for those not willing to pay anything, estimated to be 21.26% of the population.

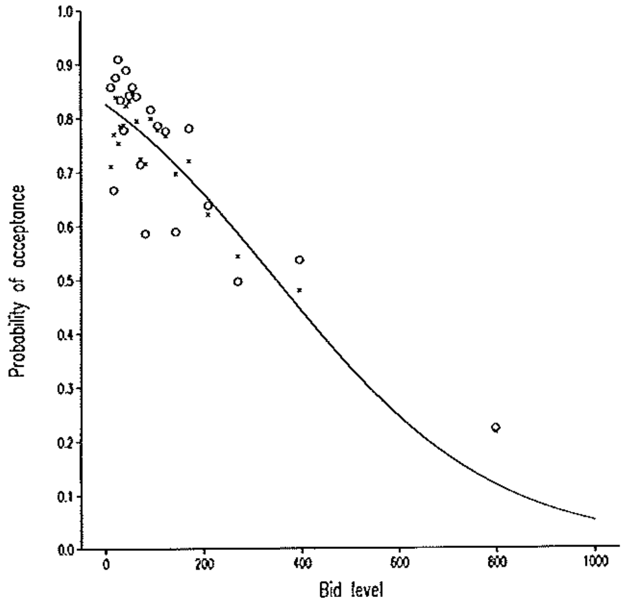

Fig. 2. Scatterplot of the proportion of respondents accepting a bid (O) and of the mean predicted probability of acceptance of a bid (×) against bid level under left-truncation of the WTP curve, fitted using the untransformed bid level: also shown is the logistic curve (———) fitted to the mean predicted probabilities

negative predicted WTP yields a reasonable estimate of mean WTP. This is because the poor fit is only at small bid levels, which leave the mean bid level largely unaffected. By contrast, if we log-transform bid levels, the logistic fit is very good except at the highest bid level, where the predicted probability of acceptance is substantially higher than the observed probability (Fig. 3). If this curve is not truncated to the right, a huge estimated mean WTP is obtained (Table 5). The usual solution of truncating the curve, say at the highest bid level, may reduce the estimated mean to believable levels but does not cure the problem. The estimation is extremely sensitive to the arbitrary and subjective choice of truncation distance. The rule of truncating at the highest bid level renders the estimation highly sensitive to the design decision of what bid levels to offer. Another 'solution' is to estimate the mean log(WTP), and then to back-transform. This provides a more 'respectable' estimate of £387, but it is a (strongly) biased estimate of mean WTP. It is a better estimate of the median WTP, which is substantially smaller than the mean under the assumed model. To estimate total WTP across the population, we require a valid estimate of the

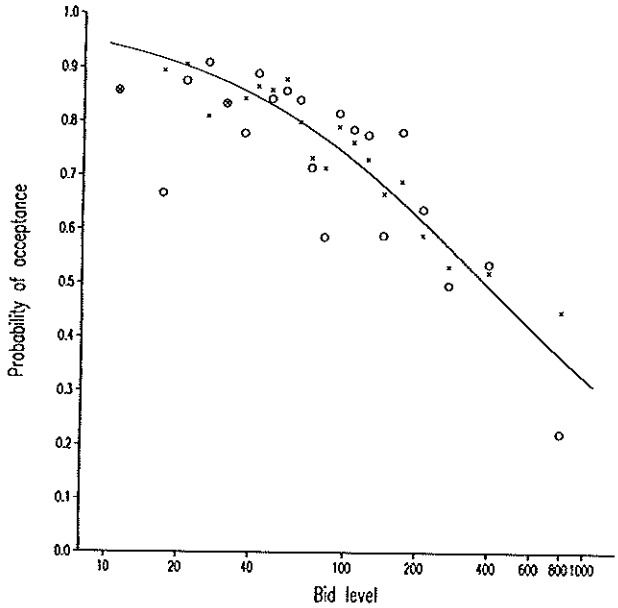

Fig. 3. Scatterplot of the proportion of respondents accepting a bid (O) and of the mean predicted probability of acceptance of a bid (×) against bid level under the logarithmic bid transformation: also shown is the logistic curve (———) fitted to the mean predicted probabilities

mean, not the median. The final method that we consider is one that is often recommended, of predicting WTP by bid level at the average of the other covariate values. In Fig. 4, we see that the estimated WTP curve at low bid levels is rather high relative to the observed proportions accepting the bid, presumably because individuals on average income are likely to accept low bid levels, whereas most on very low incomes reject these bid levels. In this study, the effect on estimated mean WTP is slight (Table 5), largely because the effect reverses at high bid levels, so that positive bias in the estimated WTP curve at low bid levels is offset by negative bias at high levels.

4. Discussion

In the theoretical development of this paper, it has been assumed that WTP is integrated over the range 0–∞. Because we rely on numerical integration, in practice, ∞ is replaced by some large finite value, chosen so that the area of the tail above that value is negligible. Several researchers have noted the sensitivity of mean WTP to the choice of model, and Duffield and Patterson (1991) have made a convincing argument for truncating the WTP curve at some smaller value. All the above results, except for the simple logistic model without left truncation or transformation, apply equally to the case of right truncation, provided that ∞ is replaced by the truncation value in the integrals. Some researchers advocate the use of median estimators, but these can be severely biased for estimating the mean (or total) WTP in the population, if the WTP curve is markedly asymmetric. Again, Duffield and Patterson (1991) have provided a useful discussion of this issue.

We have described methods that can be readily implemented using standard statistical software that provides logistic regression and logistic curve fitting facilities, together with numerical integration. Cameron (1988) has shown that it is possible to model WTP without including bid level as a covariate. Instead, she developed a type of censored logistic regression. Her more direct

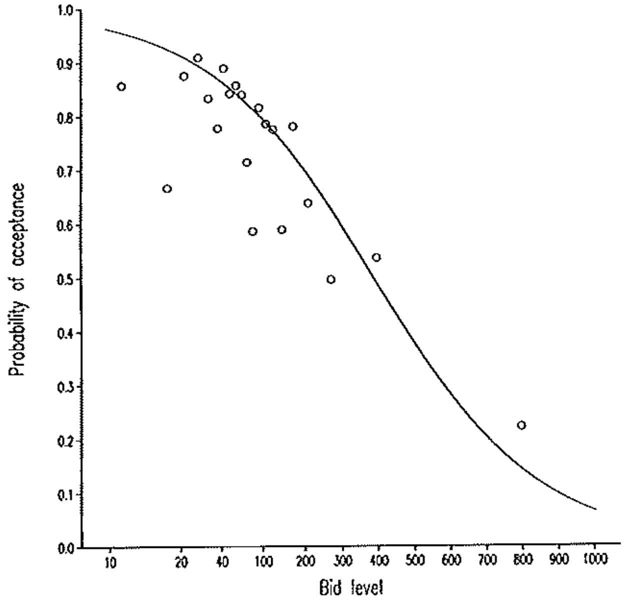

Fig. 4. Scatterplot of the proportion of respondents accepting a bid (O) against bid level under the reciprocal bid transformation: also shown is the logistic curve (———) obtained by averaging over covariates other than bid level

approach to estimating WTP has considerable appeal but requires methods that are not available in standard statistical software.

It is important in contingent valuation studies to secure a high response rate, as WTP is likely to be correlated with whether an individual responds. In our example, we found that respondents who required reminders were willing to pay less than those who responded before reminders were mailed. Hence it is likely that non-respondents were on average willing to pay less than respondents. It can also be expected that the proportion of non-respondents who were not willing to pay anything will be higher than the 21.3% estimated from respondents.

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We thank David Elston of Biomathematics and Statistics Scotland and a referee for their useful review comments. This work was partly funded by the Scottish Office Agriculture, Environment and Fisheries Department.

Appendix A

We provide here the formulae required to estimate variances under the models of Section 2. The notation is defined there. Integrals may be evaluated by any numerical integration routine. All variance and covariance expressions were obtained by using the delta method as described for example by Seber (1982).

Let x_i be random variables with expectation θ_i, $i = 1, \ldots, n$. Suppose that we wish to estimate the variance of some function $g(x_1, \ldots, x_n)$ and its covariance with another function $h(x_1, \ldots, x_n)$. Using a Taylor series expansion,

$$g(x_1, \ldots, x_n) \simeq g(\theta_1, \ldots, \theta_n) + \sum_{i=1}^{n} (x_i - \theta_i) \frac{\partial g}{\partial x_i} + \ldots$$

and similarly for h. Ignoring higher order terms,

$$\operatorname{var}\{g(x_1, \ldots, x_n)\} \simeq \sum_{i=1}^{n} \operatorname{var}(x_i) \left(\frac{\partial g}{\partial x_i} \right)^2 + 2 \sum_{i=1}^{n-1} \sum_{j>i} \operatorname{cov}(x_i, x_j) \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j},$$

$$\operatorname{cov}\{g(x_1, \ldots, x_n), h(x_1, \ldots, x_n)\} \simeq \sum_{i=1}^{n} \sum_{j=1}^{n} \operatorname{cov}(x_i, x_j) \frac{\partial g}{\partial x_i} \frac{\partial h}{\partial x_j}.$$

A.1. Model 1: logistic regression

$$\operatorname{var}(m) \simeq \frac{b_0^2 \operatorname{var}(b) + b^2 \operatorname{var}(b_0) - 2bb_0 \operatorname{cov}(b, b_0)}{b^4}, \tag{14}$$

$$\operatorname{cov}(b, m) \simeq \frac{b_0 \operatorname{var}(b)}{b^2} - \frac{\operatorname{cov}(b, b_0)}{b}. \tag{15}$$

A.2. Model 2: left truncation of the logistic curve

$$\begin{aligned}\operatorname{var}(m_1) \simeq{}& \operatorname{var}(b) \left[\int_0^\infty \frac{xu\{(bx-1) - (bx+1)u\}}{(1+u)^3} \, \mathrm{d}x \right]^2 + \operatorname{var}(m) \left\{ \int_0^\infty \frac{b^2 xu(u-1)}{(1+u)^3} \, \mathrm{d}x \right\}^2 \\ &+ 2 \operatorname{cov}(b, m) \int_0^\infty \frac{xu\{(bx-1) - (bx+1)u\}}{(1+u)^3} \, \mathrm{d}x \int_0^\infty \frac{b^2 xu(u-1)}{(1+u)^3} \, \mathrm{d}x\end{aligned} \tag{16}$$

where $u = \exp\{-b(x-m)\}$.

$$\begin{aligned}\operatorname{var}(m_0) \simeq{}& \operatorname{var}(b) \left(\int_0^\infty \frac{xu[v\{b(x-m) - u - 1\} - b\{uv(x-m) + m(1+u)\exp(bm)\}]}{v^2(1+u)^3} \, \mathrm{d}x \right)^2 \\ &+ \operatorname{var}(m) \left[\int_0^\infty \frac{b^2 ux\{2uv - (1+u)\}}{v^2(1+u)^3} \, \mathrm{d}x \right]^2 \\ &+ 2 \operatorname{cov}(b, m) \left(\int_0^\infty \frac{xu[v\{b(x-m) - u - 1\} - b\{uv(x-m) + m(1+u)\exp(bm)\}]}{v^2(1+u)^3} \, \mathrm{d}x \right) \\ &\times \int_0^\infty \frac{b^2 ux\{2uv - (1+u)\}}{v^2(1+u)^3} \, \mathrm{d}x\end{aligned} \tag{17}$$

where $v = 1 + \exp(bm)$ and u is as above.

A.3. Model 3: logarithmic transformation of bid

$$\begin{aligned}\operatorname{var}(m_1) \simeq{}& \operatorname{var}(b) \left(\int_0^\infty \frac{u[b\log(x) - 1 - \{b\log(x) + 1\}u]}{(1+u)^3} \, \mathrm{d}x \right)^2 \\ &+ \operatorname{var}(m) \left\{ \int_0^\infty \frac{b^2 u(u-1)}{(1+u)^3} \, \mathrm{d}x \right\}^2 \\ &+ 2 \operatorname{cov}(b, m) \int_0^\infty \frac{u[b\log(x) - 1 - \{b\log(x) + 1\}u]}{(1+u)^3} \, \mathrm{d}x \int_0^\infty \frac{b^2 u(u-1)}{(1+u)^3} \, \mathrm{d}x\end{aligned} \tag{18}$$

where $u = \exp[-b\{\log(x) - m\}]$.

A.4. Model 4: reciprocal bid transformation

$$\operatorname{var}(a) \simeq \frac{b_2^2 \operatorname{var}(b) + b^2 \operatorname{var}(b_2) - 2bb_2 \operatorname{cov}(b, b_2)}{b^4}, \tag{19}$$

$$\operatorname{cov}(b, a) \simeq \frac{b_2 \operatorname{var}(b)}{b^2} - \frac{\operatorname{cov}(b, b_2)}{b}, \tag{20}$$

$$\operatorname{cov}(m, a) \simeq \frac{b_0 b_2 \operatorname{var}(b) - bb_2 \operatorname{cov}(b, b_0) - bb_0 \operatorname{cov}(b, b_2) + b^2 \operatorname{cov}(b_0, b_2)}{b^4}, \tag{21}$$

$$\begin{aligned}
\operatorname{var}(m_1) \simeq{}& \operatorname{var}(b)\left[\int_0^\infty \frac{xuv\{1+u+b\log(u)(u-1)\}}{(1+u)^3}\,\mathrm{d}x\right]^2 + \operatorname{var}(m)\left\{\int_0^\infty \frac{b^2 xuv(u-1)}{(1+u)^3}\,\mathrm{d}x\right\}^2 \\
&+ \operatorname{var}(a)\left[\int_0^\infty \frac{bu\{bv(u-1)+(1+u)/x\}}{(1+u)^3}\,\mathrm{d}x\right]^2 \\
&+ 2\operatorname{cov}(b, m)\int_0^\infty \frac{xuv\{1+u+b\log(u)(u-1)\}}{(1+u)^3}\,\mathrm{d}x \int_0^\infty \frac{b^2 xuv(u-1)}{(1+u)^3}\,\mathrm{d}x \\
&+ 2\operatorname{cov}(b, a)\int_0^\infty \frac{xuv\{1+u+b\log(u)(u-1)\}}{(1+u)^3}\,\mathrm{d}x \int_0^\infty \frac{bu\{bv(u-1)+(1+u)/x\}}{(1+u)^3}\,\mathrm{d}x \\
&+ 2\operatorname{cov}(m, a)\int_0^\infty \frac{b^2 xuv(u-1)}{(1+u)^3}\,\mathrm{d}x \int_0^\infty \frac{bu\{bv(u-1)+(1+u)/x\}}{(1+u)^3}\,\mathrm{d}x
\end{aligned} \tag{22}$$

where $u = \exp\{-b(x - a/x - m)\}$ and $v = 1 + a/x^2$.

References

Bowker, J. M. and Stoll, J. R. (1988) Use of discrete choice non-market methods to value the whooping crane resource. *Am. J. Agric. Econ.*, **70**, 372–381.

Boyle, K. J., Welsh, M. P. and Bishop, R. C. (1988) Validation of empirical measures of welfare change: comment. *Land Econ.*, **64**, 94–98.

Buckland, S. T. (1984) Monte Carlo confidence intervals. *Biometrics*, **40**, 811–817.

Burnham, K. P., Anderson, D. R., White, G. C., Brownie, C. and Pollock, K. H. (1987) *Design and Analysis Methods for Fish Survival Experiments based on Release–Recapture*. Bethesda: American Fisheries Society.

Cameron, T. A. (1988) A new paradigm for valuing non-market goods using referendum data: maximum likelihood estimation by censored logistic regression. *J. Environ. Econ. Mangmnt*, **15**, 355–379.

Cooper, J. C. (1993) Optimal bid selection for discrete choice contingent valuation survey. *J. Environ. Econ. Mangmnt*, **24**, 25–40.

———(1994) A comparison of approaches to calculating confidence intervals for benefit measures from dichotomous choice contingent valuation surveys. *Land Econ.*, **70**, 111–122.

Duffield, J. W. and Patterson, D. A. (1991) Inference and optimal design for a welfare measure in dichotomous choice contingent valuation. *Land Econ.*, **67**, 225–239.

Efron, B. (1979) Bootstrap methods: another look at the jackknife. *Ann. Statist.*, **7**, 1–26.

Hanemann, W. M. (1984) Welfare evaluations in contingent valuation experiments with discrete responses. *Am. J. Agric. Econ.*, **66**, 332–341.

———(1991) Willingness to pay and willingness to accept: how much do they differ? *Am. Econ. Rev.*, **81**, 635–647.

Johansson, P. O., Kriström, B. and Mäler, K. G. (1989) Welfare evaluations in contingent valuation experiments and discrete response data: comment. *Am. J. Agric. Econ.*, **71**, 1053–1056.

Macmillan, D. C., Hanley, N. and Buckland, S. T. (1995) Valuing biodiversity losses due to acid rain deposition: a contingent valuation study of uncertain environmental gains. *Discussion Paper in Ecological Economics 95/1*. University of Stirling, Stirling.

National Oceanic and Atmospheric Administration (1993) Report of the National Oceanic and Atmospheric Administration Panel on contingent valuation. *Fed. Reg.*, **58**, 4602–4614.

Park, T., Loomis, J. B. and Creel, M. (1991) Confidence intervals for evaluating benefits estimates from dichotomous choice contingent valuation studies. *Land Econ.*, **67**, 64–73.

Seber, G. A. F. (1982) *The Estimation of Animal Abundance and Related Parameters*, 2nd edn. London: Griffin.