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Surface Models and the Resolution of n-Dimensional Cell Ambiguity

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◊ Introduction ◊

The representation of n-dimensional continuous surfaces often employs a discrete lattice of n-dimensional cube cells. For instance, the Marching Cubes method (Lorensen and Cline 1987), locates the surface lying between adjacent vertices of the n-cube edges in which the cell vertices represent discrete sample values. The volume's surface exists at a point of zero value: it intersects any cube edge whose vertex values have opposing sign.

Ambiguities occur in the cells whose vertex set show many sign alternations. Geometrically, the surface intersects one face of the n-cube through each of its four edges. It is these special cases which engenders the need for resolution as a central concern in surface modeling. This gem reviews and illustrates the disambiguation strategies described in the literature.

◊ Background ◊

In an ideal surface algorithm, the features of the surface geometry should match those of the underlying surface. In particular, if the original surface is continuous, the representational model must preserve this continuity. Most practical algorithms create spurious holes (false negatives) or additional surfaces (false positives) depending on the "eagerness" of the algorithm in joining pieces of the surface model along adjacent cube faces. This is the consequence the "ambiguous face" n-cube present in any dimension $n >= 2$ and whose vertex signs resemble a spatial "checkerboard" (Figure 1). The abutting of two cubes having such faces then introduces the possibility of false positives or negatives (Figure 2).

In this gem, we refer to the vertex classification with respect to the threshold as

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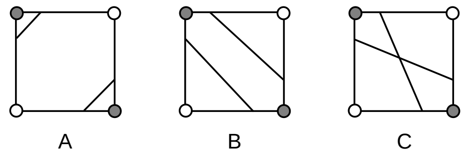


Figure 1. Ambiguous Face Choices

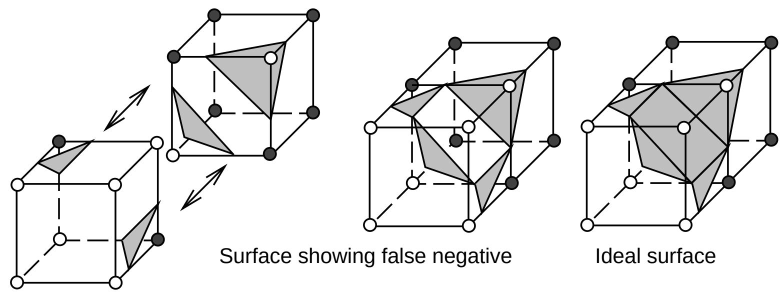


Figure 2. Ambiguous Face

inside or *outside* the surface, and the surface intersects the edge between an *inside* and an *outside* vertex, shown grey on the diagrams; linear interpolation is used to calculate this position. The ambiguous face can be estimated using the vertex classification, but can never be completely disambiguated.

The local surface contours can be represented by sections of a hyperbola and the ambiguous face can be one of three orientations (Figure 1); therefore, the cross representation is the other orientations taken to the limit, and is normally discarded.

The cells can be subdivided into further n-cubes or into simplices. A simplex is the simplest non-degenerate object in n-dimensions (Hanson 1994,Moore 1992a), *e.g.* a triangle in two dimensions and a tetrahedron in three dimensions. A simplex is always unambiguous and so can be used in an n-cube disambiguation strategy.

◊ Static Analysis ◊

To disambiguate the ambiguous face the static techniques consider only the vertex classification points; they do not introduce extra classification points. These methods are generally fast, but they do not guarantee an ideal or faithful surface.

Uniform Orientation: always present the surface at a common orientation whenever the evaluation of an ambiguous face is encountered. Computation of orientation can can be implemented using a lookup table (Lorensen and Cline 1987) or by algorithm (Wyvill et al. 1986,Bloomenthal 1988,Bloomenthal 1994). If the data resolution is high

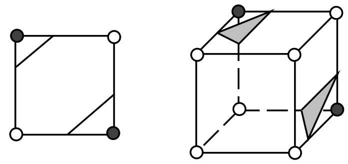


Figure 3. Fixed Orientation

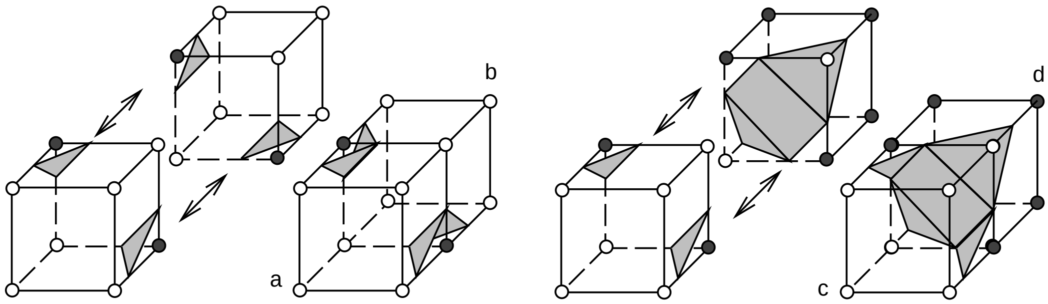


Figure 4. Adjacent Cell Disambiguation

the surface segments will be small and the anomalies unnoticeable (unless the surface is zoomed). This method is simple to implement and is fast to execute.

Face Adjacency: in some cases the adjacent cell configuration can be used to disambiguate the n-cube (Duurst 1988,Zahlten 1992); for example, if an "inverted" cube and a "normal" cube orientation are adjacent then the surface should be added (Figure 4). The new surface intersects the diagonal between the non-adjacent vertices c and d, where vertex d is *inside* and vertex c is *outside* the surface.

Simplex Decomposition: in two dimensions the square can be decomposed into two triangle segments; and treated as by the "Uniform Orientation" method. In three dimensions the cube has many decompositions into tetrahedra (Moore 1992b,Moore 1992a) (Figure 5); examples of five tetrahedra (Ning and Bloomenthal 1993) and six tetrahedra (Zahlten 1992) behave like the "Fixed Orientation" method in that they add an extra diagonal which affects the connectivity of the surface. The orientation of the diagonal is determined by the simplex decomposition. To maintain surface consistency, neighboring n-cubes should have the same diagonal orientation (mirrored simplex orientation).

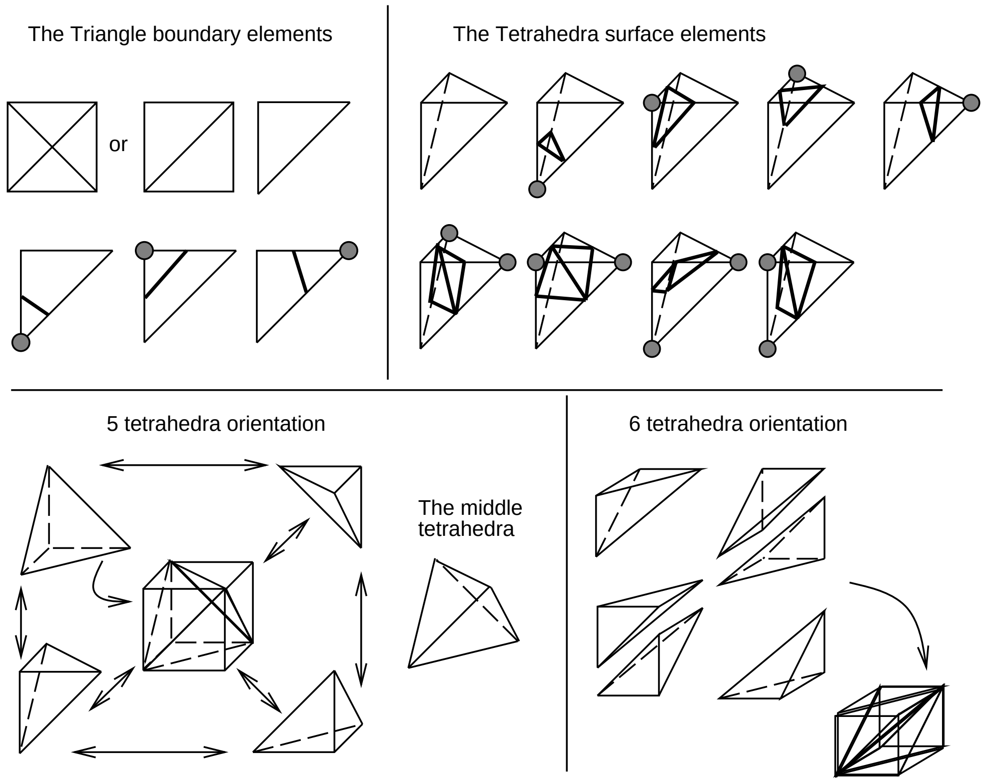


Figure 5. Simplex Decompositions, and surfaces

◊ Interpolation Analysis ◊

This section reviews disambiguation techniques that require the computation of additional values or vertices for the decision. The values are often created by methods of tri-linear interpolation (Hill 1994) and other methods such as tri-cubic interpolation (Arata 1994).

Closest Orientation: the four face intersection points are located by linear interpolation, the total length of the connecting paths calculated, and the orientation having the shortest path is chosen (Mackerras 1992). If both paths are the same length then the cross configuration is chosen (Cottifava and Moli 1969). In Figure 1, the "Closest Orientation" technique would select configuration A.

Resampling: the data is resampled at a higher resolution and solution reattempted. This is possible only when the data is algorithmically obtained or readily resampled. Moreover, ambiguities may still remain at the higher resolution.

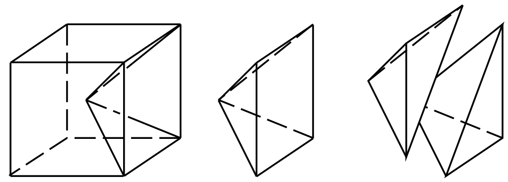


Figure 6. 12 Tetrahedra Orientation in a cube

Interpolation: the data resolution is doubled using a tri-linear interpolation (Hill 1994) or a tri-cubic interpolation. The tri-cubic interpolation considers points outside the local neighbors. As with the Resampling technique, ambiguities may still occur at the finer resolution. (A variation reinterpolates merely the ambiguous cells.)

Subdivision: all the n-cubes that are on the surface are sub-divided (using linear interpolation) until a predefined limit is reached. The limit can be the pixel size *e.g.* Dividing Cubes (Cline et al. 1988), or smaller (Cook et al. 1987). Each sub-cube is either *inside*, *outside* or *on* the surface; and may be shaded and projected onto the view plane. Tri-linear interpolation cannot introduce an ambiguous case, but might not (therefore) faithfully model the surface. However, adaptive subdivision techniques (using interpolation or resampling methods) can be used at points of great interest or high curvature (Bloomenthal 1988).

Simplex Decomposition: in two dimensions the two-cube can be decomposed into two or four triangles (Figure 5); with two triangles the method is similar to the "Uniform Orientation" strategy, but with four triangles an extra center vertex is required. This can be obtained by averaging the four vertices (*i.e.* bi-linear interpolation). If the center value is *inside* the threshold then orientation B is chosen otherwise orientation A is used (Figure 1). This method is often named "facial average" and can be used on any n-cube face when $n > 1$ (Wyvill et al. 1986,Wilhelms and Gelder 1990,Hall 1990).

In three dimensions the three-cube can be divided into twelve tetrahedra and the value at the center of the cube is required (using tri-linear interpolation) (Figure 6).

Bilinear Contours: the contours of the image can be represented (locally) by parts of a hyperbola (Nielson and Hamann 1991). The ambiguous face occurs when both parts of the hyperbola intersect a face, therefore, the topology of the hyperbola equals the connection of the contour. The correct orientation is achieved by comparing the threshold with the bilinear interpolation at the crossing point of the asymptotes of the hyperbola, given by: $\frac{P0P3+P2P1}{P0+P3-P1-P2}$ (Figure 7). If the interpolation value is less than the threshold then use orientation A otherwise use orientation B.

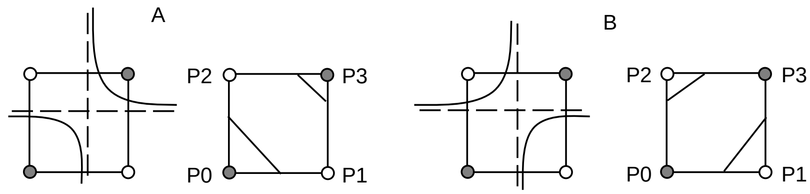


Figure 7. Bilinear Contours

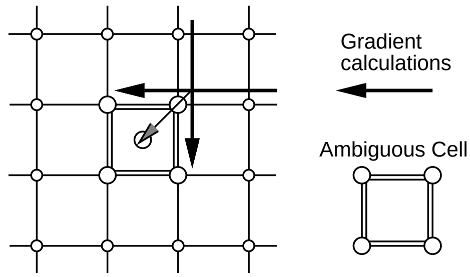


Figure 8. Gradient Disambiguation

Gradient: disambiguation of the cell can be achieved by calculating the gradient contribution (Ning and Bloomenthal 1993,Wilhelms and Gelder 1990), from the neighboring faces which point towards the center of the ambiguous face. These gradient contributions can be added to the four face vertex values and used to create a better approximation for the center of that face. This center value can then be used to disambiguate the cell (Figure 8).

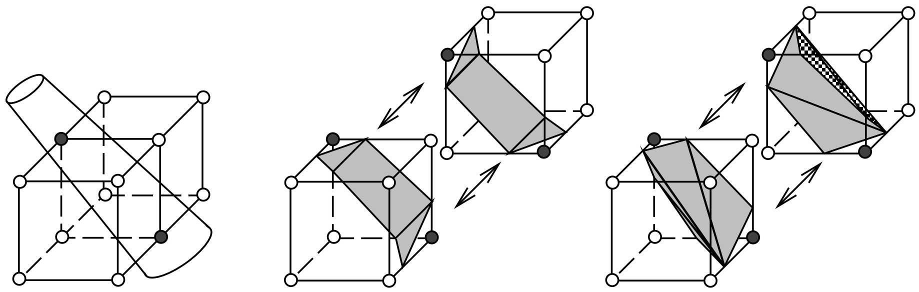
Quadratic: disambiguation can be achieved by fitting a quadratic curve to the local values (using the method of least-squares). The orientation of the curve is then used to disambiguate the face (Wilhelms and Gelder 1990,Ning and Bloomenthal 1993).

◊ Summary ◊

The n-cube with an ambiguous face can never be disambiguated by the vertex classification alone, however, at high resolutions the anomalies become unnoticeable.

The Simplex decomposition strategies work well if a center vertex is calculated, but they accrue many triangle elements.

Subdivision techniques can be used to view an enlargement of the image without false positives and negatives appearing, and the pixel sized cubes are then projected onto the viewing plane using a gradient shading based upon the four vertices. Subdivision techniques also eliminate degenerate triangle segments. Degenerate segments (very small triangle pieces) occur when the data resolution is high, or at the edge of the



Surface showing shared face   Ideal surface

Figure 9. Concave Triangle Surfaces

evaluation mesh. The degenerate triangles degrade the rendering efficiency. Degenerate triangles can also be reduced by using a "bending" technique (Moore and Warren 1992).

The Gradient and Quadratic methods are more accurate and more expensive than other methods, but they are useful if the sampling rate is low and if the data cannot be resampled.

Most disambiguation strategies, after deciding on the face orientation, place an extra surface section on the face. However, two such adjacent surfaces may share a common face. To resolve this, concave surfaces are used (Nielson and Hamann 1991) (Figure 9).

In the choice of disambiguation strategy there is a contention between speed and fidelity. Static methods are generally faster but can lead to erroneous surfaces. When the data resolution is sufficiently high these artifacts are not significant.

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