Near-Heisenberg-limit Quantum Computing

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Quantum computing is fundamentally limited by the Planck constant ($h = 6.63 \times 10^{-34}\ J \cdot s$) through the Heisenberg Limit. The energy consumption over a given time, or the speed of processing information with a specific energy budget, is a core research focus in quantum computing. To date, the smallest action (the energy-time cost) achieved is approximately $10^{-29} J \cdot s$, using a giant spin qubit composed of 20 spins. In our study, we achieved an action of $1.66 \times 10^{-34}\ J \cdot s$ to reversibly manipulate a single spin qubit through a spin-spin magnetic interaction experiment. By adhering to the principle of least action, our theoretical and experimental results establish the minimal action required. Our findings highlight the potential of spin-qubit quantum computers as accelerators for computation-intensive applications, such as AI and Post-Quantum Cryptography, since they exhibit several unique advantages: 1. High energy efficiency (by approaching the Heisenberg limit as well as the Landauer bound); 2. High-density integration (with just an atom/ion per qubit); 3. Long coherence times (tens of seconds); 4. High-fidelity (98%); and 5. Fault tolerance (through decoherence-free subspaces).

CCS Concepts: • **Computer systems organization** → **Quantum computing**;

Additional Key Words and Phrases: Quantum computing architecture, qubit, the principle of least action, artificial intelligence, post-quantum cryptography

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1 Introduction

Quantum computing relies on both reversible and irreversible operations, each serving distinct but vital roles, as shown in Figure 1. Unitary operations that are commonly used in quantum computing are reversible. In contrast, the initializations initializing the system to an entangled state and the projective measurements recovering (classical) information from the quantum computation are irreversible. In principle, the Heisenberg limit [1] limits both reversible and irreversible operations, while the Landauer bound [2] limits irreversible operations only. As can be seen in Section 4, the energy required in a reversible operation is much smaller than that required in the irreversible operation that normally needs a (relatively large) energy barrier to stop the system from returning to its initial state (this is the so-called irreversibility). Thus, the Heisenberg limit is always associated with reversible operations in this article, although it governs all quantum computing operations in principle.

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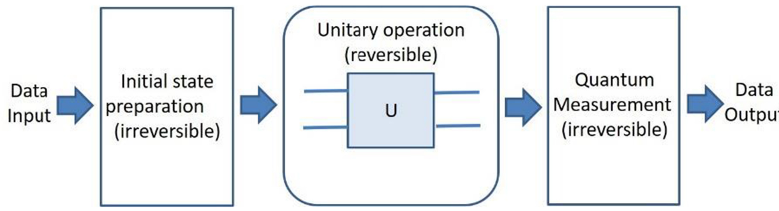

Fig. 1. Quantum computing relies on both reversible and irreversible operations.

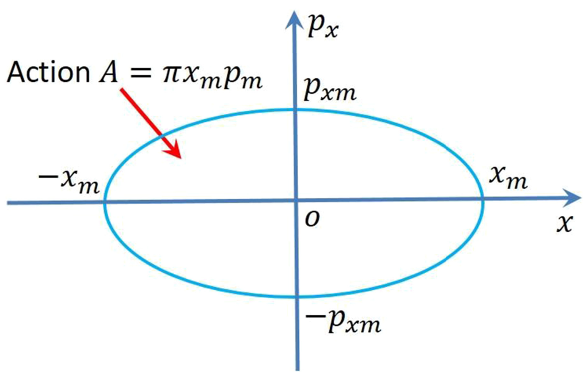

Fig. 2. In accordance with the Heisenberg limit, the principle of least action governs a system's dynamics by minimizing a specific integral known as the action, which is the product of position and momentum, or equivalently, energy and time. The phase trajectory (p_x vs. x) turned out to be an ellipse, whose area is the action $A = \pi x_m p_{xm}$.

Heisenberg initially presented his uncertainty principle in 1927 as $\Delta x \Delta p \geq h$, utilizing the position x, the momentum p, and the full Planck constant h [3]. In 1928, this inequality was modified to: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$, where $\hbar = \frac{h}{2\pi}$ represents the reduced Planck constant ($\hbar = \frac{h}{2\pi} = 1.0545 \times 10^{-34}\ J \cdot s$) [4, 5].

In 1945, Mandelstamm and Tamm extended the above position-momentum relation to the energy-time one [6]. That is, the product of the uncertainty in the energy measurement and the uncertainty in the lifetime measurement should not be smaller than the reduced Planck constant: $\Delta E \Delta t \geq \hbar$, where ΔE and Δt represent the uncertainties in the energy and lifetime measurements, respectively.

In 1945, a non-relativistic time–energy uncertainty relation was established as $\Delta E \Delta t \geq \frac{\hbar}{2}$ [6].

In 1990, Anandan and Aharonov gave a new quantum limit for an arbitrary quantum evolution from the Fubini-Study metric with a new geometric meaning to time [7]. They used the arc length (the geodesic distance) $s_{FS} = 2 \int \frac{\Delta E(t) dt}{\hbar}$ for the actual evolution between the two quantum states. Since the shortest possible distance between orthogonal states along a geodesic is π (to be shown in Figure 3), the actual distance $s_{FS} \geq \pi$ can be rewritten as $\langle \Delta E \rangle \Delta t \geq \frac{1}{4} h$, where $\langle \Delta E \rangle$ is the time-averaged uncertainty in energy during the time interval Δt. The above inequality is more stringent than the conventional time-energy uncertainty relations having $h/2$ or $\hbar/2$ on the right-hand side [7].

In this study, we will use the above Heisenberg quantum limit ($\frac{1}{4} h = \frac{\pi}{2} \hbar$) [8] from the action (energy times time) perspective [9] in terms of every information processing in quantum computing involving an action quantity (Figure 2). Simply speaking, the frequency (inversely proportional to time) is connected with the energy of a quantum state. Thus, to measure the energy with good

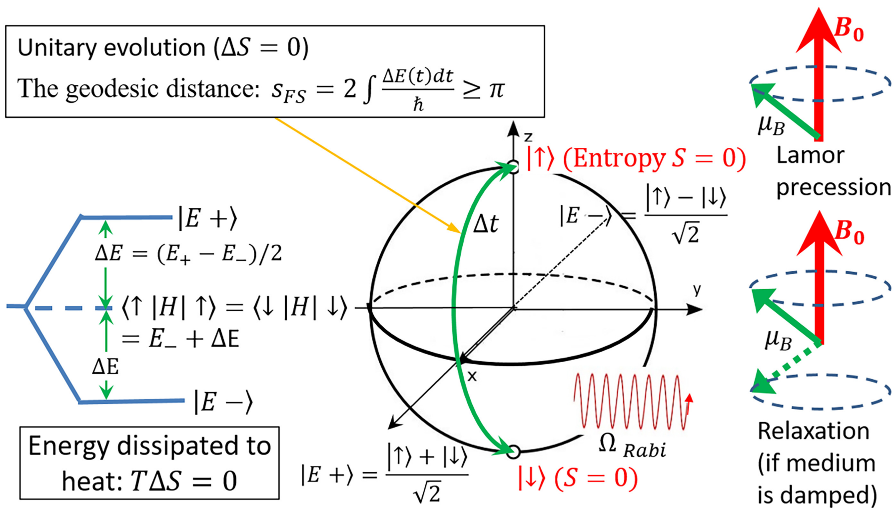

Fig. 3. Reversible operations ($|\uparrow\rangle \Leftrightarrow |\downarrow\rangle$) is ultimately limited by Heisenberg's limit, such as a unitary NOT operation performed on a spin qubit. This reversible oscillation meets the quantum interpretation of the Heisenberg limit [7-10]: a quantum state $|\uparrow\rangle$ with energy $\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow}$ takes time that is at least $\Delta t = \frac{\pi\hbar}{2\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow}}$ to evolve from an orthogonal state $|\uparrow\rangle$ to another state $|\downarrow\rangle$. Note that, in an arbitrary quantum evolution, the shortest possible distance between orthogonal states along a geodesic is π from the Fubini-Study metric.

accuracy, the state must be measured for many cycles. Against a classical saying "it takes time Δt to measure energy to an accuracy ΔE", the accurate interpretation proposed is that a quantum state with a spread in energy ΔE takes time that is at least equal to $\Delta t = \frac{\pi\hbar}{2\Delta E}$ to evolve from an orthogonal state to another. This interpretation can be further extended: a quantum system with an average energy E requires at least a time $\Delta t = \frac{\pi\hbar}{2E}$ to evolve from one orthogonal state to another [8-10].

2 Reversible Computing and Heisenberg's Limit

As a simple example in reversible computations [8], as shown in Figure 3, a (one-to-one) NOT operation on a spin qubit with logical states ($|\uparrow\rangle$ and $|\downarrow\rangle$) is performed. To perform the NOT operation, a potential $H = E_+|E_+\rangle\langle E_+| + E_-|E_-\rangle\langle E_-|$ can be applied with energy eigenstates $|E_+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ and $|E_-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$.

In practice, a static external magnetic field B_0 (that is large on the order of $k_B T$) can be activated to set the spin quantization axis and drive the spin to precess about the field direction (along a latitude of the Bloch sphere). The field-anti-aligned spin has an energy advantage of $\Delta E = \mu_B B_0$ over the field-aligned spin, so by aligning itself with the field to minimize energy, the spin will slowly reach a thermal (statistical) equilibrium. Nevertheless, the time scale of the above spin relaxation is so large that it is usually necessary to pulse another (small) resonant oscillating magnetic field with a Rabi frequency Ω_{Rabi} to flip the qubit, in which $h\Omega_{Rabi} = \mu_B B_0$ is used to supply the transition quantum energy between two possible spin states.

Because the midpoint of the $|E_+\rangle$-$|E_-\rangle$ gap is $\frac{1}{\sqrt{2}}(|E_+\rangle + |E_-\rangle) = \frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) + \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)] = |\uparrow\rangle$ or $\frac{1}{\sqrt{2}}(|E_+\rangle - |E_-\rangle) = \frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) - \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)] = |\downarrow\rangle$, each logical state $|\uparrow\rangle$, $|\downarrow\rangle$ has a spread in energy $\Delta E = \frac{1}{2}(E_+ - E_-)$. After a length of time $\Delta t = \frac{\pi\hbar}{2\Delta E}$, the qubit evolves so that $|\uparrow\rangle \Rightarrow |\downarrow\rangle \Rightarrow |\uparrow\rangle \Rightarrow |\downarrow\rangle \ldots$. That is, the application of the above potential is equivalent to a NOT operation [8]. The average energy E of the spin qubit during the logical operation is $\langle\uparrow|H|\uparrow\rangle = \langle\downarrow|H|\downarrow\rangle = \frac{1}{2}(E_+ + E_-) = E_- + \Delta E$. If $E_- = 0$, then $E = \Delta E$.

3 Irreversible Computing and Landauer's Bound

In the 1950s, von Neumann thought that, in a computer, every logical operation increases entropy by $k_B \ln 2$ (k_B is the Boltzmann constant), thereby dissipating energy $k_B T \ln 2$ at temperature T [11]. This speculation proved to be wrong by Landauer in 1961, who demonstrated that reversible operations (e.g., NOT) can be performed without dissipation whereas irreversible operations (e.g., many-to-one AND or one-bit ERASE operations) need to dissipate at least $k_B \ln 2$ for each bit of information lost [2]. The so-called Landauer erasure is a logical operation that erases a bit, 0 or 1, to 0 [3]. In 2009, Landauer's principle was derived from the second law of thermodynamics [12], which means that, on the one hand, Landauer's principle is technically correct, but, on the other hand, it may not be as fundamental and general as Heisenberg's limit. So far, on various information carriers [13-19], where a single spin [19] is minimal, Landauer's bound has been experimentally verified.

For a bit of (classical) position-encoded information [13-15], representing a bit of (random) datum, the information carrier is equally probable to be located in the L chamber or R chamber, i.e., the probabilities are $P(L) = P(R) = 1/2$, the entropy is thus $S = k_B \ln 2$. After the erasure, the carrier is assuredly erased to a designated reference state (L here): $P(L) = 1$; $(R) = 0$; $S = 0$. Note that, according to the principle of least action (Figure 2), the area enclosed by the phase trajectory of this position-encoded information carrier, no matter its oscillation frequency, is always given by a multiple of the quantum of action (the smallest change of action one can ever get).

For a bit of (classical) orientation-encoded information, the information carrier is a (single-domain) nanomagnet that has more than 10^4 spins [16]. It is still large enough to be considered classical [2]. Its (magnetization) orientation fluctuates due to thermal agitation and takes either 'up or 'down' along the easy axis ($S = k_B \ln 2$).

For a bit of qubit in quantum computing, encoding the two possible spin quantizations induced by an external magnetic field B_0, the information carrier is either a giant spin [17] or a single spin [19]. At effective low temperatures, quantum spin tunnelling via the ground state becomes the dominant spin relaxation channel [17].

In the Landauer principle of the above standard version [2], a very large reservoir is assumed by default, which means that the erasure process hardly changes the states of the reservoir, so that the mutual information and the relative entropy of the reservoir can be simply ignored, Figure 4 schematizes the improved Landauer principle with a finite-sized reservoir [15, 20]. In the initial maximally mixed state $\updownarrow$ (the center of the Bloch sphere) with an equal probability admixture of $|\uparrow\rangle$ and $|\downarrow\rangle$, we have a maximal entropy S = $k_B \ln 2$. After the erasure, the spin qubit eventually decays to a polarized pure quantum state $|\uparrow\rangle$ (a point on the Bloch sphere's surface) due to decoherence, which has a zero entropy S=0, and the heat $\Delta Q = ST$ is dissipated to the reservoir. The spin qubit is realized by encoding the two internal energy levels of the ion whereas the finite-size reservoir is realized by quantizing the z-axis micromotion of the ion, which can be measured by counting the phonon number n. By using a red-sideband laser to couple the (spin) qubit and the reservoir in this way ($|\downarrow, n\rangle \Leftrightarrow |\uparrow, n+1\rangle$), the flip of the spin will increment or decrement the phonon number n during the erasure process. The irreversibility required for the Landauer erasure [2] is automatically ensured by the decreased entropy ($\Delta S < 0$).

4 Qubit Encoded by a Single Spin in Quantum Computing

A spin–spin magnetic interaction experiment [21] was used to study a qubit encoded by a single spin in quantum computing. For most of the experimental time, the state evolution is limited to

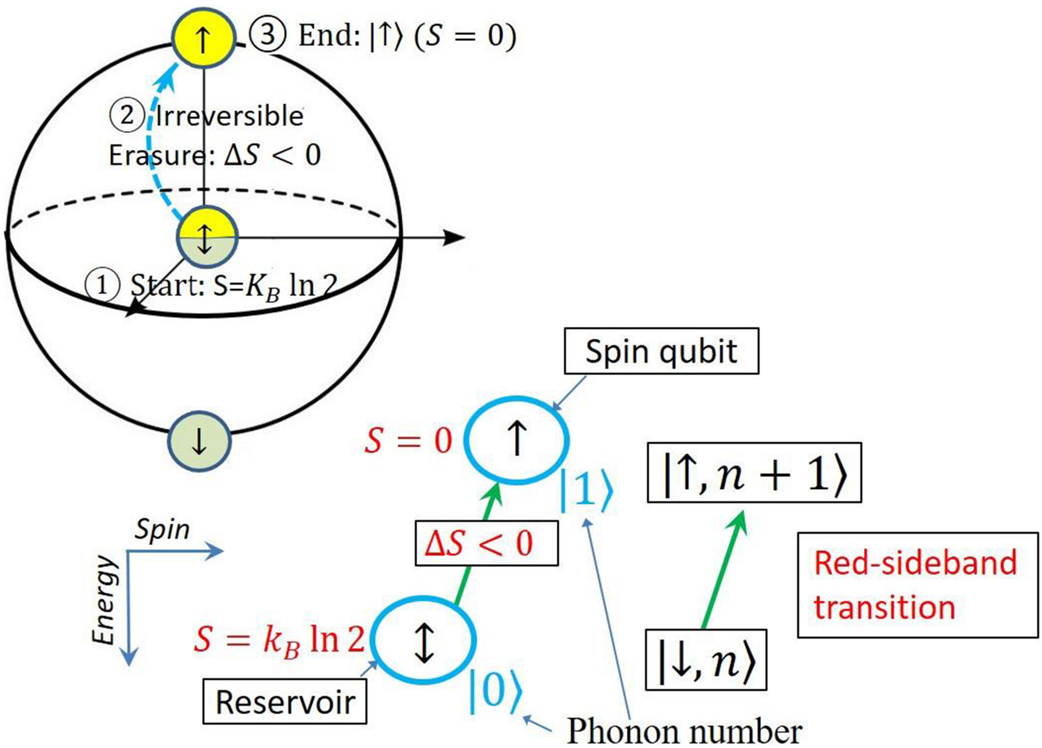

Fig. 4. Irreversible operations ($\updownarrow \ \Rightarrow |\downarrow\rangle$) is governed by the Landauer principle. The Landauer erasure process begins at the center of the Bloch sphere and irreversibly evolves towards a point on the surface of the Bloch sphere, leading to a decrease in entropy ($\Delta S < 0$).

the $|\uparrow\downarrow\rangle \Leftrightarrow |\downarrow\uparrow\rangle$ subspace, which is geometrically pictured in the Bloch sphere spanned by the $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ manifolds in Figure 5. An almost ideal $|\uparrow\downarrow\rangle \Leftrightarrow |\downarrow\uparrow\rangle$ rotation around x is observed at a rate of 4ξ, in which the spin–spin magnetic interaction strength is $\xi = \frac{\mu_0 (g\mu_B/2)^2}{4\pi\hbar d^3}$ with the vacuum permeability μ_0, the electron Lande g-factor $g \approx 2$, the Bohr magneton μ_B, and the ion-ion separation $d = (2.18 \sim 2.76)\ \mu m$ [21]. These rotations are essentially reversible because the damping of the spin oscillations is very weak [21]. That is, this state has low energy dissipation [21]. In fact, the evolution of the rotation between the $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ states is completely confined to the energy degenerate subspace [21]. All the above collective operations have typical fidelities of more than 98% [21].

The two-spin Hamiltonian [21] can be written as

$$H = \underbrace{0.5\hbar(\omega_{A,1}\sigma_{z,1} + \omega_{A,2}\sigma_{z,2}) +}_{Irreversible\,rotation\,by\,pushing\,(MHZ)} \quad +2\hbar\xi\sigma_{z,1}\sigma_{z,2} \quad \underbrace{-\hbar\xi(\sigma_{x,1}\sigma_{x,1} + \sigma_{y,1}\sigma_{y,2})}_{Reversible\,rotation\,due\,to\xi(mHz)} \quad , \tag{1}$$

in which the spin Larmor frequency is $\omega_{A,i} = 2\mu_B B_0/2\hbar$ with the Bohr magneton μ_B, and B_0 is an external magnetic field. The four eigenstates of this Hamiltonian in Equation (1) are $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and the two entangled Bell states $|\psi\pm\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$. Due to decoherence, the spin qubit will eventually decay from a random state $|\uparrow\downarrow\rangle$ with S = $k_B \ln 2$ to a ground state $|\uparrow\rangle$ with S = 0. Such irreversible decoherence ($\updownarrow \Rightarrow |\uparrow\rangle$, $\Delta S < 0$) is equivalent to Landauer erasure. As mentioned in Section 3, the symbol "$\updownarrow$" indicates that we have no information about the individual spin qubit,

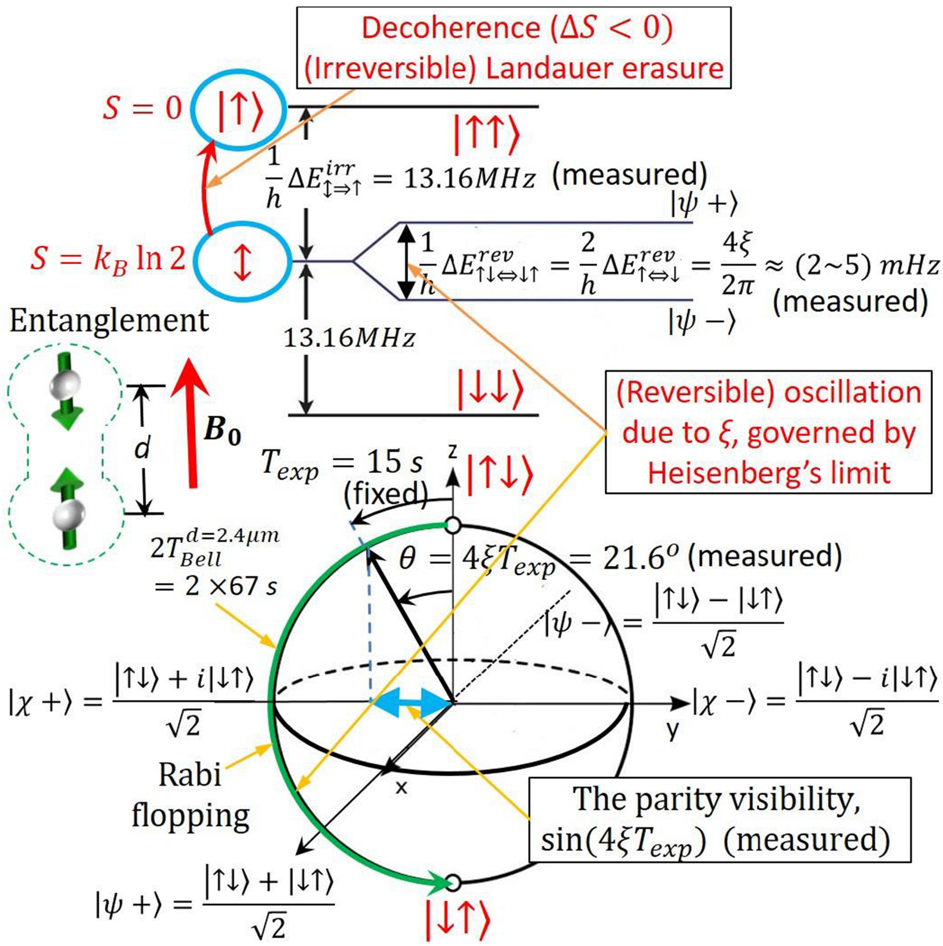

Fig. 5. A spin–spin magnetic interaction experiment [21] was used to measure the minimal action of manipulating a spin qubit for quantum computing, in terms of one spin being viewed as a qubit and another spin being viewed as a magnet to manipulate the (spin) qubit via the spin-spin magnetic interaction. The spin–spin magnetic interaction ξ imposes a reversible oscillation (in green) between $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ [21]. This reversible oscillation meets the quantum interpretation of the Heisenberg limit [7-10]: a quantum state $|\uparrow\downarrow\rangle$ with $\Delta E^{rev}_{\uparrow\downarrow\Leftrightarrow\downarrow\uparrow}$ takes time that is at least $\Delta t = \frac{\pi\hbar}{2\Delta E^{rev}_{\uparrow\downarrow\Leftrightarrow\downarrow\uparrow}}$ to evolve from an orthogonal state $|\uparrow\downarrow\rangle$ to another orthogonal state $|\downarrow\uparrow\rangle$. In spite of the indistinguishability between $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ in this experiment, the measured energy splitting $[4\xi = 2\pi(2 \sim 5)\ mHz]$ between $|\psi+\rangle = \frac{|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle}{\sqrt{2}}$ and $|\psi-\rangle = \frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}$ and the measured coherence time $T^{d=2.4\mu m}_{Bell} = 67\ s$ at $d = 2.4\ \mu m$ can still be used to verify the Heisenberg limit (which controls such a reversible oscillation ($|\uparrow\downarrow\rangle \Leftrightarrow |\downarrow\uparrow\rangle$)). On the other hand, the midpoint of the $|\psi+\rangle$-$|\psi-\rangle$ gap is $\frac{1}{\sqrt{2}}(|\psi+\rangle + |\psi-\rangle) = \frac{1}{\sqrt{2}}([\frac{|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle}{\sqrt{2}}] + [\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}]) = |\uparrow\downarrow\rangle$, in which a spin is equally possible to be in $|\uparrow\rangle$ or $|\downarrow\rangle$ and this random state "$\updownarrow$" has a (maximal) entropy $S = k_B \ln 2$. The measured energy splitting $(2 \times 2\pi \times 13.16MHz)$ between $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ under an external magnetic field $B_0 = 1.30 \times 10^{-3}\ T$ can be used to verify (irreversible) Landauer erasure ($\Delta S < 0$). Note that, in this quantum metrology, the energy (mHz) required in the reversible operation ($|\uparrow\downarrow\rangle \Leftrightarrow |\downarrow\uparrow\rangle$) is 10^{-9} times that (MHz) required in the irreversible operation ($\updownarrow\ \Rightarrow |\uparrow\rangle$). A redraw courtesy of Dr. Shlomi Kotler from the Hebrew University of Jerusalem.

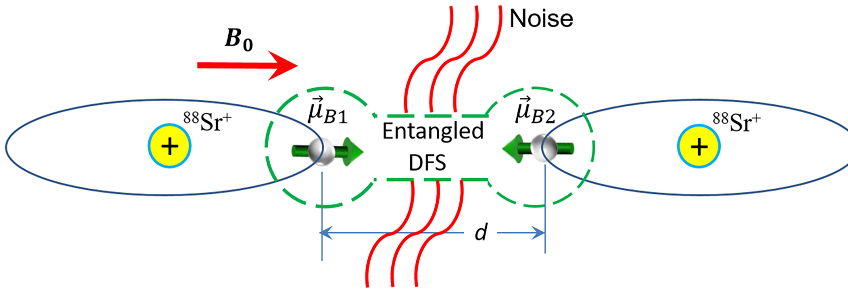

Fig. 6. Fault-tolerant quantum computing: Two entangled spin qubits create a decoherence-free Subspace (DFS) that is immune to stray magnetic field noises within the whole Hilbert space $\mathcal{H} = span(|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle)$.

whether it is in $|\uparrow\rangle$ or $|\downarrow\rangle$, corresponding to a maximum entropy $S = k_B \ln 2$. In contrast, the spin–spin interaction ξ gives rise to reversible oscillations ($|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$, $\Delta S = 0$), which are ultimately governed by the Heisenberg limit [1].

The external magnetic field ($B_0 = 1.30 \times 10^{-3} \pm 1.0 \times 10^{-7}$) T not only sets the spin quantization axis, but also lifts the degeneracy between the probe states by $f_0 = 13.16\ MHz$ [21]. As also shown in Figure 5, the coherence between $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ is dependent on the magnetic field B_0. Obviously, the energy splitting between $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ is twice the energy splitting between $|\uparrow\rangle$ and $|\downarrow\rangle$, i.e., $\Delta E_{\uparrow\uparrow\Rightarrow\downarrow\downarrow} = E_{\uparrow\uparrow} - E_{\downarrow\downarrow} = 2\Delta E_{\uparrow\Rightarrow\downarrow} = 2(E_\uparrow - E_\downarrow)$.

As shown in Figure 5, the irreversibility of the decoherence ($\updownarrow \Rightarrow |\uparrow\rangle$) (equivalent to Landauer erasure) is automatically ensured by the decreased entropy ($\Delta S < 0$) in a manner similar to the single-atom demonstration [15] in Figure 4. The corresponding energy is a linear Zeeman shift as follows:

$$\Delta E^{irr}_{\updownarrow\Rightarrow\uparrow} = \mu_B B_0 = 9.274 \times 10^{-24}\ {}^{J}\!/_{T} \times 1.3 \times 10^{-3}\ T = 1.21 \times 10^{-26}\ J. \tag{2}$$

Spin rotations were performed by pulsing a (resonant) oscillating magnetic field that is perpendicular to the mentioned quantization axis z. The above energy can also be estimated by the aforementioned resonant frequency $f_0 = 13.16\ MHz$ [21]:

$$\Delta E^{irr}_{\updownarrow\Rightarrow\uparrow} = hf_0 = 6.626 \times 10^{-34} J \cdot s \times 13.16\ MHz = 8.72 \times 10^{-27}\ J. \tag{3}$$

The above double-checked energy scale of $\Delta E^{irr}_{\updownarrow\Rightarrow\uparrow}$ is also in reasonable agreement with the single-spin demonstration of the Landauer bound [19]. The relaxation time is extremely long (measured $T^{d=2.4\mu m}_{Bell} = 67\ s$) [21]. Therefore, to reset this spin, we need to use an auxiliary energy level and optical pumping. Such a design indeed results in an irreversible process [21].

Note that the motivation of the cited experiment [21] is completely different from what we have here: spin–spin magnetic interactions are found to obey the inverse-cube law. Such an interaction was fully used in our study: a spin ($\vec{\mu}_B$) generates a magnetic field at the location ($\vec{r}$) of another spin (Figure 6), which is

$$\vec{B} = \frac{\mu_0}{4\pi r^3}\left[\frac{3\,(\vec{\mu}_B \cdot \vec{r})\,\vec{r}}{r^2} - \vec{\mu}_B\right]. \tag{4}$$

The magnetic interaction between the two spins (aligned linearly) across the separation $d = 2.4\ \mu m$ is

$$B_{spin-spin} = \frac{\mu_0}{4\pi}\frac{2\mu_B}{d^3} = \frac{4\pi \times 10^{-7} T \cdot m/A}{4\pi}\frac{2 \times 9.2740 \times 10^{-24}\ J/T}{(2.4\ \mu m)^3} = 1.34 \times 10^{-13}\ T. \tag{5}$$

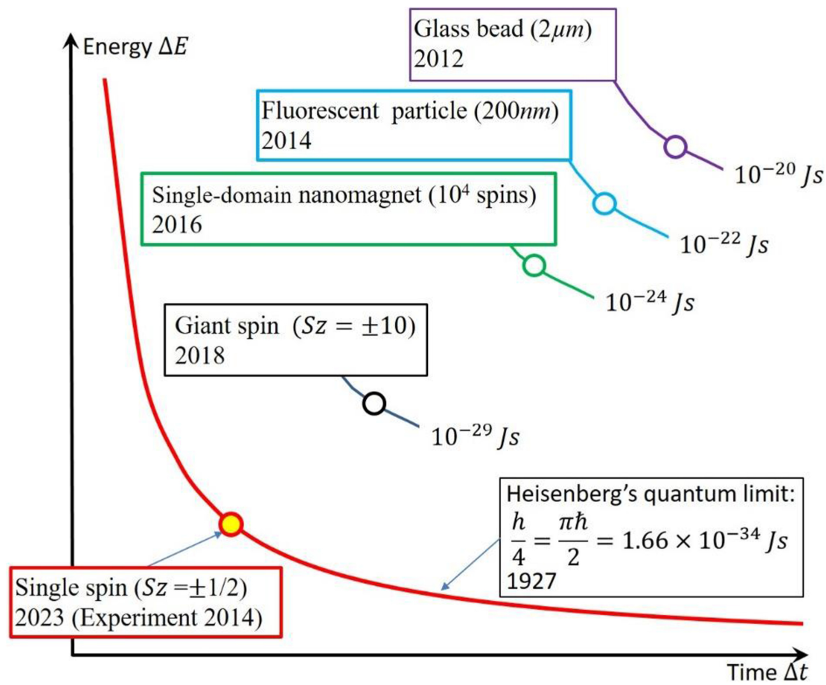

Fig. 7. Various information carriers in several experiments [12-21]. In this study, a single spin qubit serves as the smallest information carrier in quantum computing, with an energy-time cost that closely approaches the Heisenberg quantum limit, $\frac{h}{4} = \frac{\pi\hbar}{2} = 1.66 \times 10^{-34}$ Js.

where the vacuum permeability is $\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$, and the Bohr magneton is $\mu_B = 9.2740 \times 10^{-24}\ J/T$.

Accordingly, the energy of reversibly flipping a single spin is

$$\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow} = \mu_B B_{spin-spin} = 9.2740 \times 10^{-24} \frac{J}{T} \times 1.34 \times 10^{-13}\ T = 1.24 \times 10^{-36}\ J. \quad (6)$$

Taking the energy spread ($1.24 \times 10^{-36}\ J$) in Equation (6) that corresponds to $d = 2.4\ \mu m$ and the measured evolution time $\Delta t = 67s \times 2 = 134\ s$ at the same separation ($d = 2.4\ \mu m$) [21], the action of manipulating a single spin qubit reversibly ($|\uparrow\rangle \Leftrightarrow |\downarrow\rangle$) at $d = 2.4\ \mu m$ can be estimated as

$$\Delta A = \Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow} \Delta t = 1.24 \times 10^{-36}\ J \times 134s = 1.66 \times 10^{-34}\ Js, \quad (7)$$

which is the same as the Heisenberg quantum limit of $\frac{h}{4} = \frac{\pi\hbar}{2} \approx 1.66 \times 10^{-34} J \cdot s$ [7, 8] among various information carriers (so far the smallest action energy-time cost is $10^{-29} J \cdot s$ achieved on a giant spin containing 20 spins) [17]), as shown in Figure 7.

In addition to the above calculation, this energy time cost can also be double-checked by the measured energy splitting [$4\xi = 2\pi(2 \sim 5)\ mHz$] in the spin-spin magnetic interaction experiment [21]. Bearing in mind that this measured energy gap of $(2 \sim 5)\ mHz$ [21] between two entangled Bell states $|\psi\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$ is equivalent to the energy of flipping two spins, we have

$$\Delta E^{rev}_{\uparrow\downarrow\Leftrightarrow\downarrow\uparrow} = 2\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow} = h\frac{4\xi}{2\pi} = 6.63 \times 10^{-34} J \cdot s \times (2 \sim 5)\ mHz = (1.33 \sim 3.31) \times 10^{-36}\ J. \quad (8)$$

Table 1. Comparison of the Three Experiments. Both the Heisenberg Limit and the Landauer Bound Can Be Approached Closely via a Single Spin Qubit in Quantum Computing

Info carrier	Single spin [19, 21]	10 spins [17]	10^4 spins [16]
The moment size	$1\mu_B$	$20\mu_B$	$2 \times 10^4\ \mu_B$
Energy $\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow}$ (reversible)	$1.24\ \times 10^{-36}\ J$ at $d = 2.4\ \mu m$		
Time Δt	134 s at $d = 2.4\ \mu m$	$> 100\ s$ if $B_y = 0$ [17].	Seconds [16]
$\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow}\Delta t$	$1.66\ \times 10^{-34}\ J \cdot s$	$\sim 10^{-29}\ J \cdot s$ [17]	$\sim 10^{-24}\ J \cdot s$ [16]
Heisenberg's quantum limit $\frac{h}{4} = \frac{\pi\hbar}{2}$	$1.66\ \times\ 10^{-34} J \cdot s$	$1.66\ \times\ 10^{-34} J \cdot s$	$1.66\ \times\ 10^{-34} J \cdot s$
Approaching factor (Heisenberg)	1	10^5	10^{10}
Energy $\Delta E^{irr}_{\updownarrow\Rightarrow\uparrow}$ (irreversible)	$1.21\ \times\ 10^{-26} J^{19}$	$(1.1 \pm 0.3) \times 10^{-23} J$ [17]	$4.2\ \times 10^{-21} J$ [16]
Landauer's bound at T	$9.60\ \times\ 10^{-27} J$ at $T = 1\ mK$	$9.60\ \times\ 10^{-24} J$ at $T = 1\ K$	$2.90 \times 10^{-21} J$ at $T = 300\ K$
Approaching factor (Landauer)	1.25	1.15	1.45

The energy splitting $[4\xi = 2\pi(2 \sim 5)mHz]$ and coherence time $T^{d=2.4\mu m}_{Bell} = 67\ s$ (at $d = 2.4\ \mu m$) mentioned in the Figure 5 caption were actually measured via a parity observable that represents the coherence between $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$. To estimate this coherence, the system state was initialized to $|\uparrow\downarrow\rangle$ and then evolved under spin–spin interaction to $|\psi(T)\rangle = \cos(2\xi T)|\uparrow\downarrow\rangle + i\sin(2\xi T)|\downarrow\uparrow\rangle$. In this parity analysis [21], the angle $\theta = 21.6^o$ (for the fixed actual experimental duration $T_{exp} = 15\ s$) is estimated by parity visibility, $\sin(4\xi T)$ (a function of the spin-spin magnetic interaction strength, ξ, and the experimental time, T). This parity visibility is the projection (in blue) of the Bloch vector on the equatorial plane in Figure 5. Actually, the above angle, $\theta = 21.6^o = \frac{21.6^o\pi}{180^o} = 0.38\ rad$ (at $d = 2.4\ \mu m$), reasonably falls within the range specified by

$$4\xi T_{exp} = 2\pi\,(2 \sim 5)\,mHz\ \times\ 15\ s = (0.19 \sim 0.47)\ rad, \tag{9}$$

which corresponds to the separation range $d = (2.18 \sim 2.76)\ \mu m$. In Figure 5, $\theta = 4\xi T_{exp}$, which means that this is a rotation with a constant (angular) speed 4ξ.

From the above $T^{d=2.4\mu m}_{Bell} = 67\ s$ experiment, we can estimate the time $\Delta t = 2T^{d=2.76\mu m}_{Bell}$ for the two entangled spins to evolve from $|\uparrow\downarrow\rangle$ to $|\downarrow\uparrow\rangle$ at $d = 2.76\ \mu m$ [note that $T_{exp} = 15\ s$ is fixed across $d = (2.18 \sim 2.76)\ \mu m$], which corresponds to the smallest energy $\Delta E^{rev}_{\uparrow\downarrow\Leftrightarrow\downarrow\uparrow}$ required for this evolution, as follows:

$$\Delta t = 2T^{d=2.76\mu m}_{Bell} = 2\ \times\ 67\ s\ \times\ \frac{0.38\ rad}{0.19\ rad} = 268\ s. \tag{10}$$

Taking the lowest energy ($\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow} = \frac{1}{2}\Delta E^{rev}_{\uparrow\downarrow\Leftrightarrow\downarrow\uparrow} = 6.65\ \times 10^{-37}\ J$) in Equation (8) that corresponds to $d = 2.76\ \mu m$ and the time $\Delta t = 268\ s$ in Equation (10) for the two entangled spins to evolve from $|\uparrow\downarrow\rangle$ to $|\downarrow\uparrow\rangle$ at $d = 2.76\ \mu m$, the energy-time cost of manipulating a single spin qubit reversibly ($|\uparrow\rangle \Leftrightarrow |\downarrow\rangle$) at $d = 2.76\ \mu m$ can be estimated as

$$\Delta A = \Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow}\Delta t = 6.65\ \times 10^{-37}\ J \times\ 268s = 1.78\ \times 10^{-34}\ Js, \tag{11}$$

which is at the same order of magnitude as the calculated action in Equation (7). Thus, the minimal action (energy times time) of manipulating a single spin qubit is double-checked.

In theory, the energy time cost does not changes across $d = (2.18 \sim 2.76)\ \mu m$, i.e., $\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow}\Delta t = $ constant, since $\Delta E^{rev}_{\uparrow\Leftrightarrow\downarrow} \propto \xi$ [see Equation (8)] and $\Delta t \propto 1/\xi$ [see Equations (9) and (10)].

Table 1 summarizes three typical experiments that encode the orientation as information: the single spin experiment [21], the 10 spin experiment [17] and the 10^4 spin experiment [16]. The

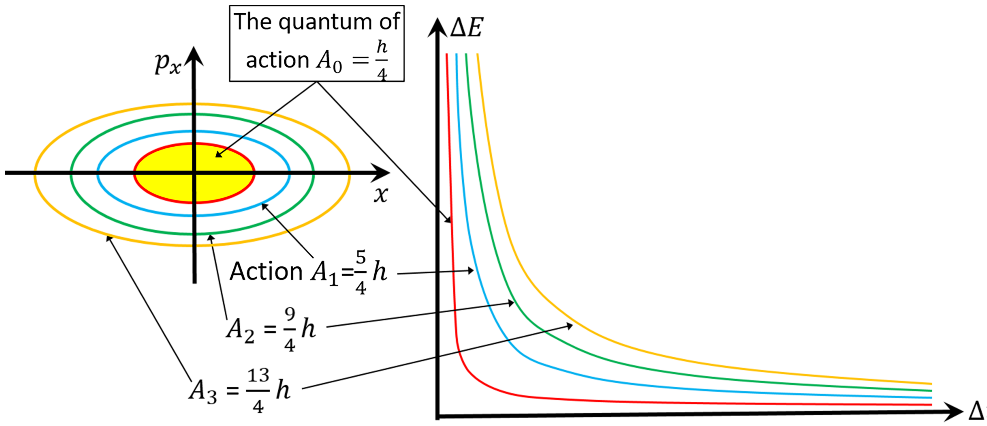

Fig. 8. Quantization of the action of a qubit. Its phase trajectory 'at rest' is not a point at origin, but an ellipse (in red/yellow) with area $A_0 = \frac{h}{4}$, which is the quantum of action and adheres to Heisenberg's quantum limit.

approaching factor is defined as

$$\text{The approaching factor} = \frac{\text{The experimentally} - \text{verified energy} - \text{time cost } (J \cdot S) \text{ or energy } (J)}{\text{Heisenberg limit or Landauer bound at the corresponding temperature}}, \quad (12)$$

which is always larger than one unless one can experimentally break Heisenberg's limit or Landauer's bound. Our aim is to generally quantify the energy required to invert spins reversibly (ultimately governed by the Heisenberg limit [1]) and irreversibly (equivalent to Landauer erasure [2]).

5 Qubit and Quantized Action

Planck introduced the now famous Planck constant $h = 6.626 \times 10^{-34} J \cdot s$ [3] and predicted: A harmonic oscillator with frequency f_0 can change its energy only by an amount $\Delta E = hf_0$ (the so-called energy quantum) [22]. That is,

$$E = (n + \alpha)\, hf_0, \quad (13)$$

where $n = 0, 1, 2, \ldots$, and α can be any number between zero and one (here, we take $\alpha = \frac{1}{4}$).

The corresponding action can then be written as

$$A_n = (n + \alpha)\, hf_0 \cdot \frac{1}{f_0} = (n + \alpha)\, h. \quad (14)$$

It means that a quantum oscillator with frequency f_0 cannot follow an arbitrary phase trajectory. Only the ellipses with strictly defined areas are allowed (Figure 8). This is a remarkable fact. The oscillator energy quantization was not universal: different oscillators would have different frequencies and therefore different energy quanta. The area enclosed by the phase trajectory of any quantum oscillator, no matter its frequency, is always given by a multiple of the quantum of action (the smallest change of action one can ever get) [9, 22]:

Adhering to Heisenberg's quantum limit, a qubit has the minimal energy (so-called zero-point energy) $E_0 = \frac{1}{4}\hbar\omega_0 = \frac{1}{4}hf_0$ and the quantum of action $A_0 = \frac{1}{4}h$. The phase trajectory for a qubit 'at rest' is therefore not a point at origin, but an ellipse with area $\frac{h}{4}$, which is the smallest area that can be enclosed by its phase trajectory [9]. This is one manifestation of a quantum property of Nature: the product of uncertainties of two conjugate variables cannot be less than (quarter of) the Planck constant. There are lots of pairs of conjugate variables in physics, but the energy and time are the most relevant ones for our current purpose.

6 Spin-Qubit Quantum Computers

The single spin qubit Schrödinger equation [23] is

$$i\hbar \frac{d}{dt}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}\varepsilon & \delta \\ \delta & -\varepsilon\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}-\frac{1}{2}\varepsilon\alpha - \frac{1}{2}\delta\beta \\ -\frac{1}{2}\delta\alpha + \frac{1}{2}\varepsilon\beta\end{pmatrix}, \tag{15}$$

where the two complex numbers, α and β, satisfy $|\alpha|^2 + |\beta|^2 = 1$ ($|\alpha|^2$ and $|\beta|^2$ are, respectively, the probabilities of finding the spin qubit in $|0\rangle$ and $|1\rangle$), ε is the bias caused by B_0, and δ is the tunnelling splitting with respect to the ground state [23].

In the form of $|\Psi_E\rangle = (\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}) exp(-\frac{iEt}{\hbar})$ where E is a (definite) energy [22], an energy eigenstate satisfies Equation (15). Thus,

$$-\frac{1}{2}\begin{pmatrix}\varepsilon & \delta \\ \delta & -\varepsilon\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = E\begin{pmatrix}\alpha \\ \beta\end{pmatrix}, \tag{16}$$

From the energy eigenstate $(\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}) exp(-\frac{iEt}{\hbar})$, we found that it is the energy-time product that determines the behavior of a spin qubit, that is, setting the highest speed at which a spin can change its energy with a given amount. The time for a bit to flip from $|\uparrow\rangle$ to $|\downarrow\rangle$ is given by

$$\Delta t = \frac{\pi\hbar}{2E^{rev}_{\uparrow\Leftrightarrow\downarrow}}, \tag{17}$$

where $E^{rev}_{\uparrow\Leftrightarrow\downarrow} = \mu_B B_{spin-spin}$ [Equation (6)] or $E^{rev}_{\uparrow\Leftrightarrow\downarrow} = \frac{1}{2}h\frac{4\xi}{2\pi}$ [Equation (8)].

The aforementioned static magnetic field B_0 and resonant oscillating magnetic field can be combined into a rotating magnetic field B with a controlled frequency of ω:

$$B = B_0\hat{z} + B_1(\cos\omega t\hat{x} - \sin\omega t\hat{y}). \tag{18}$$

Now, we let the qubit be in $|\uparrow\rangle$ at time $t = 0$. Then, at time t, the probability of it being found in $|\downarrow\rangle$ is given by

$$P_{\uparrow\Rightarrow\downarrow}(t) = \left(\frac{\omega_1}{\Omega_{Rabi}}\right)^2 sin^2\left(\frac{\Omega_{Rabi}t}{2}\right), \tag{19}$$

where the Rabi frequency is $\Omega_{Rabi} = \sqrt{(\omega-\omega_0)^2 + \omega_1^2}$ with $\omega_0 = \gamma B_0$ and $\omega_1 = \gamma B_1$. The energy difference between $|\uparrow\rangle$ and $|\downarrow\rangle$ is $\hbar\omega_0$. $\gamma = \frac{B_0}{\omega_0}$ is the electron gyromagnetic ratio. In this (reversible) Rabi flopping [24] (Figure 3), the qubit oscillates between the $|\uparrow\rangle$ and $|\downarrow\rangle$ states, which is ultimately governed by Heisenberg's limit. The maximal amplitude for oscillation is achieved at $\omega = \omega_0 = 2\pi f_0$ (the condition for resonance). At resonance, the transition probability is given by $P_{\uparrow\Rightarrow\downarrow}(t) = sin^2(\frac{\omega_1 t}{2})$. To flip from $|\uparrow\rangle$ to $|\downarrow\rangle$, the time t needs to be $t = \frac{\pi}{\omega_1}$ such that B acts as a π pulse. If $t = \frac{\pi}{2\omega_1}$, we have a $\frac{\pi}{2}$ pulse, its effect is: $|\uparrow\rangle \Rightarrow \frac{|\uparrow\rangle + i|\downarrow\rangle}{\sqrt{2}}$ and is a well-used operation in quantum computing [5, 6].

Owing to their unique properties described above, spin-qubit quantum computers present several advantages for computation-intensive applications such as AI and Post-Quantum Cryptography (PQC) [5, 6]. Specifically, in the realm of PQC (where cryptographic algorithms are designed to be secure against an attack by a quantum computer), the following benefits are noteworthy:

(1) High Energy Efficiency: Spin qubits can significantly reduce the energy and time required to implement complex quantum cryptographic protocols, making them more efficient.
(2) Small Footprint/High-Density Integration: With a size comparable to just an atom or ion for each spin, spin qubits enable high-density integration on chips. This scalability is vital for constructing quantum computers that must have millions of qubits to run algorithms like Shor's algorithm to factorize large integers, such as a 2048-bit RSA key [5, 6].
(3) Long Coherence Times: Coherence times of tens of seconds [21] are essential for performing intricate quantum operations and ensuring the reliability of cryptographic protocols.
(4) High-Fidelity Operations: Demonstrated gate operations with fidelities as high as 98% [21] are crucial for minimizing errors in quantum computations and enhancing the robustness of cryptographic algorithms.
(5) Fault-Tolerance: Spin qubits can be utilized in quantum key distribution protocols, which facilitate the secure sharing of cryptographic keys. The principles of quantum mechanics ensure that any eavesdropping attempts can be detected, thereby enhancing security [5, 6].

7 Conclusions and Discussions

In this article, we attempt to answer a number of basic questions in quantum computing: what are the ultimate physical limits to quantum computing? How closely can we approach them in quantum computing? Can a single spin be more energy-efficiently manipulated than a collective giant spin in quantum computing and why?

Our theoretical and experimental verification shows that a single spin is so physically ultimate that both the Heisenberg limit [1] and the Landauer bound [2] can be approached. It is noteworthy that the energy-time cost of (reversibly) manipulating a single spin qubit is $1.66 \times 10^{-34} J \cdot s$, which is, among the various spin information carriers, the closest to the Heisenberg quantum limit of $\frac{h}{4} = \frac{\pi\hbar}{2} \approx 1.66 \times 10^{-34} J \cdot s$ [3, 7-10] [13-19]. So far, the minimum energy-time cost is $10^{-29} J \cdot s$ obtained on a giant spin ($S_z = 10$) [17], which means that we have improved the approach by a factor of 10^5.

As an inherent aspect of anything, the Heisenberg limit manifests how energy-efficiently information can be processed given a certain amount of time or how quickly information can be processed using a certain amount of energy in quantum computing. In short, Heisenberg's limit is still absolute, and no experiment has ever approached it using an object of this size ($1\ \mu_B$). Our approaching factor is nearly one. Our established minimal action ($1.66 \times 10^{-34}\ J \cdot s$) for manipulating a spin qubit is the same as the theoretical Heisenberg quantum limit.

Quantum computing relies on both reversible and irreversible operations, each serving distinct but vital roles. Our prior publication [19] demonstrated that the Landauer bound can be closely approached using a single spin qubit in irreversible quantum computing. In contrast, this study reveals that the Heisenberg limit can also be closely approached using a single spin qubit in reversible quantum computing. Furthermore, this study may be more fundamental and general than our previous article [19]. This is because Landauer's principle, which underpins the Landauer bound, can be derived from the second law of thermodynamics [12]. In comparison, Heisenberg's limit, which pertains to the precision of quantum measurements, is a more fundamental and general concept in quantum computing, in the sense that the Heisenberg limit [1, 8] limits both reversible and irreversible operations, while the Landauer bound [2] limits irreversible operations only.

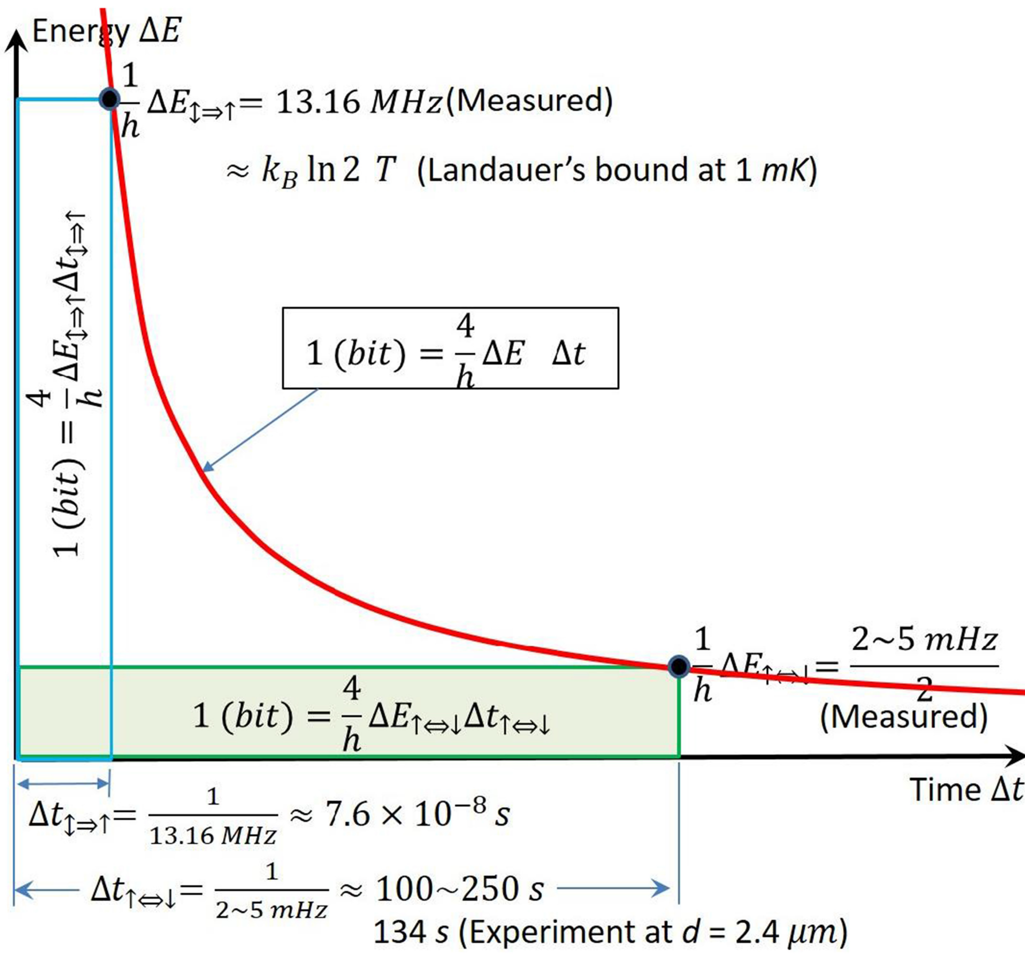

Fig. 9. From the action perspective, one bit of information [$1\ (bit) = \frac{4}{h}\Delta E\ \Delta t$] represents the smallest error when quantifying the action (the measured energy times the measured time). Note that $\frac{4}{h}\Delta E\Delta t$ is dimensionless, maintaining consistency with the definition of information in terms of units. In this context, we employed the two measured energy gaps ($2 \sim 5\ mHz$ and $13.16\ MHz$) from the spin-spin experiment [21] as two extreme examples for reversible and irreversible operations in quantum computing, respectively.

To demonstrate the universality of the Heisenberg limit, we can now use a new definition of information [$1\ (\mathrm{bit}) = \frac{4}{\mathrm{h}}\Delta \mathrm{E}\ \Delta \mathrm{t}$] to calculate the time needed to write a qubit at the Landauer bound (at room temperature):

$$\Delta t = \frac{h}{4\Delta E} = \frac{6.63 \times 10^{-34}\ J \cdot s}{4 \times k_B \ln 2\ T} = \frac{6.63 \times 10^{-34}\ J \cdot s}{4 \times 1.38 \times 10^{-23} \frac{J}{K} \times \ln 2 \times 300K} = 5.8 \times 10^{-14}\ s. \quad (20)$$

This calculation result is reasonably consistent with the picosecond timescale demonstrated by ultrafast reversal in magnetics and electronics [24, 25]. As a matter of fact, a modern computer uses millions of times as much energy as the Landauer bound [2].

If the write time is as short as the Planck time (5.39×10^{-44} s) that is the shortest validly measurable time length [23], the corresponding energy needed to write a qubit is

$$\Delta E = \frac{h}{4\Delta t} = \frac{6.63 \times 10^{-34}\ J \cdot s}{2 \times 5.39 \times 10^{-44}\ s} = 3.1 \times 10^{9}\ J, \tag{21}$$

which is 30 orders of magnitude larger than the Landauer bound [2].

Notably, as depicted in Figure 9, the relationship between input energy and the time required to write/erase a qubit is inversely proportional—the higher the input energy, the shorter the required time, and vice versa. This new definition of information holds particular significance in our theory, especially within the context of the action remaining constant. This constant action, vividly illustrated in the areas outlined in blue or green, theoretically allows for the energy required to irreversibly erase a spin qubit to approach $k_B T$, a notion experimentally validated. In contrast, the energy needed to reversibly flip a qubit is 10^{-6} times lower than the former. Our new information definition, grounded in Heisenberg's principle, enables us to discern the tradeoff between energy and the speed of manipulating a qubit.

In essence, our research is expected to contribute to the understanding of the fundamental limitations of quantum computing and the potential of single-spin-encoded quantum information processing as the most energy-efficient quantum computing paradigm. In addition to its significant practical importance in quantum computing, understanding the basic limits of what we can achieve with our computing machines is nothing more than understanding the limits of the world we live in and preparing for a revolution/breakthrough in the future.

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References

[1] W. Heisenberg. 1927. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Zeitschrift für Physik* 43, 3–4 (1927), 172–198.

[2] R. Landauer. 1961. Irreversibility and Heat Generation in the Computing Process. *IBM J. Res. Develop.* Issue 5 (1961), 183–191.

[3] M. Planck. 1900. Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum. *Verhandlungen der Deutschen Physikalischen Gesellschaft* Issue 2 (1900), 237–245.

[4] R. Feynman, R. Leighton, and M. Sands. 1965. Feynman Lectures on Physics. s.l.: Basic Books, Vols. I, II and III, 2010. Addison Wesley.

[5] A. Michael, I. Nielsen, and L. Chuang. 2010. *Quantum Computation and Quantum Information.* Cambridge University Press.

[6] J. Sakurai and J. Napolitano. *Modern Quantum Mechanics.* Cambridge University Press.

[7] J. Anandan and Y. Aharonov. 1990. Geometry of quantum evolution. *Physical Review Letters* 65, 14 (1990), 1697–1700.

[8] S. Lloyd. 2001. Ultimate physical limits to computation. *Nature* 406, 1 (2000), 1047–1054.

[9] L. Xu. 1961. *The Principle of Least Action and the Development of Physics.* Chengdu: Sichuang Educational Publisher.
[10] Y. Aharonov and D. Bohm. 1961. Time in the quantum theory and the uncertainty relation for the time and energy domain. *Phys. Rev.* Issue 122 (1961), 1649–1658.
[11] J. Von Neumann. 1966. *Theory of Self-Reproducing Automata Lect. 3, Univ.* Illinois Press, Urbana and London, IL, 1966.
[12] F. Cao and M. Feito. 2009. Thermodynamics of feedback controlled systems. *Physical Review E.* 79, 4 (2009), 041118.
[13] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz. 2012. Experimental verification of Landauer's principle linking information and Thermodynamics. *Nature* 483 (2012), 187–189.
[14] Y. Jun, M. Gavrilov, and J. Bechhoefer. 2014. High-precision test of Landauer's principle in a feedback trap. *Physical Review Letters* 113, 19 (2014), 190601.
[15] L. L. Yan, T. P. Xiong, K. Rehan, F. Zhou, D. F. Liang, L. Chen, J. Q. Zhang, W. L. Yang, Z. H. Ma, and M. Feng. 2018. Single-atom demonstration of the quantum Landauer principle. *Phys. Rev. Lett.* Issue 120 (2018), 210601.
[16] J. Hong, B. Lambson, S. Dhuey, and J. Bokor. 2016. Experimental test of Landauer's principle in single-bit operations on nanomagnetic memory bits. *Science Adv.* 2, 3, (2016), 1–6.
[17] R. Gaudenzi, E. Burzurí, S. Maegawa, H. S. J. van der Zant, and F. Luis. 2018. Quantum Landauer erasure with a molecular nanomagnet. *Nature Physics* 14, 6 (2018), 565–568.
[18] U. Çetiner, O. Raz, and S. Sukharev. 2020. Dissipation During the Gating Cycle of the Bacterial Mechanosensitive Ion Channel Approaches the Landauer's Limit. DOI: https://doi.org/10.1101/2020.06.26.174649
[19] F. Z. Wang. 2024. Spin-encoded quantum computer near ultimate physical limits. *Quantum Information Processing, Nature Springer* 23, Article No. 152 (2024).
[20] D. Reeb, and M. M. Wolf. 2014. An improved Landauer principle with finite-size corrections. *New J. Phys.* Issue 16 (2014), 103011.
[21] S. Kotler, N. Akerman, N. Navon, Y. Glickman, and R. Ozeri. 2014. Measurement of the magnetic interaction between two bound electrons of two separate ions. *Nature* 510, 7505 (2014), 376–380.
[22] A. Einstein and O. Stern. 1905. Einige argumente für die annahme einer molekularen Agitation beim absoluten Nullpunkt. *Annelen der Physik* 345 (1905), 551–560. https://doi.org/10.1002/andp.19133450309,1913
[23] A. Zagoskin. 2015. *Quantum Mechanics: A Complete Introduction: Teach Yourself.* John Murray Press.
[24] Y. Yang. 2017. Ultrafast magnetization reversal by picosecond electrical pulses. *Sci. Adv.* Issue 3 (2017), 1603117–1603122.
[25] C. Heide, P. D. Keathley, and M. F. Kling. 2024. Petahertz electronics. *Nature Reviews Physics* Issue 6 (2024), 648–662.

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