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Three-dimensional flame temperature reconstruction through adaptive segmentation weighted non-negative least squares and light field imaging

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Abstract: Existing flame temperature reconstruction algorithms experience significant performance degradation when subjected to radiation intensity noise interference, resulting in limited accuracy in low-temperature regions of flames with broad temperature distributions. This study proposes a novel 3-D flame temperature reconstruction algorithm by integrating adaptive segmentation weighted non-negative least squares with light field imaging (LFI). Building on the NNLS framework, the proposed algorithm introduces an adaptive strategy to improve the temperature reconstruction accuracy of low-temperature regions of flames. It also incorporates adaptive weight factors to reduce measurement errors caused by the extensive temperature range, enabling precise 3-D temperature reconstruction. To validate the algorithm, numerical simulations of a bimodal asymmetric flame were performed to evaluate its noise tolerance and compare its performance with other existing algorithms. The simulation results indicated that the proposed algorithm demonstrates strong noise resistance, achieving approximately 70% higher reconstruction accuracy than the LSQR algorithm. Experiments were carried out on bimodal flames to reconstruct the temperature under various combustion conditions. The reconstructed temperature showed good agreement with trends reported in the literature. The results of this study demonstrate the viability and robustness of the proposed algorithm for reconstructing broader temperature distributions of flames.

Keywords: Light field imaging; Flame temperature; Reconstruction algorithm; Adaptive segmentation

1. Introduction

High-temperature combustion is a fundamental process in contemporary aerospace and energy systems. In aerospace, it generates the thrust required by jet engines and rockets, whereas in energy production, devices such as gas turbines and power plant boilers utilize this process to transform fuel into electrical or mechanical energy [1-3]. In these systems, flame temperatures can fluctuate drastically, often exceeding 1000 K [4], and exhibit high-frequency oscillations. Measurement techniques must cover a wide temperature range and have a high spatial resolution to capture localised temperature variations accurately during combustion processes. Such capabilities are essential for advancing the design and optimization of combustion devices, improving combustion efficiency, and effectively managing pollutant emissions [5, 6].

Recent developments in laser, optical, and information technologies have led to the widespread adoption of non-contact optical diagnostic methods across multiple disciplines [7-9]. One prominent application is in the measurement of temperatures in flames, where techniques based on flame emission are favored because they do not disturb the system, respond quickly, and are relatively simple to integrate [10]. A standout among these methods is light field imaging (LFI) [11]. This technique uniquely captures spatial and angular information of flames in a single exposure, offering an innovative approach to measuring flame temperatures and analyse intricate flow patterns. LFI has proven effective in generating three-dimensional (3-D) images, which have been particularly useful in applications such as tracking the movement of particles in a fluid, studying the dynamics of fuel injection, and estimating depth in various environments [12-14]. In reconstructing a flame's temperature, LFI divides the flame into 3-D voxels. Each voxel's temperature is estimated by applying algorithms that solve the equations governing light propagation within the flame. These equations describe how light is emitted, absorbed, and scattered [15]. The challenge arises because soot particles within the flame both emit radiation and interact with the light in ways that are strongly influenced by the local temperature. This interdependence creates a complex, nonlinear inversion problem. To address this challenge, researchers have developed numerous hybrid algorithms to tackle the complex task of simultaneously reconstructing flame temperature and soot radiation properties. These algorithms typically fall into two categories: gradient-based methods [16-18] and intelligent optimization techniques [19, 20]. In gradient-based approaches, the process involves computing Jacobian matrices, a task that is notably computationally intensive. For example, when applying the LMBC (Levenberg-Marquardt with Boundary Constraint)-NNLS (Non-Negative Least Squares) algorithm [17] on a setup consisting of 480 voxels, the processing time exceeded 3 hours [21]. Similarly, the intelligent optimization methods use extensive stochastic search techniques, which contribute to an even greater computational burden. These heavy resource demands not only reduce overall efficiency but also limit the achievable spatial resolution in temperature measurement.

To improve the efficiency and accuracy of LFI-based temperature measurement, some researchers have incorporated prior information on soot radiation characteristics to transform the nonlinear inverse problem into a linear one. This approach enables the use of linear algorithms for 3-D flame temperature reconstruction. For instance, the least-squares QR decomposition (LSQR) algorithm is employed with the soot absorption coefficient of 0.8 m^{-1} for reconstructing the flame temperature distribution[15]. Qi et al. [22, 23] utilized the NNLS algorithm to reconstruct temperature by assuming an absorption coefficient of 10 m^{-1}. However, the dynamic composition of soot particles [24] can introduce significant errors in such simplifications [25, 26]. Shi et al. [27] proposed a line-of-sight attenuation (LOSA) approach to measure the soot absorption coefficient and used it as prior information for temperature measurement. The measurement of soot absorption coefficients and temperature is achieved by applying the Landweber algorithm through numerical simulations. Ling et al. [21] also combined the LOSA method with the LFI technique and employed the NNLS algorithm to reconstruct the 3-D temperature distribution of laminar diffusion flames.

The aforementioned methods effectively addressed the nonlinearity challenges in applying LFI techniques to flame temperature reconstruction, improving both reconstruction accuracy and efficiency. However, the reconstructed temperature fields still exhibit errors, particularly under conditions where the detection of radiation intensity is severely affected by noise, such as sensor thermal noise, electronic noise, and environmental noise [28]. The primary concern arises from the extensive dynamic range of the flame radiation captured by the light field camera (LFC). During algorithm iterations and matrix decomposition processes, larger numerical values can overshadow smaller ones, leading to error accumulation. Additionally, the solving process tends to prioritize fitting larger numerical components, resulting in lower fitting accuracy for smaller components [29]. Since smaller components contribute less to the overall residual, their associated errors may be overlooked during the solving process. Consequently, the accuracy of temperature reconstruction is particularly low in the low-temperature regions of the combustion field. To resolve this, various studies introduced weighting strategies into the inverse algorithms. By considering the quality of the measurement signals, these strategies have enhanced the flame reconstruction accuracy. For instance, Qi et al. [30] proposed a weighted non-negative least squares (WNNLS) algorithm, which assigns different weights to radiation intensity values based on the strength of the sampled signals. This approach ensures the equitable consideration of each radiation signal during reconstruction, thereby enhancing temperature reconstruction accuracy to a certain extent. However, numerical simulations validated that the weighting strategy has been limited to a temperature range of 1700 K- 2050 K, and its applicability across a broader temperature range remains to be further investigated. Therefore, there is a need to develop a more generalized weighting strategy capable of accurately reconstructing flame temperatures across a wider temperature range.

Moreover, most current tomographic reconstruction algorithms are designed for flames within specific temperature ranges, such as ethylene-air diffusion flames at the 800 K-2000 K range [31], soot flames from nanofluid fuels within the 800 K-2000 K range [32], and luminous combustion gases in the range of 1000 K-2200 K [33]. In wide-temperature-range flame temperature reconstruction tasks, the dynamic range of radiation LF image signals is even broader, leading to significantly lower reconstruction accuracy for existing algorithms compared to their performance in narrow temperature ranges. For instance, Niu et al. [34] demonstrated that, despite incorporating regularisation terms, the reconstruction accuracy within a broad temperature range (800 K- 2800 K) is significantly inferior to that within a narrow temperature range (1200 K- 2000 K). This finding underscores the substantial room for enhancement in the reconstruction accuracy of current algorithms for wide-temperature-range flame temperature reconstruction. The image thresholding technique [35] segments an image by comparing pixel values with a predefined threshold, dividing the image into multiple regions. The segmented areas can then be processed separately for different treatments and subsequent applications. This technique has broad applicability in image preprocessing, target detection, and pattern recognition [36-38]. Inspired by this, a segmentation strategy can be implemented to partition the flame field into high- and low-temperature regions. The refinement process would then be applied specifically to the low-temperature areas, effectively mitigating the influence of the high-temperature zone on the low-temperature zone.

This paper presents an adaptive segmentation weighted non-negative least squares (ASWNNLS) algorithm for flame temperature reconstruction. This algorithm addresses the challenges of severe noise interference in radiation intensity detection and low reconstruction accuracy in low-temperature regions for wide-temperature-range flames. Building upon the NNLS algorithm, the ASWNNLS algorithm incorporates an adaptive segmentation iterative solving strategy and an adaptive weighting factor to enhance reconstruction accuracy. To verify the feasibility and applicability of the algorithm, numerical simulations of bimodal flames across different temperature ranges were conducted, and its performance was compared with that of presently employed LF temperature reconstruction algorithms. Experimental studies on ethylene laminar diffusion flames were further performed to validate the algorithm's performance under various combustion conditions.

2. Methodology

2.1 LFC imaging model

Soot particles in flames emit thermal radiation that spans the entire visible spectrum [39]. This radiation can be captured using a LFC, which is composed mainly of a main lens, a microlens array (MLA) and a sensor. Compared to traditional cameras, the unique structure of the LFC enables it to simultaneously capture both the intensity and direction of the flame radiation in a single exposure [40]. The schematic diagram of flame radiation sampling using the LFC is shown in Fig. 1. The radiation from the flame passes through the main lens, is divided by the MLA, and reaches the CCD sensor, producing a LF image. In this image, each pixel corresponds not only to a specific spatial point but also carries directional information about the light. By applying ray tracing techniques [41], one can retrace the light's journey starting from the CCD pixel, through the microlens array and primary lens, and ultimately back to its origin in the flame.

The relationship between different points in the imaging system follows the principles of lens optics. The pixel point $G(x_G,\ y_G,\ z_G)$ on the image detector corresponds to a virtual image point $P(x_P,\ y_P,\ z_P)$, with both being conjugated relative to the center of the microlens $N(x_N,\ y_N,\ z_N)$. Similarly, the virtual image point P and the actual object point $O(x_O,\ y_O,\ z_O)$ in the flame are conjugate to the main lens center $S(x_S,\ y_S,\ z_S)$. The point where the radiation ray intersects the main lens plane is labeled as $Q(x_Q,\ y_Q,\ z_Q)$. By applying lens imaging principles, Eqs. (1)-(6) determine the spatial and directional characteristics of the flame radiation ray.

$$\frac{1}{L_{np}}+\frac{1}{L_{ng}}=\frac{1}{f_m} \tag{1}$$

$$\frac{z_N-z_G}{z_N-z_P}=\frac{y_N-y_G}{y_N-y_P}=\frac{L_{ng}}{L_{np}} \tag{2}$$

$$\frac{1}{L_{oq}}+\frac{1}{L_{qn}-L_{np}}=\frac{1}{f} \tag{3}$$

$$\frac{z_O-z_S}{z_P-z_S}=\frac{y_O-y_S}{y_P-y_S}=\frac{L_{oq}}{L_{qn}-L_{np}} \tag{4}$$

$$\theta=\arccos\left[\frac{z_Q-z_O}{\sqrt{(y_Q-y_O)^2+(z_Q-z_O)^2+(x_Q-x_O)^2}}\right] \tag{5}$$

$$\varphi=\begin{cases}\arctan\left(\dfrac{y_Q-y_O}{x_Q-x_O}\right), y_Q\geq y_O\\ \arctan\left(\dfrac{y_Q-y_O}{x_Q-x_O}\right)+2\pi, y_Q<y_O\end{cases} \tag{6}$$

where L_{ng} and L_{np} represent the distances of the MLA and virtual image plane from the sensor, respectively. f_m and f denote the focal length of the microlens and main lens, respectively. L_{oq} and L_{qn} indicate the distances of the flame and MLA from the main lens, respectively. θ and φ are the polar and azimuthal angles of the sampled ray, respectively.

2.2 Flame radiative transfer model

This study focuses on diffusion flames that produce minimal soot. In such cases, the absorption coefficient is often assumed to be equal to the extinction coefficient, especially for soot fractal aggregates with primary particles in the Rayleigh range [42]. To analyze these flames, the radiation intensity detected by each pixel is interpreted as the intensity of the matching light ray. This intensity is determined through the radiative transfer equations (RTE) [43]

$$\frac{dI_\lambda(s,\boldsymbol{\Psi})}{ds}=-K_\lambda^{(a)}(s)I_\lambda(s,\boldsymbol{\Psi})+K_\lambda^{(a)}(s)I_{b\lambda}(s) \tag{7}$$

where $I_\lambda(s,\boldsymbol{\Psi})$ is the radiation intensity at position s and direction $\boldsymbol{\Psi}$, $I_{b\lambda}$ is the blackbody radiative intensity, $K_\lambda^{(a)}$ is the spectral absorption coefficient at wavelength λ.

The spectral radiation intensity of the flame $I_\lambda(s,\boldsymbol{\Psi})$ along the detection path $(s,\boldsymbol{\Psi})$ is achieved by solving Eq. (7) using a discretized numerical method as follows

$$
\begin{aligned}
I_\lambda(s,\boldsymbol{\Psi}) = {} & I_{b\lambda}^{nn}\left[1-\exp\left(-r_{nn}K_{\lambda nn}^{(a)}(s)\right)\right] \\
& + \sum_{i=1}^{nn-1}\left[\exp\left(-\sum_{j=i+1}^{nn} l_j\,K_{\lambda j}^{(a)}(s)\right)-\exp\left(-\sum_{j=i}^{nn} l_j\,K_{\lambda j}^{(a)}(s)\right)\right]I_{b\lambda}^{i}
\end{aligned}
\tag{8}
$$

where nn is the total number of voxels that the flame radiation traverses along the detection path. Each voxel is assumed to maintain a uniform temperature. The indices i and j correspond to specific voxels along the detection path. l represents the segment length of the detection path within the voxel.

The total outgoing radiative intensity observed across the entire detection field is obtained by integrating Eq. (8) over all possible detection directions:

$$
\boldsymbol{I_\lambda} = \boldsymbol{A_\lambda}\cdot\boldsymbol{I_{b\lambda}} \tag{9}
$$

where $\boldsymbol{I_\lambda}$ is the flame radiative intensity vector; $\boldsymbol{I_{b\lambda}}$ is the blackbody radiative intensity vector; $\boldsymbol{A_\lambda}$ is the coefficient matrix. The dependence of the blackbody radiative intensity $\boldsymbol{I_{b\lambda}}$ on the temperature T is expressed mathematically by Planck's law:

$$
I_{b\lambda}(s) = \frac{c_1\lambda^{-5}}{\pi\left[e^{c_2/\lambda T(s)}-1\right]} \tag{10}
$$

where c_1 and c_2 are the first and second radiation constants, respectively.

2.3 Proposed algorithm

According to the NNLS method, the ASWNNLS algorithm introduces two innovations:

(1) Adaptive segmentation iterative solving strategy: The NNLS algorithm adaptively segments the flame field into high- and low-temperature regions. Corrections are applied to the low-temperature regions during each iteration.

(2) Adaptive weighting factors: The weighting factors are defined based on the residuals calculated by the deviations between the resolved results and the observed data. For vectors with larger residuals, their weights are reduced to mitigate the influence of noise or unreliable data on the solution. Conversely, for vectors with smaller residuals, their weights are increased to enhance the contribution of reliable data to the reconstruction.

The procedures of the ASWNNLS algorithm are shown in Fig. 2 and described in detail as follows:

Step 1: Discretize the flame into a computational grid along the circumferential (N_φ), radial (N_r) and axial (N_z) directions. Ray tracing [discussed in Section 2.1] is performed on the flame LF images to determine the length matrix $\boldsymbol{l}$, which represents the path lengths of rays traversing the grid cells.

Step 2: Input the collected flame radiation intensity vector $\boldsymbol{I_\lambda}$, along with prior information on the soot absorption coefficient $k_{a,\lambda}$ [44]. Based on Eq. (8), the coefficient matrix $\boldsymbol{A_\lambda}$ is then computed and established.

Step 3: Utilize the NNLS algorithm [45] to solve for the blackbody radiation intensity $\boldsymbol{I_{b\lambda\_old}}$ based on Eq. (11). Subsequently, compute the initial temperature distribution T_{old}.

$$
Minimize\ \left\|\boldsymbol{A_\lambda}\cdot\boldsymbol{I_{b\lambda\_old}}-\boldsymbol{I_\lambda}\right\|\ subject\ to\ \boldsymbol{I_{b\lambda\_old}}\geq 0 \tag{11}
$$

Step 4: Based on the T_{old} obtained in Step 3, the 3-D flame temperature field is adaptively segmented into $T_e(T_i > 0)$

and $T_z(T_i = 0)$. This segmentation arises from the NNLS algorithm's inherent characteristic of iteratively selecting variables that have the most significant impact on the objective function, while components with minimal or no contribution are set to zero. Subsequently, the radiative intensities contributed by the T_e and T_z grids, $\boldsymbol{I}_{\boldsymbol{\lambda},\boldsymbol{e}}$ and $\boldsymbol{I}_{\boldsymbol{\lambda},\boldsymbol{z}}$, are calculated using Eqs. (12) and (13), respectively.

$$\boldsymbol{I}_{\boldsymbol{\lambda},\boldsymbol{e}} = \boldsymbol{A}_{\boldsymbol{\lambda},\boldsymbol{e}} \cdot \boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{e}} \tag{12}$$

$$\boldsymbol{I}_{\boldsymbol{\lambda},\boldsymbol{z}} = \boldsymbol{I}_{\boldsymbol{\lambda}} - \boldsymbol{A}_{\boldsymbol{\lambda},\boldsymbol{e}} \cdot \boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{e}} \tag{13}$$

where $\boldsymbol{A}_{\boldsymbol{\lambda},\boldsymbol{e}}$ represents the coefficient matrix for rays passing through the T_e grid and $\boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{e}}$ denotes the blackbody radiation intensity corresponding to the temperature T_e.

Step 5: Use the NNLS algorithm to solve for the blackbody radiation intensity $\boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{z}}$ corresponding to the temperature T_z based on Eq. (14). Update T_{old} and substitute it into Step 4. Continue the calculations from Step 4 to Step 5.

$$Minimize \left\|\boldsymbol{A}_{\boldsymbol{\lambda},\boldsymbol{z}} \cdot \boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{z}} - (\boldsymbol{I}_{\boldsymbol{\lambda}} - \boldsymbol{A}_{\boldsymbol{\lambda},\boldsymbol{e}} \cdot \boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{e}})\right\| \; subject\ to\ \boldsymbol{I}_{\boldsymbol{b\lambda},\boldsymbol{z}} \geq 0 \tag{14}$$

where $\boldsymbol{A}_{\boldsymbol{\lambda},\boldsymbol{z}}$ represents the coefficient matrix for rays passing through the T_z grid.

Step 6: The iteration is terminated when T_{old} it consists entirely of non-zero values, and the updated temperature distribution is then used for subsequent calculations. The purpose of this process is to correct the errors in the low-temperature regions of the wide-temperature-range 3-D temperature field.

Step 7: Calculate the residual $\boldsymbol{r}$ based on the blackbody radiation intensity $\boldsymbol{I}_{\boldsymbol{b\lambda\_old}}$ corresponding to the initial solution T_{old}.

$$\boldsymbol{r} = \boldsymbol{I}_{\boldsymbol{\lambda}} - \boldsymbol{A}_{\boldsymbol{\lambda}} \cdot \boldsymbol{I}_{\boldsymbol{b\lambda\_old}} \tag{15}$$

Step 8: Based on the residual $\boldsymbol{r}$, construct the diagonal weighting matrix $\boldsymbol{W}$ such that components with smaller residuals are assigned higher weights in the next iteration of the solution process.

$$\omega_i = \frac{1}{|r_i| + \epsilon} \tag{16}$$

$$\boldsymbol{W} = \mathrm{diag}(\omega_i) \tag{17}$$

where r_i represents the ith component of the residual $\boldsymbol{r}$, ω_i is the adaptive weighting factor, and the small constant ϵ is introduced to prevent division by zero.

Step 9: Use the WNNLS algorithm to solve for the new blackbody radiation intensity $\boldsymbol{I}_{\boldsymbol{b\lambda\_new}}$ based on Eq. (18), and then compute the corresponding temperature T_{new}.

$$Minimize \left\|\boldsymbol{W} \cdot \boldsymbol{A}_{\boldsymbol{\lambda}} \cdot \boldsymbol{I}_{\boldsymbol{b\lambda\_new}} - \boldsymbol{W} \cdot \boldsymbol{I}_{\boldsymbol{\lambda}}\right\| \; subject\ to\ \boldsymbol{I}_{\boldsymbol{b\lambda\_new}} \geq 0 \tag{18}$$

Step 10: Compare the new solution T_{new} with the previous solution T_{old}. If the maximum change between the two is smaller than a pre-set convergence threshold or the maximum number of iterations is reached, stop the iteration process. Otherwise, update $T_{old} = T_{new}$ and return to Step 4.

Step 11: Compute and output the final 3-D temperature field T_{new}.

3. Numerical validation

The proposed algorithm's performance is evaluated through numerical simulations. The simulation focuses on a bimodal asymmetric ethylene diffusion flame, with a radius (R) of 8 mm and a height (Z) of 25 mm. The flame is divided into $N_z \times N_r \times N_\varphi = 6 \times 10 \times 12$ voxels. The LFC parameters used in the simulations are summarised in Table 1. Eq. (19) [46] defines the flame temperature and soot absorption coefficient distributions. Temperature and absorption coefficient are uniformly distributed at the voxel centre within each voxel.

$$f(x,y,z) = \frac{a}{3}\left(\begin{array}{c} \exp\left\{ \begin{array}{l} -40[(750x+7.5)/9-1.1]^2 \\ -25[(750y+8.5)/9-0.8]^2 \end{array} \right\} \\ +0.8\exp\left\{ \begin{array}{l} -25[(750x+7.5)/9-0.8]^2 \\ -35[(750y+8.5)/9-1.2]^2 \end{array} \right\} \end{array} \right) + b\left(1-\frac{100z}{3}\right) + c \tag{19}$$

where x, y, z represent the 3-D spatial coordinates of the cylindrical flame. a, b, c are control parameters that define the overall range and spatial gradient of the flame parameters, with temperature and absorption coefficient units set to K and m^{-1}, respectively. Specifically, the parameter *a* controls the amplitude of temperature variations in the *x*-*y* plane. The coefficient *b* determines the vertical temperature gradient by setting the rate at which the flame warms or cools with increasing *z*. Finally, *c* establishes the uniform baseline temperature, shifting the entire field upward or downward.

Two flame models with different temperature ranges, wide and narrow, are defined to evaluate the algorithm's applicability for reconstructing flame temperature. For the flames with a narrow temperature range, the absorption coefficient control parameters are (a, b, c) = (30, 6, 6), and the temperature control parameters are (a, b, c) = (2400, 480, 1012). For the wide-temperature-range flame, the absorption coefficient control parameters are (a, b, c) = (40, 16, 2), and the temperature control parameters are (a, b, c) = (3100,1240,712). The original temperature distribution of the simulated flames is shown in Fig. 3.

The effects of varying noise levels and flame temperature ranges were analyzed and discussed. The relative error of the temperature field reconstruction (ΔT) and the mean relative error (ΔT_{mean}) were used as metrics to assess reconstruction quality. These metrics are defined as,

$$\Delta T_i = \frac{\left|T_{rec,i} - T_{ori,i}\right|}{T_{ori,i}} \tag{20}$$

$$\Delta T_{mean} = \frac{1}{N}\sum_{i=1}^{N} \Delta T_i \tag{21}$$

where rec, i and ori, i represent the reconstructed and original values of the ith flame voxel, respectively; N is the total number of voxels into which the flame is divided. The reconstruction results reported in this study are derived from the average values obtained over ten simulation runs.

3.1 Effects of noises

Noise resistance analysis is crucial for evaluating an algorithm's robustness. Two flame models, one with a broader temperature range and the other with a narrow temperature range, were considered to investigate the noise resistance of the proposed algorithm under varying flame radiation signal noise (γ). Noise levels were introduced using a method:

$$I_{mea} = (1 + \sigma\xi) I_{exa} \tag{22}$$

$$\sigma = \frac{I_{exa} \times \gamma}{2.576} \tag{23}$$

where, I_{mea} represents the measured flame radiative intensity, I_{exa} is the true flame radiative signal without noise, σ is the standard deviation of the noise, and ξ is a random variable following a standard normal distribution.

The temperature reconstruction accuracy of the proposed algorithm was compared under three different noise levels: γ =0 %, γ =1 % and γ =3 %. The reconstructed temperature distribution and corresponding relative error for the narrow-temperature-range flame under different noise levels are shown in Fig. 4. The reconstruction error increases with noise levels because the proposed algorithm's core is based on the NNLS method, a form of maximum likelihood estimation. Consequently, the solution tends to shift toward the noise components, leading to a decrease in accuracy. However, the reconstructed temperature remains in good agreement with the original distribution. Even at γ =3 %, the mean relative error is only 3.75 %, demonstrating the excellent noise resistance of the proposed algorithm.

Fig. 5 illustrates the reconstructed temperature distribution and relative error of the wide-temperature-range flame under various noise levels. Similar to the narrow-temperature-range flame, both the average and maximum relative errors tend to increase with higher noise levels. Notably, the increase is more significant for the wide-temperature-range flame. This can be attributed to its broader dynamic signal range, which encompasses both strong and weak signal regions. In weak signal regions, noise constitutes a relatively higher proportion, sometimes approaching or even exceeding the signal magnitude. Such noise may be overlooked or insufficiently fitted during the optimization process, leading to localized deviations in the solution (e.g., low-temperature voxels in the range of indices 480-720). Nonetheless, even at γ =3 %, the mean relative error of temperature reconstruction is only 5.46 %, further demonstrating the excellent noise resistance of the proposed algorithm and its robustness in reconstructing temperatures for wide-temperature-range flames.

3.2 Performance analysis

To validate the performance of the proposed algorithm, comparative studies were conducted, including two cases.

Case 1: Temperature field reconstruction using the LSQR algorithm [47].

Case 2: Temperature field reconstruction using the proposed algorithm but excluding steps 7-10 in Section 2.3.

The reconstructed temperature distributions for narrow- and wide-temperature-range flames are shown in Figs. 6 and 7, respectively. Table 2 summarises the reconstruction performance for different cases. The reconstruction times were calculated under identical conditions, including temperature and absorption coefficient distributions, voxel divisions, and noise level (γ = 1%). All computations were performed on the same platform, operating system, configuration, and programming libraries.

The proposed algorithm achieves the smallest reconstruction errors for both narrow- and wide-temperature-range flame models, with particularly accurate temperature distribution reconstruction at the flame's top region. For the narrow temperature-range model, the ΔT_{mean} is reduced by 77 % and 71 % compared to Case 1 and Case 2, respectively. For the wide-temperature-range model, the ΔT_{mean} is decreased by 66 % and 63 %, respectively.

In Case 1, the higher reconstruction error arises because the LSQR algorithm imposes weak physical constraints on the solution and lacks targeted handling of variations in the dynamic range of the measured signals. This is particularly evident in low-temperature regions with high noise levels, where the reconstruction accuracy is poor [34]. In contrast, the proposed algorithm balances the contributions of noise and signal through a weighting matrix and enhances the physical plausibility of the solution using non-negativity constraints, significantly improving the reconstruction accuracy of temperature fields with a wide dynamic range. In Case 2, the initial solution obtained using the NNLS algorithm is refined through threshold segmentation and iterative correction, which slightly improves reconstruction accuracy compared to Case 1. However, this method still suffers from the suppression of weak signals by strong signals, especially during the reconstruction of wide-temperature-range flames. The low-temperature signal components are often overlooked, leading to higher reconstruction errors. The proposed algorithm addresses this issue by introducing an adaptive weighting factor defined by the residuals between the initial and refined solutions. This dynamically adjusts the contributions of different data components, adapting to variations in data distribution and noise characteristics, and thereby improves the reconstruction accuracy in low-temperature regions. These results demonstrate that the proposed algorithm effectively reconstructs wide-temperature-range flames.

A comparison of the computational efficiency of the three cases reveals that the proposed algorithm is slower, with computation time approximately 25 times longer than the LSQR algorithm. This is due to the additional computational complexity introduced by the weight matrix adjustment and the non-negative constraint in the proposed algorithm's solution process. Further comparison of the proposed algorithm’s performance in different temperature-range flame models, based on the maximum value of the temperature change between two successive iterations $\max|T_{new} - T_{old}|$, is shown in Fig. 8. For the narrow-temperature-range flame, the temperature reconstruction requires 13 iterations, with a reconstruction time of approximately 298 s. For the wide-temperature-range flame, the reconstruction requires 18 iterations, with a reconstruction time of approximately 327 s. This increased iteration count and reconstruction time for the wide-temperature-range flame is due to the additional time needed to process noise, dynamic range differences, and the bias introduced by non-negativity constraints. For the narrow-temperature-range flame, the coefficient matrix $\boldsymbol{A_\lambda}$ has a lower condition number, and the gradient direction is more distinct, leading to fewer iterations. Although the proposed algorithm requires more computational time than existing methods, it demonstrates higher accuracy and robustness in temperature field monitoring during combustion processes, highlighting its significant practical application value.

4. Experiments on bimodal flames

Experiments were carried out to verify the proposed algorithm by reconstructing the 3-D temperature field of bimodal ethylene laminar flames. Key parameters of the setup are as follows: The primary lens has a focal length of 50 mm and is equipped with a band-pass filter centred at a wavelength of 660 nm. The MLA has dimensions of 100 × 100 µm and a f-number of 4.2. The optical sensor (JHUM 204B) has a resolution of 920 (H) × 1200 (V) pixels and a pixel size of 5.86 µm.

A replaceable dual-nozzle plate was installed at the burner's centre to generate a bimodal ethylene laminar diffusion flame. The burner has two 8 mm fuel injection nozzles and a 50 mm concentric annular tube for co-flow air. Constant fuel and airflow rates were maintained by two mass flow controllers. Table 3 summarises the operating parameters for three combustion conditions, with corresponding flame LF images shown in Fig. 9.

The LFC's radiation intensity calibration was performed using a blackbody furnace (LANDCAL R1500T) to establish a relationship between the grayscale radiation intensity at temperatures between 900-1450°C, with 50°C intervals.

At each temperature, twenty frames were captured to enhance the signal-to-noise ratio (SNR) of the measurements. The results of the radiation intensity calibration are presented in Fig. 11. The fitted calibration function relates the pixel grayscale value x to the corresponding radiation intensity y.

The flame was divided into N_z×N_r×N_φ=15×20×20 voxels. The 3-D soot absorption coefficient distribution was determined using the method described [44] and was utilized as prior information for reconstructing the temperature field. To improve the SNR, 10 consecutive LF images were captured for each experimental condition. The reconstructed flame temperature distribution is shown in Fig. 12.

The experimental conditions yield temperature ranges spanning from 1000 K to 2100 K, with peak values of 2131 K, 2118 K, and 2107 K, respectively. In the region near the fuel nozzle outlets, temperatures remain relatively low due to the incomplete mixing of fuel and air, as well as the sluggish chemical reaction rates. Here, heat generation primarily stems from preheating at the initial combustion stage rather than from active combustion. As the fuel and oxidizer mix more thoroughly in the mid-flame region, the chemical reaction rates and heat release intensify, causing a rapid temperature rise to its peak. Beyond this combustion zone, as the reaction products (e.g., water vapor and carbon dioxide) disperse into the surrounding air, heat is gradually lost through convection and radiation. Consequently, the temperature steadily decreases until it reaches ambient levels. The reconstructed temperature distribution obtained in this study aligns closely with previously reported results [48], further confirming the accuracy and reliability of the proposed algorithm.

5. Conclusions

This paper proposes an ASWNNLS algorithm to address the challenges faced by existing flame temperature field reconstruction methods, including their susceptibility to radiation intensity noise and low reconstruction accuracy in low-temperature flame regions with wide temperature ranges. The proposed algorithm's effectiveness was validated through numerical simulations, particularly analysing reconstruction performance under varying radiation intensity noise levels and temperature ranges. Additionally, a comparative evaluation with other solution algorithms was conducted. Finally, experimental studies were performed to verify the applicability of the proposed method. The key findings are as follows:

- The proposed algorithm demonstrates strong noise resistance and can accurately reconstruct flame temperature distributions under various temperature ranges. At a noise level of γ =3 %, the mean relative reconstruction errors for narrow and wide temperature range flames remain low at 3.75 % and 5.46 %, respectively.
- Compared with existing flame temperature field reconstruction algorithms, the proposed method achieves higher reconstruction accuracy across various temperature ranges, albeit with a longer reconstruction time. Specifically, it improves reconstruction accuracy by approximately 70% compared to the LSQR algorithm but incurs a computational efficiency reduction of about 25 times.
- Under various experimental conditions, the reconstructed temperature distributions of bimodal ethylene laminar diffusion flames exhibit trends that closely correspond with the results reported in the literature, thereby demonstrating the substantial practical applicability of the proposed algorithm.

Future research endeavours will extend the application of this algorithm to various types of flames and combustion environments. Additionally, performance evaluation will be conducted to assess its capabilities in reconstructing temperature fields under diverse fuel compositions, flame structures, and combustion conditions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Tianxiang Ling: Conceptualization, Methodology, Software, Writing – original draft. **Md. Moinul Hossain:** Funding acquisition, Data curation, Formal analysis, Writing – review & editing. **Guoqing Chen:** Formal analysis, Investigation. **Qi Qi:** Data curation, Validation. **Biao Zhang:** Software, Resources, Writing – review & editing. **Chuanlong Xu:** Supervision, Writing – review & editing.

Data availability

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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Tables

Table 1. Overview of the LFC parameters used in this study.

Symbol	Value	Description
L_{oq}/mm	505	the distance between the flame centerline and the main lens
L_{qn}/mm	53.1	the distance between the main lens and MLA
L_{ng}/mm	480	the distance between the MLA and the photosensor
f/mm	50	the focal length of the main lens
$f_m/\mu m$	600	the focal length of the microlens
$d_g/\mu m$	8	the length of the pixel
$d_m/\mu m$	95	the diameter of each microlens
N_m	60	the number of microlenses
N_g	12	the number of pixels covered by each microlens

Table 2. Overview of the reconstruction error and the time under different reconstruction methods.

Case	Narrow temperature range			Wide temperature range		
	ΔT_{mean}/%	ΔT_{max}/%	Reconstruction time/s	ΔT_{mean}/%	ΔT_{max}/%	Reconstruction time/s
Proposed	1.64	7.08	298	3.18	12.64	327
Case 1	6.98	52.05	10.02	9.31	89.2	12.1
Case 2	5.68	43.28	19.95	8.59	74.54	14.81

Table 3. Operation conditions of bimodal flames.

Condition	Ethylene (L/min)	Air (L/min)
C1	0.1	5
C2	0.12	5
C3	0.15	5

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Fig. 1. Example illustration of the methodology for flame radiation sampling employing the LFC.

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