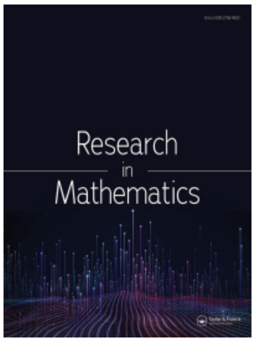

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RESEARCH ARTICLE

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A computation of implied volatility leveraging model-free option-implied information

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ABSTRACT
This paper takes inspiration from recent model-free techniques for estimating the risk-free rate and dividend yield from European-style option prices. It proposes a methodology for computing implied volatility (IV) that integrates this option-derived information. Instead of relying on traditional inputs like treasury yields and historical dividend yields, our approach incorporates forward-looking estimates of the dividend-adjusted underlying asset price and the implied discount factor into the IV computation. This results in a simpler yet more informative adjustment that may prove useful in updating the computation of IV.

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Implied volatility; implied discount factor; forward price; option-implied; repeated median; Cboe

1. Introduction

The Black-Scholes-Merton (BSM73, henceforth) formula is one of the most recognized result in modern finance. Within a simple set of assumptions, the formula offers a framework for pricing European options based on just six inputs: the underlying asset's spot price, the option's strike price, the risk-free rate of interest, the dividend yield, the time to maturity of the option, and a constant volatility parameter. Among these parameters, volatility is the only variable that cannot be observed directly, hence has to be estimated.

One approach in the estimation of this volatility parameter, σ, entails the use of historical price data of the underlying asset over a period of the same length as the option's time to maturity, see in detail (Björk, 2020) and(Hull, 2018). This follows from the underlying assumption in the BSM73 model where the dynamics of the underlying asset (under the physical measure $\mathbb{P}$) are given by the stochastic differential equation $dS_t = \mu S_t dt + \sigma S_t dW_t$ where, μ denotes the constant drift parameter, σ represents the standard deviation of the returns of the log-asset price and W_t is a standard Wiener process. Under this approach, volatility is constant, and the estimate for the volatility obtained is backward-looking in nature as it is over a time period in the past, not for a future period (over the life of the option).

Another approach, which is more robust and often preferred due to its application to hedging and risk management in general, is to invert the BSM73 formula and extract the volatility variable, given that all other variables can be observed from the market (or estimated). Since option prices are already quoted in the market, this implies that the market has some level of information with regard to this unobserved volatility. Using the BSM73 formula, we can solve for σ by considering the option price quoted in the market and substituting the observable variables. This volatility derived from the quoted price for a single option is called the (BSM) implied volatility (BSMIV, henceforth).

BSMIV, however, cannot be calculated explicitly and studies resort to either closed-form approximations such as (Brenner & Subrahmanyam, 1988) and others (see (Fengler, 2012) for a review and comparison) or iterative procedures such as Newton-Raphson, see (Manaster & Koehler, 1982), or its variants.

Of the five parameters needed in the BSMIV extraction, the strike price and the time to maturity are specified by the terms of the option contract, whereas the remaining three: the underlying asset's spot price, the dividend yield, and the risk-free rate of interest are obtained from other markets either directly, indirectly through proxies or estimated.

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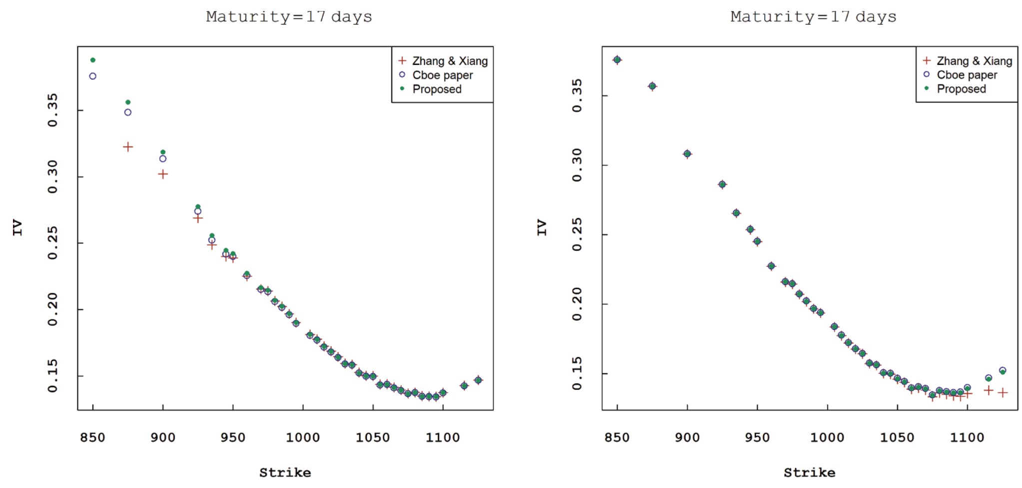
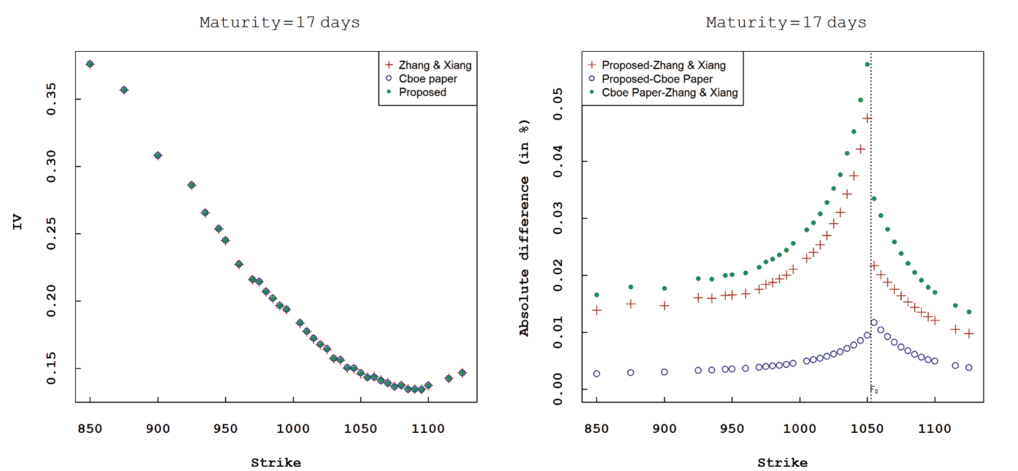


(a) Call options (b) Put options

Figure 1. Comparison of the implied volatility obtained from the three approaches for SPX call and put options that mature on 21 November 2003.



(a) IV smirk for out-of-the-money options. (b) Absolute differences among the approaches for the IV smirk given on the left panel.

Figure 2. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches that mature on 21 November 2003.

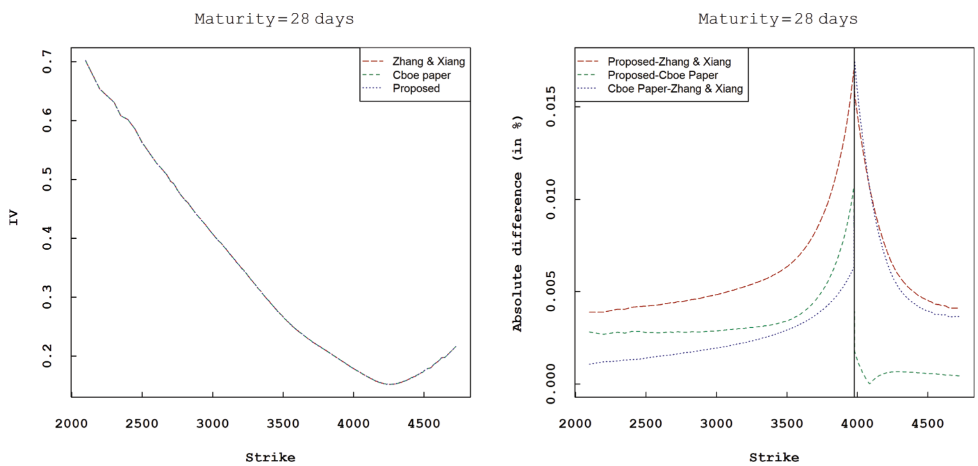
In the extraction of the BSMIV, the treasury rate[1] whose term is close to the option maturity is usually used as a proxy for the risk-free rate, and if none is close enough one interpolates or extrapolates between available treasury rates such as in (Zhang & Xiang, 2008). An alternative proxy for the risk-free rate as suggested in recent literature, which would not require interpolation or extrapolation as it matches the option's maturity precisely, is the option-implied risk-free interest rate, which is extracted from the market prices of options. Indeed, given the option's time to maturity, the implied discount factor, e^{-rT}, is determined.

The underlying asset price is observed from the stock market while the dividend yield has to be estimated. Due to technicalities in obtaining an accurate estimation of the future dividend yield, market practitioners prefer to use Black's formula rather than the BSM73 formula because the dividend component does not have to be estimated explicitly as all information regarding dividends is captured by the implied forward price, see, (Hull, 2018).

The derivation of the implied forward price varies across the literature. In (Zhang & Xiang, 2008), the implied forward price is calculated from an at-the-money (ATM, henceforth) strike, the corresponding call and put option prices at that strike with treasury rates used for discounting. In contrast, the Cboe European-Style Options Implied Volatility Calculation

Table 1. Parameter estimation results on the market data of SPX options (standard) maturing on 17 February 2023, 17 March 2023, 21 April 2023, 19 May 2023, 21 July 2023, and 19 January 2024. $\%'_{[Y}$ represents the discount factor obtained via the approach in (Zhang & Xiang, 2008) and is calculated using the one-month, two-months, three-months, four-months, six-months and one-year treasury yield curve rates given respectively by 4.69%, 4.64%, 4.72%, 4.75%, 4.8% and 4.68%. $\%'_{[Y}$ and $\%'_{LN}$ are the implied discount factors obtained using the approach in the Cboe paper and our proposed approach, respectively

Time to maturity	$'^{[Y}_0$	$\%'_{[Y}$	$'^{DQ}_0$	$\%'_{DQ}$	$'^{LN}_0$	$\%'_{LN}$
28	3977.31	0.9964087	3977.20	0.9961106	3977.16	0.9964120
56	3985.96	0.9929064	3986.09	0.9923645	3986.02	0.9925000
91	4000.15	0.9883013	4000.33	0.9879649	3999.92	0.9880973
119	4009.70	0.9846330	4010.01	0.9845612	4009.79	0.9844826
182	4034.47	0.9763499	4035.35	0.9755800	4035.35	0.9763463
364	4106.03	0.9544006	4105.81	0.9520073	4105.95	0.9529330



(a) IV smirk for out-of-the-money options that mature on 17 February 2023.

(b) Absolute differences among the approaches for the IV smirk given on the left panel. The continuous vertical line is the point $K = F_0^{km}$.

Figure 3. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options that mature on 17 February 2023.

Methodology paper[2] (Cboe paper, henceforth) considers options with strikes centered around an ATM strike, then apply ordinary least squares estimation to find the implied dividend and the implied discount factor. These estimates are in turn used, together with the underlying asset price, to calculate the implied forward price.

With all variables available, IV is extracted using numerical procedures.

In this present paper, we propose a computation of the BSMIV that incorporates the model-free estimates of the implied discount factor and the dividend-adjusted underlying asset price presented in (Kamau & Mwaniki, 2023). Thus, in lieu of treasury yields, we use option-implied interest rates and incorporate a forward-looking estimate of the dividend-adjusted underlying asset price. Therefore, in this approach, the implied forward price does not need to be computed explicitly. Further, in our estimation we take into consideration all available quoted options data not just those near an ATM strike. Hence, we rely on the repeated median approach which is more robust to outliers than the ordinary least squares estimation used in the Cboe paper. This paper therefore contributes to the literature on IV computation as it presents an approach that makes exclusive use of market option data to extract BSMIV.

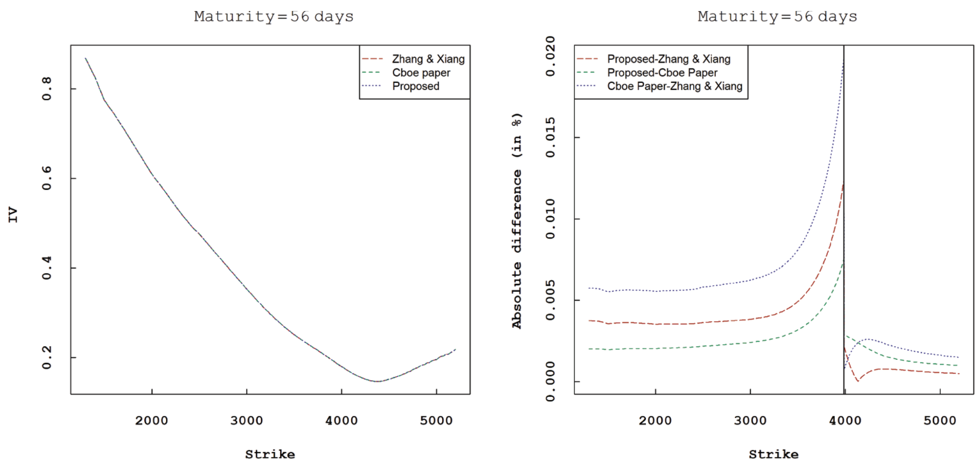
The rest of the paper is organized as follows: Section 2 outlines the methodology used in the paper with Section 2.1 describing what is in the literature (our benchmark) and Section 2.2 presenting the approach proposed in this paper, Section 3 provides a description of the data used, the empirical results obtained, and a discussion of the results, and Section 4 concludes.

2. Methodology

2.1. Using Black's formula

2.1.1. Definition

Let F_0 denote the forward price of an index for a contract with time to maturity T, r the risk-free rate of interest, σ the volatility, and K the strike price.



(a) IV smirk for out-of-the-money options that mature on 17 March 2023.

(b) Absolute differences among the approaches for the IV smirk given on the left panel. The continuous vertical line is the point $K = F_0^{km}$.

Figure 4. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options that mature on 17 March 2023.

Black's formula for calls and puts is given, respectively, by

$$C_B = F_0 e^{-rT}\Phi(d_1) - Ke^{-rT}\Phi(d_2) \quad \text{and}$$
$$P_B = Ke^{-rT}\Phi(-d_2) - F_0 e^{-rT}\Phi(-d_1), \tag{1}$$

where $d_1 = \frac{\log\left(\frac{F_0}{K}\right)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

As noted before, the strike price and the time to maturity are specified by the terms of the option contract whereas the risk-free rate of interest is either estimated or determined via a proxy. Therefore, the task is to determine the only remaining variable, F_0.

2.1.2. *Computation of the implied forward price*

There are different approaches that have been proposed in the literature on the calculation of the implied forward price, F_0. We highlight two.

Let C_i and P_i denote the mid-prices of each option's bid/ask quotation, for calls and puts, respectively, with strike K_i (for $i = 1, 2, \dots, n$).

(a) Zhang & Xiang (2008)
In their approach, for a given set of call and put options data with the same time to maturity, T, but with n different strike prices, (Zhang & Xiang, 2008) first identify an ATM strike. This is defined to be the strike at which the absolute difference between the call and put prices is the smallest.

The implied forward price under this approach, F_0^{zx}, is then determined by

$$F_0^{zx} = K_{ATM} + e^{rT} \times (C_{ATM} - P_{ATM}), \tag{2}$$

where C_{ATM} and P_{ATM} are the call and put prices, respectively, at an ATM strike K_{ATM}, and r is the risk-free rate.

(b) Cboe paper
In this approach, as before, first identify an ATM strike. Then select the strikes and the corresponding option prices that are ATM − 8% to ATM + 8%. By put-call parity, for call and put options with the same strike and time to maturity, we have the relation

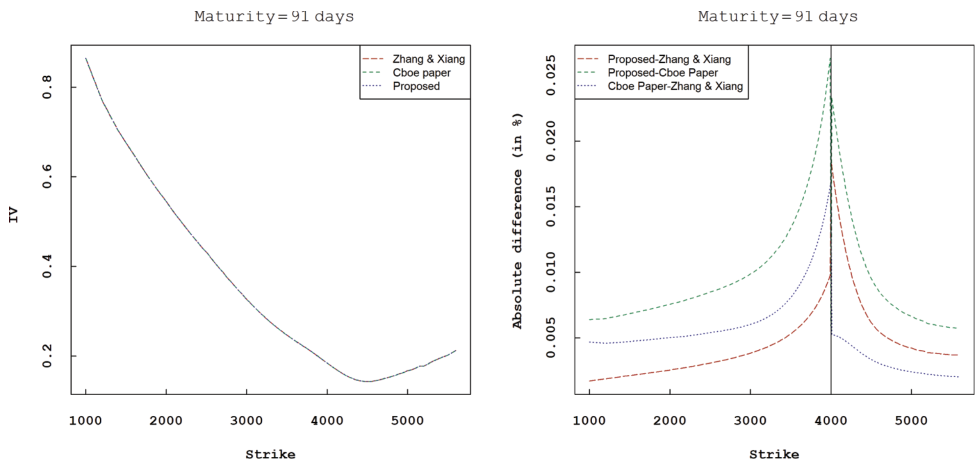
$$C - P = (S - D) - Ke^{-rT}, \tag{3}$$

where S is the underlying index value, D is the discrete dividend amount, and e^{-rT} is the discount factor.
Next, fit the linear model[3]

$$Y = a + bX, \tag{4}$$

where $Y_i = P_i - C_i + S$, and $X_i = K_i$.
Using ordinary least squares estimation, the estimators are

$$\hat{b} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - \overline{X})^2} \quad \text{and} \quad \hat{a} = \overline{Y} - \hat{b}\overline{X}, \tag{5}$$



(a) IV smirk for out-of-the-money options that mature on 21 April 2023.

(b) Absolute differences among the approaches for the IV smirk given on the left panel. The continuous vertical line is the point $K = F_0^{km}$.

Figure 5. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options that mature on 21 April 2023.

where

$$\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} \quad \text{and} \quad \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}.$$

The implied forward price under this approach, F_0^{cp}, is obtained by

$$F_0^{cp} = \frac{S - \hat{a}}{\hat{b}}, \tag{6}$$

where S is the underlying index value, $\hat{a}$ is an estimate of the discrete dividend amount, D and $\hat{b}$ is an estimate of the implied discount factor.

2.1.3. Computation of the implied volatility

Given market prices of European options with their corresponding strike prices and respective time to maturity, and with the implied forward price determined and an estimate or a proxy for the implied discount factor obtained, σ is then obtained under the two approaches by inverting Black's formula given by Equation (1) and extracting IV using numerical methods. Here, we consider the Newton-Raphson procedure.

With $\mathbb{G}_B$ denoting the model option price and $\mathbb{G}_{mkt}$ denoting the observed market option price, the root-finding problem to be solved is

$$f(\sigma) = \mathbb{G}_B - \mathbb{G}_{mkt} = 0. \tag{7}$$

We start with an initial value for σ (σ_0) defined as

$$\sigma_0 = \sqrt{\frac{2}{T}\left|\log\left(\frac{F_0}{K}\right)\right|}. \tag{8}$$

Let σ_k denote the value obtained after k iteration steps, then the next value σ_{k+1} is

$$\sigma_{k+1} = \sigma_k - \frac{f(\sigma_k)}{f^{\prime}(\sigma_k)}, \qquad \text{for } k = 0, 1, 2, \dots, \tag{9}$$

where $f^{\prime}(\sigma_k)$ is the option vega computed at σ_k and $f(\sigma_k)$ is as defined in (7), where

$$\mathbb{G}_B = \begin{cases} F_0e^{-rT}\Phi(d_1) - Ke^{-rT}\Phi(d_2), & \text{for call options} \\ -F_0e^{-rT}\Phi(-d_1) + Ke^{-rT}\Phi(-d_2), & \text{for put options.} \end{cases}$$

The iteration repeats until a tolerance such as $|\mathbb{G}_{mkt} - \mathbb{G}_{k+1}| \leq \epsilon$ is attained.

2.2. Using Black-Scholes-Merton formula

2.2.1. Definition

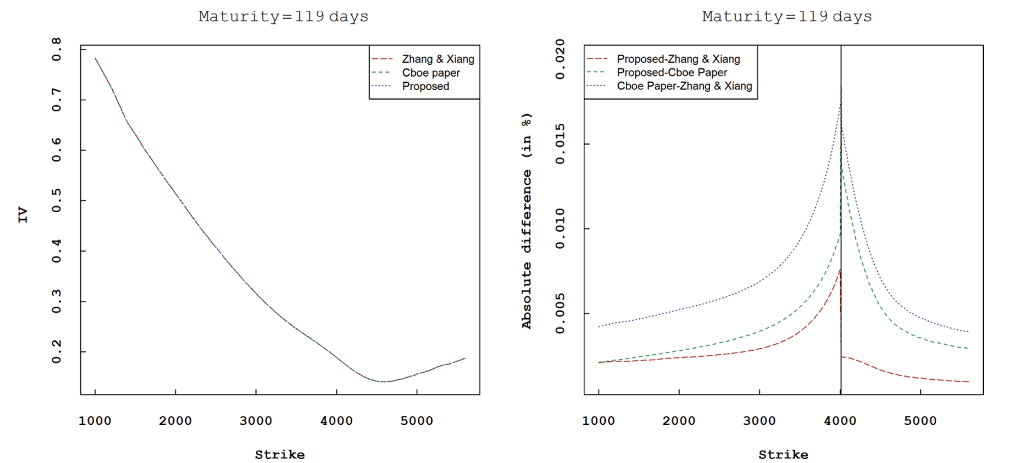
Let C_{BSM} denote the BSM73 price of a call option, δ the dividend yield, S_0 the current underlying price, r the risk-free rate of interest, and σ the volatility of the underlying, then for a European call option with time to maturity T and with strike price K, C_{BSM} is defined as

$$C_{BSM} = S_0e^{-\ T}\Phi(d_1) - Ke^{-rT}\Phi(d_2), \tag{10}$$

where $\Phi(x) = \int_{-Ü}^{x} \frac{1}{\sqrt{2\pi}} \exp(-\frac{u^2}{2})du$; $d_1 = \frac{\kappa + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$, where $\kappa = \log\left(\frac{S_0e^{-\delta T}}{Ke^{-rT}}\right)$.

By put-call parity, $C_{BSM} + Ke^{-rT} = P_{BSM} + S_0e^{-\ T}$, the corresponding put option pricing formula is given by

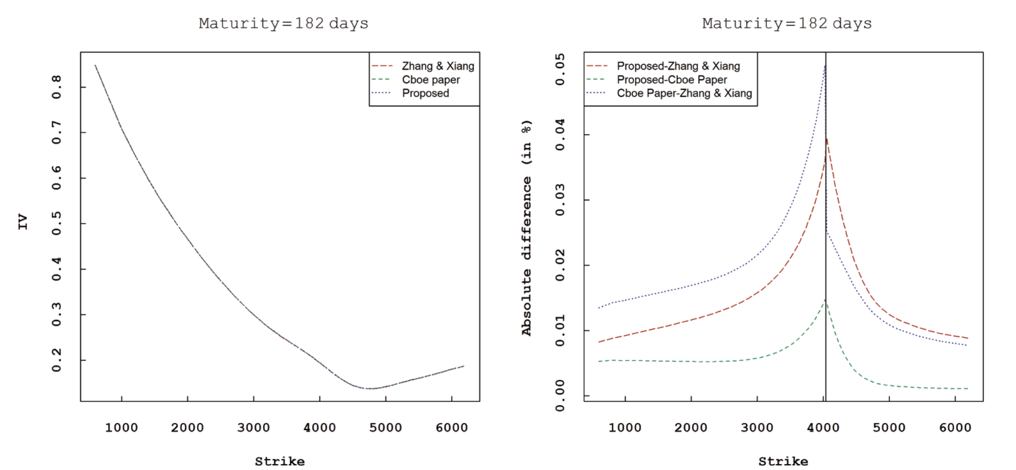
$$P_{BSM} = -S_0e^{-\ T}\Phi(-d_1) + Ke^{-rT}\Phi(-d_2). \tag{11}$$



(a) IV smirk for out-of-the-money options that mature on 19 May 2023.

(b) Absolute differences among the approaches for the IV smirk given on the left panel. The continuous vertical line is the point $K = F_0^{km}$.

Figure 6. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options that mature on 19 May 2023.



(a) IV smirk for out-of-the-money options that mature on 21 July 2023.

(b) Absolute differences among the approaches for the IV smirk given on the left panel. The continuous vertical line is the point $K = F_0^{km}$.

Figure 7. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options that mature on 21 July 2023.

2.2.2. Model-free estimation of the implied discount factor and the dividend-adjusted underlying price

For each maturity T, Kamau and Mwaniki (Kamau & Mwaniki, 2023) fit the line

$$P_i - C_i = \alpha + \beta K_i + \epsilon_i, \tag{12}$$

where by put-call parity, $\beta = e^{-rT}$ and $\alpha = -S_0 e^{- \ T}$. Therefore, the slope estimate yields the implied discount factor and the intercept estimate (in absolute terms), the dividend-adjusted underlying asset price.

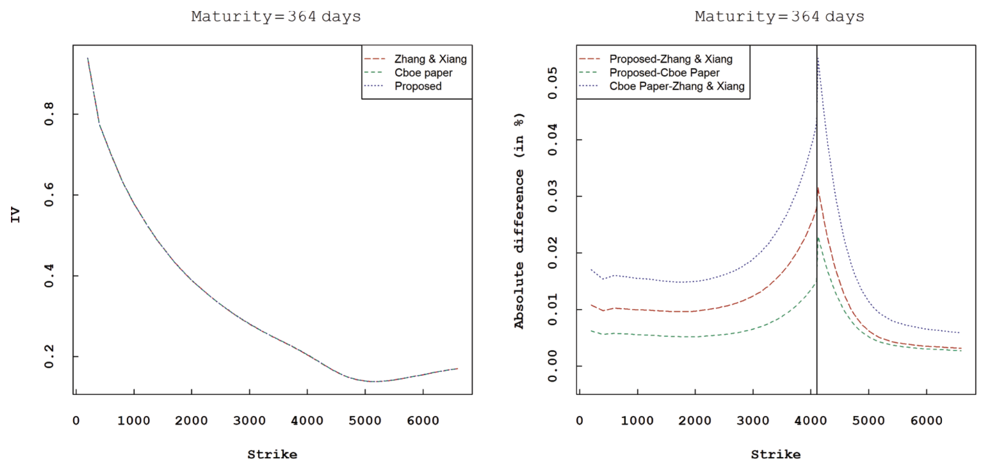
The slope and intercept estimators are estimated using the repeated median approach as follows.

For each time to maturity T, all $\frac{n\&n-1'}{2}$ slopes of the lines and intercepts connecting each pair of points $(K_i, (P_{i,T} - C_{i,T}))$ and $(K_j, (P_{j,T} - C_{j,T}))$, where, $K_i \neq K_j$ are computed. Respectively,

$$\beta(i,j) = \frac{(P_{i,T} - C_{i,T}) - (P_{j,T} - C_{j,T})}{K_i - K_j} \quad \text{and}$$

$$\alpha(i,j) = \frac{K_i(P_{j,T} - C_{j,T}) - K_j(P_{i,T} - C_{i,T})}{K_i - K_j}.$$

These $\beta(i,j)$ and $\alpha(i,j)$ values can be visualized as the elements in the $n \times n$ symmetric matrices $\mathbf{B}$ and $\mathbf{A}$, respectively, both having no entries in the main diagonal.



(a) IV smirk for out-of-the-money options that mature on 19 January 2024.

(b) Absolute differences among the approaches for the IV smirk given on the left panel. The continuous vertical line is the point $K = F_0^{km}$.

Figure 8. Comparison of the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options that mature on 19 January 2024.

The repeated median estimators for the slope and intercept are obtained by taking the median of the row medians (or column medians, by symmetry) of matrices **B** and **A**, respectively, leading to

$$\hat{\beta} = \underset{j}{\text{median}}\{\underset{i \neq j}{\text{median}}\ \beta(i,j)\} \quad \text{and}$$
$$\hat{\alpha} = \underset{j}{\text{median}}\{\underset{i \neq j}{\text{median}}\ \alpha(i,j)\}. \tag{13}$$

We thus estimate e^{-rT} with $\hat{\beta}$ in the BSM73 formula and $S_0 e^{-\ T}$ with $|\hat{\alpha}|$, then extract BSMIV.

2.2.3. BSMIV extraction

Given market prices of European-style options, the volatility parameter, σ, given in Equations (10) and (11) can be extracted, given that all other variables are available. An exact closed-form formula to extract σ from the market price of an option does not exist, thus one has to resort to either approximation or numerical methods. In this paper, we consider the numerical approach via the Newton-Raphson procedure.[4]

With $\mathbb{C}_{model}$ denoting the model option price and $\mathbb{C}_{mkt}$ denoting the observed market option price, the root-finding problem to be solved is

$$f(\sigma) = \mathbb{C}_{model} - \mathbb{C}_{mkt} = 0. \tag{14}$$

In this procedure, we start with an initial value for σ (σ_0) as suggested in Manaster and Koehler (Manaster & Koehler, 1982), but modified as

$$\sigma_0 = \sqrt{\frac{2}{T}\ |\kappa|}, \qquad \text{where} \quad \kappa = \log\left(\frac{S_0 e^{-\ T}}{K e^{-rT}}\right). \tag{15}$$

And, denoting σ_k as the value obtained after k iteration steps, the next value σ_{k+1} is

$$\sigma_{k+1} = \sigma_k - \frac{f(\sigma_k)}{f'(\sigma_k)}, \qquad \text{for } k = 0, 1, 2, \dots, \tag{16}$$

where $f'(\sigma_k)$ is the option vega computed at σ_k and $f(\sigma_k)$ is as defined in (14), where

$$\mathbb{C}_{model} = \begin{cases} S_0 e^{-\delta T}\Phi(d_1) - K e^{-rT}\Phi(d_2), & \text{for call options} \\ -S_0 e^{-\delta T}\Phi(-d_1) + K e^{-rT}\Phi(-d_2), & \text{for put options.} \end{cases}$$

The iteration repeats until a tolerance such as $|\mathbb{C}_{mkt} - \mathbb{C}_{k+1}| \leq \epsilon$ is attained. The iteration procedure (16) may fail if vega is close to zero, which often occurs for very deep in-the-money and very deep out-of-the-money options.

Remark 1. The dividend component is not estimated explicitly but rather by relying on information obtained, exclusively, from the options market, an estimate of the dividend-adjusted underlying asset price is obtained. This estimate can be seen to capture all information regarding dividends, similar to the implied forward price used in Black's formula. Therefore, under this approach, using either the BSM73 formula or Black's formula results in the same IV. It follows that the implied forward price under this approach, F_0^{km}, is given by $F_0^{km} = \frac{|\hat{\alpha}|}{\hat{\beta}}$.

3. Data and empirical results

3.1. Data set 1: Data in Zhang and Xiang (2008)

3.1.1. Data description and processing

The first data set that we use is the market data of SPX options on 4 November 2003 on pg. 268 of Zhang and Xiang (2008). The data has columns consisting of the strike price, last sale, volume, bid, ask, and the mid-values for both calls and puts. The maturity date is 21 November 2003. The time to maturity is 17 days and the S&P 500 index level is $S_0 = 1053.25$. The risk-free rate, obtained by extrapolation using the one-month and three-months US treasury yield curve rates, is 0.9743%.

From this data set, the ATM strike is $K_{ATM} = 1055$ with $C_{ATM} = 11.9$ (the mid-price of the call's bid/ask quotation) and $P_{ATM} = 14.2$ (the mid-price of the put's bid/ask quotation). The implied forward price is therefore, by formula (2), given by

$$F_0^{zx} = 1055 + e^{0.9743\%\Omega 17\text{-}365} \times (11.9 - 14.2) = 1052.70$$

With this implied forward price and all other variables available, IV is extracted using Equation (9).

Similarly, using the Cboe approach and performing the ordinary least squares estimation, we find $\hat{a} = 1.94$ and $\hat{b} = 0.9985941$ which implies that $F_0^{cp} = 1052.79$. Again, with this implied forward price and all other variables available, IV is extracted using Equation (9).

In our proposed approach, we estimate the implied discount factor and the dividend-adjusted underlying asset price using the repeated median approach and obtain $|\hat{\alpha}| = 1051.212$ and $\hat{\beta} = 0.998524$ which implies that $F_0^{km} = 1052.77$. Finally, we extract BSMIV using Equation (16).

A comparison of the implied volatility obtained from the three approaches is displayed in Figure 1 for both calls and puts.

3.1.2. Results and discussion

From Figure 1, there is a notable difference in the implied volatility obtained for in-the-money calls and in-the-money puts among the three approaches, but no significant difference in the implied volatility obtained for out-of-the-money calls and puts. This corresponds to the assertion in the literature that in-the-money options are more sensitive to model changes than their out-of-the-money counterparts. Therefore, as in (Zhang & Xiang, 2008), we construct our IV smirk using out-of-the-money options only. We therefore select put options with strike prices that are below the implied forward price, and select call options with strike prices that are above the implied forward price. The subsequent implied volatility smirk obtained exclusively from out-of-the-money options is displayed in Figure 2 under the three approaches.

3.2. Data set 2: CBOE delayed option quotes data

3.2.1. Data description and processing

Secondary index option data containing standard call and put option quotes on the S&P 500 (SPX) Index was obtained from the Chicago Board Options Exchange (CBOE) delayed option quotes page.[5] The data has columns consisting of the expiration date, strike price, last sale, open interest, volume, bid, ask, IV, delta and gamma for both calls and puts maturing on 17 February 2023, 17 March 2023, 21 April 2023, 19 May 2023, 21 July 2023, and 19 January 2024.

We clean the raw data as follows. First, for each maturity, the lowest strike price is selected from the first out-of-the-money put with a non-zero bid price and the highest strike from the first out-of-the-money call with a non-zero bid price. Second, we find the mid-prices of each option's bid/ask quotation and take this to be the price of the call (put).

For each maturity, we then find the implied forward price and the (implied) discount factor using the three approaches. These estimates are compiled and presented in Table 1. As earlier noted, the computation of the implied forward price under our proposed approach is not required in the extraction of IV, however, we include it in Table 1 for completeness and for comparison purposes.

3.2.2. Results and discussion

With these estimates, we then extract IV using Equations (9) and (16). As before, we construct our IV smirk using out-of-the-money options only. The IV smirk for each maturity under the three approaches is displayed in Figures 3–8.

In comparing the IV smirk for out-of-the-money options obtained from the three approaches for SPX call and put options, it can be seen that the absolute differences among the approaches across all maturities are relatively low with the highest being in the neighborhood of 0.05%.

4. Conclusion

This paper has proposed the incorporation of the forward-looking model-free option-implied discount factor and the dividend-adjusted underlying asset price estimates in the computation of BSMIV. The proposed approach makes exclusive use of market options data to extract BSMIV. In this approach, different from past approaches, the implied forward price does not need to be computed explicitly nor does the proxy for the risk-free rate require interpolation or extrapolation in instances where the terms are not very close to or do not match the option's maturity. Further, in the comparison of the resulting IV obtained from the proposed approach with IV from previous approaches, it was found that the differences were more pronounced for both in-the-money call and put options and far less pronounced for out-of-the-money call and put options. Therefore, if interest is on constructing the IV curve using out-of-the-money options only, as is standard practice, the computed IV via the proposed approach could be used as an alternative to previous approaches. In conclusion, the approach of this paper may therefore prove useful in updating or adjusting the computation of BSMIV.

Notes

1. Among other rates, such as OIS rates.
2. For further details, see, Cboe European-Style Options Implied Volatility Calculation Methodology paper, available at: https://cdn.cboe.com/api/global/us_indices/governance/Cboe_European-Style_Option_%20Implied_Volatility_Calculation_%20Methodology.pdf.
3. We have adjusted what appears to have been a typo in the definition of the term $Y_i = P_i - C_i + S$ given in the Cboe paper.
4. Other methods such as the secant method can also be used.
5. See: https://www.cboe.com/delayed_quotes/spx/quote_table.

Disclosure statement

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References

Björk, T. (2020). *Arbitrage theory in continuous time*. Oxford University Press.

Brenner, M., & Subrahmanyam, M. G. (1988). A simple formula to compute the implied standard deviation. *Financial Analysts Journal*, *44*(5), 80–83. https://doi.org/10.2469/faj.v44.n5.80

Fengler, M. R. (2012). Option data and modeling BSM implied volatility. In J. Duan, W. Härdle, & J. Gentle (Eds.), *Handbook of computational finance* (pp. 117–142). Springer-Verlag.

Hull, J. C. (2018). *Options, futures, and other derivatives.* (10th Edition ed.). Pearson.

Kamau, M., & Mwaniki, I. J. (2023). On a robust estimation of option-implied interest rates and dividend yields. *Cogent Economics & Finance*, *11*(2), 2260658. https://doi.org/10.1080/23322039.2023.2260658

Manaster, S., & Koehler, G. (1982). The calculation of implied variances from the Black-Scholes model: A note. *The Journal of Finance*, *37*(1), 227–230. https://doi.org/10.1111/j.1540-6261.1982.tb01105.x

Zhang, J. E., & Xiang, Y. (2008). The implied volatility smirk. *Quantitative Finance*, *8*(3), 263–284. https://doi.org/10.1080/14697680601173444