The Public Provision of Goods in Democracies: Do Age and Inequality matter?

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ABSTRACT

We build a multi-dimensional model of political decision-making with endogenous political parties to analyse the effect of inequality and demography on public spending. Voters differ in terms of income and age. Political competition determines in equilibrium the tax rate and the allocation of revenue between income redistribution and two forms of public spending — a capital good and a neutral good. All agents value the neutral good equally but the young like capital spending more than the old do. We show that the effect of age (resp., inequality) on equilibrium public spending can go in any direction based on the underlying level of inequality (resp., age). Our findings reconcile a large body of seemingly contradictory stylised empirical findings in public economics.

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1. INTRODUCTION

The topic of income redistribution has been one of the central themes in the political economy literature. A closely related issue is the public provision of goods and services, (e.g., education, healthcare, drinking water, public transport, etc.) since these are viewed as a form of redistribution in the positive literature. Moreover, the public provision of various goods and services is a central feature of governance. This *per se* highlights the need to understand the various determinants of such public spending. Here we focus on the demographic and economic determinants – specifically, the age composition of the electorate and economic inequality – of the public provision of private goods in democracies. The choice of these specific determinants is motivated by the potential conflicts of interest along these dimensions. Take the case of provision of education. Education may be considered as redistribution from either the rich to the poor (see e.g., Glomm and Ravikumar (1998)) or from the poor to the rich in the case of higher education, where the poor are financially constrained from attending universities (a la Fernández and Rogerson (1995)). The issue could be perceived from another angle — namely, age cohorts, as done in Gradstein and Kaganovich (2004) who posit public education as redistribution from the old to the young.[2]

Aside from education whose benefits clearly have an age-specific aspect, there are a set of publicly provided goods over which there are *no* conflicts of interest in the sense that all agents irrespective of age or income value them equally.[3] Security services (police, etc.), supply of drinking water, transport systems (including construction of roads, etc.), emergency services (fire, etc.), parks and public libraries are some such examples. How the pattern of spending on such *neutral* goods (alongside age-specific goods) may depend upon the severity of the conflict of interest along the two dimensions of age and income is an open question which highly policy-relevant. The present paper endeavours to study precisely this issue.

We develop a theory based on the seminal contributions of Epple and Romano (1996) and (more closely) Levy (2005). While Epple and Romano (1996) analyses a model in which the *only* possible form of redistribution available to society is redistribution in kind (i.e., public provision of healthcare), Levy (2005) makes a significant advancement by allowing for society to use income redistribution as an additional policy tool aside from public provision of education. We adapt the framework in Levy (2005) for our purposes chiefly by allowing for public spending on neutral goods alongside the age-specific one. Therefore, we are able to engage with a broader set of questions than hitherto possible. Here we address the following questions: how does

[2]The argument is that the young's income in the future is dependent on their current education while it is not so for the old.

[3]To be clear, all agents derive the same utility from *consuming* these goods and services; their financing through public or private means is a separate matter.

economic inequality affect the pattern of public spending, on both the age-specific and the neutral goods/services? What role does the age distribution of the electorate play? Moreover, is there any critical interplay between these two factors — namely, inequality and demography? Our answers to these questions speak to a number of stylised empirical findings in the extant literature, which we discuss in more detail below.

The voters in our model differ in terms of income and age. The first marker (i.e., income) drives the conflict in preferences over the tax rate where the poor (who are assumed to be more numerous) ideally desire maximum taxation while the rich want it as minimal as possible. The age dimension symbolizes another form of conflict in tastes. We posit that there is a good – call it capital spending – from which the young gain more than the old do.[4] This capital spending could encompass a wide-range of activities (say, physical infrastructure spending) which augments the market activity and hence the earnings of the young. It can be also viewed as some legal capacity investments like in Besley and Persson (2010) which supports markets and in general production-related activities. The old agent's consumption possibilities do not depend as much upon such current market-augmenting measures by the government. As mentioned earlier, we also allow for another form of in-kind public provision. This is a good which is valued equally by all agents and in particular is devoid of the young-versus-old conflict. As in Epple and Romano (1996) and Levy (2005), the agents may supplement their public consumption by purchases from the private market.

There is a political process which determines the tax rate and the allocation of the revenues between income redistribution and public spending. Our modeling of the political process follows the one in Levy (2005) closely. The setup builds on the "citizen-candidate" model *a la* Besley and Coate (1997) and Osborne and Slivinski (1996). Notably, it allows both for endogenous entry of politicians and for endogenous political parties. Here parties choose which platforms to offer, where each platform specifies the tax rate and the division of the tax revenues under the following heads — income transfers, neutral public spending and capital spending. There is a restriction on the platforms any party may advance — it can only offer credible platforms, that is, policies in the Pareto set of their members. Given the platforms that are offered, the citizens cast their vote for the platform they like most and the political outcome is determined by plurality.

In this setup, we require our equilibrium outcome to be "stable" in the following sense: given the political outcome, the members of any political party do not wish to split from their party and thereby induce a different political outcome. The equilibrium analysis specifies the composition

[4]Capital spending could in principle include education-related spending. More on this later.

of political parties along with public spending, income transfers, capital spending and the total size of government.

Our analysis reveals that both sets of factors – namely, age and income inequality – are crucial in determining the nature of the equilibrium coalitions and the winning platforms. This is perhaps unsurprising. However, what *is* striking is the how inter-twined these two factors are. Specifically, we find that the effect of *either* factor (age or inequality) on public spending depends critically upon the ambient level of the *other* factor. In other words, the "marginal effect" of one factor on public spending is not independent of the underlying level of the other factor. In particular, the following key results emerge: (i) When the young are a majority, the public spending on the neutral good tends to be lower on average than when the young are a minority for all levels of inequality. The same, however, cannot be said for capital spending, which depends upon the ambient inequality. (ii) When the old are a majority, the equilibrium public spending depends quite fundamentally upon the extent of income inequality — in particular, if income inequality is above a certain threshold then the equilibrium provision of *both* types of public spending can actually be higher. In sum, the effect of age (resp., economic inequality) on the equilibrium level and composition of public spending can go in any direction based on the underlying level of economic inequality (resp., age). This broad finding helps to understand why a significant number of seemingly contradictory empirical findings exist in public economics.

First, consider some stylised empirical facts regarding the effect of the share of the elderly on redistributive policies. Ladd and Murry (2001) and Harris, Evans and Schwab (2001) find that the elderly have no significant effect on public education in the United States. On the one hand, Case, Hines and Rosen (1993) find that a larger proportion of elderly residents reduces per capita expenditures on education and health. On the other hand, Alesina, Baqir and Easterly (1999) find a positive effect of the elderly share on education spending per pupil in U.S. municipalities.[5] To the extent that capital spending in our setup captures some form of human capital spending, our analysis suggests that some these seemingly contradictory empirical results could potentially be explained by the role played by the underlying income inequality.

As regards the empirical evidence on the effect of economic inequality on public spending, there seems to be no consensus either. On the one hand, in a cross-section of countries the results tend to show that countries with high levels of inequality choose lower amounts of public spending. See, e.g., Lindert (1994) and (1996), Moene and Wallerstein (2005), Schwabish, Smeeding, and Osberg (2006).[6] On the other hand, comparisons across U.S. states and within states over

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[5]We cite only a few examples out of the very many empirical studies on this topic in the interest of brevity.
[6]However, Shelton (2007) provides an exception to this pattern.

time find that rising income inequality is accompanied by higher government expenditures and increasing progressivity in the state tax code (e.g., Chernick (2005), Schwabish (2008)). However, Alesina, Baqir and Easterly (1999) find that inequality has a negative effect on education spending per pupil. Boustan et al (2013) find that rising income inequality is associated with larger increases in tax revenues and faster growth in public expenditures at municipality and school district levels in the US. They do, however, observe that additional funds do not necessarily imply either a greater quantity or superior quality of public goods. This could potentially be explained by our theoretical results which stress the role of age cohorts in this relationship.

Our theoretical predictions underline the need to simultaneously account for inequality, age *and* their interaction in the determination of public spending not just on age-specific goods (like education) but also neutral goods/services in empirical studies. We have deliberately kept the model close to the well-known framework of Levy (2005). This allows one to appreciate the mechanisms behind the main results – and how they depart from Levy (2005) – more comprehensively.[7]

For the core intuition behind our results, we first direct attention to the implications of changes in inequality. When income inequality is sufficiently low, then the preferences of the young rich and the young poor agents are closely aligned; both would prefer positive levels of capital spending which would boost their consumption possibilities. When, however, income inequality is higher the poor segments of society tend to converge in terms of preferences regardless of age. Thus, the issue of age-wise alignment versus income-wise alignment for the poor is a key factor in determining the equilibrium outcomes.

Now consider the situation where the young are a majority. Here it is the young poor voters who represent majoritarian interests, advocating maximum taxation as well as positive levels of capital spending and provision of the neutral good. The rich agents attempt to counter this by forming a coalition with the old poor. They offer a platform with a lower tax rate and lower capital *and* neutral public spending which yields a payoff to the old poor in excess of what the young poor's platform offers. As we show below, the rich are able to safeguard against complete redistribution by joining forces with the old poor. Notice, the extent of income inequality does *not* affect the core logic of this alliance formation, although it has the potential to change the composition of the platforms offered in equilibrium. In particular, as Proposition 2 states, when income inequality is sufficiently low there will be a positive level of capital spending in equilibrium.

[7]A discussion of the differences between our findings and those in Levy (2005) is relegated to the last part of Section 3.3.

Next, consider the case when the old are a majority. Here, income inequality assumes special importance. When income inequality is sufficiently low so that the age-wise alignment among the young is salient, the old poor agents can no longer win by proposing maximum taxation and provision of only the neutral good — this is so as the support from the young poor is absent. Here, in fact, it is the young poor who stand to win if no coalitions are formed among the other three groups much like in the young majority case. To preclude such a possibility, the rich combine forces with the old poor like in the scenario discussed above. However, the main difference is that it is possible to have zero capital spending in equilibrium. Specifically, the old being a majority can ensure this. This is a major difference with the young majority case for low income inequality. Hence, we observe a positive cohort size effect here.

Things look markedly different when income inequality is high enough to align all the poor agents together under the old majority scenario. Here it is the old poor voters who represent majoritarian interests, advocating maximum taxation as well as *zero* capital spending and positive levels of the neutral good. This is the direct analytical counterpart of the young majority case. The rich agents attempt to form a coalition with the young poor which involves a lower tax rate and some capital spending with some public provision of the neutral good. By a suitable choice of tax rates and public spending, the rich can guarantee themselves higher utility by joining forces with the young poor. The latter are happy to join as long as the party's proposal gives them greater utility than the old poor agent's policy. We show that such compromises always exist within our framework. Moreover, a comparison with the high-inequality young majority case reveals the possibility of a negative cohort size effect courtesy Proposition 2.

Dhami (2003) examines the political economy of redistribution when voters have asymmetric information about the redistributive preferences of politicians and the latter cannot make credible policy commitments. The main finding there regarding the effect of inequality on redistribution is that it depends in important ways on the incentives and constraints facing politicians. Our setting involves endogenous party formation unlike the two-party competition framework in Dhami (2003). Fernández and Levy (2008), like us, highlight the implications of the trade-off between general redistribution and targeted transfers. They focus on goods that are explicitly targeted to many small interest groups, such as local public goods, and study the effect of diversity on redistribution. In particular, they do not focus on the interplay of income inequality and age like we do here.

The remainder of the paper is organised as follows. Section II describes the basic model while Section III reports the main results of the analysis. Section IV concludes. All proofs and derivations are collected in the Appendix.

2. THE MODEL

We start with a description of the economic environment outlining the various agents and their preferences.

2.1. **The Economic Environment.** There is a unit mass of agents in the economy. These agents are different in two dimensions — namely, income and age. We first focus on the former marker.

We will assume that there are two levels of income in the economy. The poor have income w_p and the rich have income w_r where $w_r > w_p > 0$. Also, we will assume – as is standard in the literature – that the poor are more numerous. Hence, letting π denote the mass of the poor we have $\pi > 1/2$. So, the average income in the economy is given by w where

$$w = \pi w_p + (1 - \pi) w_r.$$

There are two types of goods in this economy. One is a numeraire good – denoted by x – which is liked by all agents. The other – denoted by H – represents a set of goods over which there are no conflicts of interest (either generational or class-based); hence, termed neutral.[8] For a typical agent, the utility function is given by $u(x, H)$ which is assumed to be strictly increasing in both arguments, strictly concave and twice differentiable. We assume that it represents homothetic preferences. Specifically, we have the following:

ASSUMPTION 1. $u_x, u_H > 0$, $u_{xx}, u_{HH} < 0$ with $u_{xH} \geq 0$ and $u(x, 0) = 0\ \forall x \geq 0$.

Society chooses a tax level $t \in [0, 1]$ via the political process described in Section 2.2 which is levied on all agents. Tax revenues thus raised may finance three things: (i) income transfer in a lump sum way which we denote by $T \geq 0$, (ii) the public provision of H denoted by $h \geq 0$ and (iii) infrastructure/capital spending which we denote by $k \geq 0$. The prices of H and k in terms of the numeraire x are assumed to be unity.[9] Thus, the budget constraint is given by

$$tw = T + h + k.$$

What is the purpose of k? The answer to this question relates *directly* to our second source of heterogeneity among the agents (i.e., age). We posit that k has the ability to influence the (post-redistribution) consumption of the numeraire good differently across age groups. We assume that there are two age groups — the "young" and the "old". The former value k more than

[8]We wish to clarify that our usage of the term "neutral good" is purely in the sense of the good lacking any conflict along the age or income dimension in terms of the agents' preferences. This is distinct from the traditional notion of neutral goods in economics where the consumption of such a good is unaffected by income, etc.

[9]This is without loss of generality in terms of the qualitative results.

the latter. Specifically, upon the implementation of a policy (t, h, k), the maximum possible numeraire consumption of a young agent is given by

$$x_y^i = f(k)[w_i(1-t) + tw - h - k]$$

where $i \in \{p, r\}$ and $f(.)$ satisfies the following:

ASSUMPTION 2. $f(0) = 1$, $f' > 0$, $f'' < 0$ with $f'(0) = +\infty$.

For the old agents, k has no such effect. So, the maximum possible numeraire consumption of an old agent is given by

$$x_o^i = w_i(1-t) + tw - h - k$$

where $i \in \{p, r\}$. The assumption that $f(k)$ for an old agent is unity for all values of k is made for simplicity. Our core results are substantively unchanged if we instead assume that $x_o^i = f(\delta k)[w_i(1-t) + tw - h - k]$ for $\delta \in (0, 1)$. The key point is that k benefits the young more than the old.

As mentioned earlier, k denotes capital spending (say, physical infrastructure spending) which augments the market activity and hence the earnings of the young. Our k may also be viewed more broadly as some legal capacity investments like in Besley and Persson (2010) which supports markets and in general production-related activities. The old agent's consumption possibilities do not depend as much upon such current market-augmenting measures by the government.

The consumption of the neutral good H may be supplemented by purchases in the private market as in Epple and Romano (1996) and Levy (2005); we denote this private spending by s. Therefore, the consumption of H for this individual is given by $h + s$. This may be exercised by both young and old agents as this good is equally valued by both groups. There is, however, no option of supplementing k — hence, whatever k is provided publicly is all there is for the agents to do with. Again, this does not mean that there is no private capital spending in this economy. It is just that k is the essential public spending required for a market economy to function.

The four groups in the population are then the old rich (r_o), the young rich (r_y), the old poor (p_o), and the young poor (p_y). Like in Levy (2005), we assume that none of the four groups composes a majority in the population.

We denote the mass of the young agents by $\theta \in (0, 1)$.

As mentioned earlier, we assume that the poor form a majority (i.e., $\pi > \frac{1}{2}$). We also assume that this is true within each age group. For simplicity, we analyse the case where the proportion of the poor within the young and the old are the same. We later discuss the implications of relaxing this assumption. In particular, we show that allowing the old be to richer on average than the young does not affect our results in any significant manner.

2.1.1. *Ideal policies.* By construction, the set of feasible policies is given by

$$Q \equiv \{(t, k, h) : tw \geq h + k, t \in [0, 1], h, k \geq 0\}.$$

We now characterise – for each of the four segments of the population – the ideal policies within this set Q. Let $q^*(i)$ denote the ideal policy of group i where $i \in \{r_o, p_o, r_y, p_y\}$.

Start with the old poor agents — i.e., p_o. Clearly, p_o would like $t = 1$ and $k = 0$ as it entails maximum redistribution and hence the best possibility of consuming both goods (x and h). As k reduces the consumption of the numeraire without delivering any additional gains, p_o would ideally want $k = 0$. Hence, the problem simplifies to the maximisation of $u(w - h, h)$ by choosing $h \in [0, w]$. By Assumption 1, the optimal h – call it $h^*(p_o)$ – lies in the interior.

Now consider the p_y segment of the population. Such an agent would also ideally have $t = 1$. Then the problem simplifies to the maximisation of $u(f(k)[w - k - h], h)$ by choosing $k, h \geq 0$ with $w \geq k + h$. As $f'(0) = +\infty$, it follows that $k^*(p_y) > 0$. Hence $q^*(p_y) \equiv (t = 1, k^*(p_y), h^*(p_y))$ denotes this ideal policy.

Next, consider the old rich agents — i.e., r_o. Like p_o, these agents will also ideally like $k = 0$. Also, they would ideally set lump sum redistribution $T(= tw - h)$ equal to 0. As for the choice of the tax rate, the r_o agent would like $t = 0$ and hence $h = 0$. To see why, observe that any public provision implies $t > 0$, and this means for obtaining $h = tw$ the rich must pay tw_r; hence, it effectively costs the rich more than unity (the price in the private market) per unit of H. Hence, they will rather choose $t = 0$ and purchase in the private market — therefore, $s > 0$. This defines their ideal policy, namely, $q^*(r_o)$.

Finally, we come to the r_y segment of the population. By $f'(0) = +\infty$ in Assumption 2, it must be that r_y sets $t, k > 0$. Like r_o, they would ideally set lump sum redistribution $T(= tw - k - h)$ equal to 0. By the same logic as for r_o, this agent will also set $h = 0$ and $s > 0$. This describes their ideal policy $q^*(r_y)$.

Observe that setting $t = 1$ and $T = w$ would yield the p_o agent as much utility as $q^*(p_o)$ provided he is able to purchase $h^*(p_o)$ privately. Analogously, setting $t = 1$ and $T = w - k^*(p_y)$ would yield the p_y agent as much utility as $q^*(p_y)$ provided he is able to purchase $h^*(p_y)$ privately. Therefore, in principle these two policies would also be ideal for the p_o and p_y agents, respectively.[10] Notice, that the above would imply that *no* agent in society would ideally want

[10]In a setting where the government is unable to provide any H, these would be the ideal policies for these two groups given the relative prices.

any public spending on the neutral goods. This would be in sharp contrast with the fact that there *is* indeed spending by governments everywhere on several public goods.

One way to rule out the above theoretical possibility would be to assume that the poor agents *ceteris paribus* prefer public provision of the neutral good through taxation over cash transfers (T) with which they can privately obtain them at the same cost. This can be justified by noting that the poor may be sceptical about the quality of such goods in the private market (less confident of challenging private providers in consumer courts, etc.) or that there exist some infinitesimal fixed cost (physical, information, psychological etc.) of accessing these private markets which the poor may find discouraging. There exist several empirical studies which show that recipients prefer to receive benefits in kind over cash transfers (see Liscow and Pershing (2022) among others). This is particularly true in developing countries (see e.g., Khera (2014) in the context of India, and Hirvonen and Hoddinott (2021) in the context of Ethiopia). These studies provide suggestive evidence these choices (of in-kind over cash transfers) are in part driven by self-control concerns.

The key features of the above discussion along with some additional observations are collected in the following lemma.

LEMMA **1.** *The ideal policies of the four segments display the following properties:*

(i) $k^*(p_y)$ *and* $k^*(r_y)$ *are strictly positive;*
(ii) $k^*(p_o) = k^*(r_o) = 0$*;*
(iii) $h^*(p_o) < h^*(p_y)$.

By part (i) of Lemma 1, we have that both $f(k^*(p_y))$ and $f(k^*(r_y))$ are strictly greater than unity. This implies, within each income category, the numeraire consumption of the young can be higher than the old's when each agent is allowed to choose their ideal policy. Recognising the issues surrounding interpersonal utility comparisons, the above statement does *not* mean that the old agents must necessarily be worse-off in comparison to the younger ones in the aggregate — specifically, the possibility that the old agents may be richer than the young ones on average (in terms of what proportion of the cohort earns w_r as opposed to w_p) can be easily accommodated within our framework, although the baseline model assumes identical income distributions for each age cohort.

Part (iii) of the preceding lemma reports an asymmetry among the poor in terms of their ideal public provision of the neutral good — specifically, the young want higher spending than the old do. The idea behind this derives from the following logic: the poor ideally want maximum taxation and the young poor also want positive capital spending as this enables them to augment their numeraire consumption (through acquisition of new skills, etc.). In effect, the young poor

	Poor	Rich
Young	t=1, $k^*(p_y)>0$, $h^*(p_y)>0$	0<t<1, $k^*(r_y)>0$, h=0
Old	t=1, k=0, $h^*(p_o)>0$	t=k=h=0

FIGURE 1. The ideal policies.

are able to enjoy a larger numeraire consumption relative to the old poor. Given the standard homotheticity assumption on preferences, the above implies $h^*(p_o) < h^*(p_y)$. Thus, the capital spending acts like an "income effect" for the young. It is important to bear in mind that this does not necessarily mean that the younger citizens must be "richer" than their older counterparts *in terms of their income endowments*. As mentioned earlier and examined in detail later, the substantive implications of the model are unchanged when we allow for the income distribution to differ by age cohorts so that the old are richer than the young in the sense of first-order stochastic dominance.

By $h^*(r_o) = h^*(r_y) = 0$ and part (iii) of Lemma 1, it is fair to say that the "demand" for public spending on the neutral good from the old agents is actually lower than that from their younger counterparts. *In spite of this, we show that it is possible for the equilibrium public provision of this good to be higher when the old are a majority.*

In the analysis of the political model described below, the focus will be on pure strategy equilibria. To guarantee the existence of pure strategy equilibrium in this general economic environment, we impose the following restrictions on the parameters of the utility function. For $i \in \{p_y, p_o, r_y, r_o\}$, let $v_i(q)$ denote the indirect utility function of i, for any q in the set of feasible policies Q. We will assume that p_o prefers $q^*(p_y)$ over $q^*(r_o)$ and r_o prefers $q^*(r_y)$ over $q^*(p_o)$. In other words, we make the following assumption.

ASSUMPTION 3. (i) $v_{p_o}(q^*(p_y)) > v_{p_o}(q^*(r_o))$, and (ii) $v_{r_o}(q^*(r_y)) > v_{r_o}(q^*(p_o))$.

Notably, part (i) of the above assumption is also made in Levy (2005) where in fact it is assumed that the poor "stick" together so that the young poor prefer the ideal point of the old poor over that of the young rich. We do not impose this latter restriction.

An immediate corollary of Assumption 3 is that a p_o agent prefers $q^*(p_y)$ over $q^*(r_y)$. To see why, note that $v_{p_o}(q^*(r_o)) > v_{p_o}(q^*(r_y))$. This is because $q^*(r_y)$ involves a positive tax rate but no provision of either per-capita transfers (T) or h, while $q^*(r_o)$ has a tax rate of nil, thus enabling the old poor to enjoy a higher numeraire consumption.

Henceforth, we will take Assumptions 1 — 3 as operative unless otherwise stated. Table 1 depicts the ideal policies of the four groups for ease of reference.

We next describe the political process which determines the equilibrium policy for the society.

2.2. **The Political Process.** The political process is essentially the same as the one in Levy (2005) which in turn is based on Levy (2004). The two main features of this process are the *endogenous* formation of parties and the *stability* of the political outcomes. We discuss both features – which are closely inter-related – in some detail below.

As regards a political party's platform, the key idea is that each party can only offer *credible* policies — namely, policies in the Pareto set of its members. By a Pareto set for a party, we mean a set of feasible policies whose elements have the following feature — there is no other feasible policy which leaves all the members of the party weakly better off and some strictly so. When a politician runs as an individual candidate he can only offer his ideal policy, as in the "citizen-candidate" model.[11] This means that if a p_o agent runs as a candidate without forming an alliance with any of the other three segments of the population, then the only platform this agent can credibly offer is $q^*(p_o)$. The same consideration naturally applies to each of the other three segments of society — i.e., r_o, r_y and p_y.

If, however, heterogeneous politicians join together to form a party, then matters are quite different. The Pareto set of such a party is larger than the set of the ideal policies of the individual members. For example, the party of the old rich and the old poor can offer all policies with $k = 0$ and different tax rates, $t \in [0, 1]$ and correspondingly $h \in [0, h^*(p_o)]$. In a similar vein, the party of the old rich and the young rich can offer $t \in [0, t^*(r_y)]$ with $h = 0$ and some level of capital spending ranging from 0 to $k^*(r_y)$, and so on. The details regarding the construction of the Pareto set for each possible coalition is contained in the Appendix. This particular structure on policy platforms of the parties reflects the idea that parties allow different groups to come to (efficient) internal compromises.[12]

The party formation process is the first step towards determining the equilibrium policy outcome(s). Given the two markers in our economy, assume that there are four politicians participating in the political process, each representing a different group of voters. Specifically, politician i has the preferences of group $i \in \{r_o, p_o, r_y, p_y\}$.

[11]See e.g., Besley and Coate (1997) and Osborne and Slivinski (1996).

[12]The assumption about heterogeneous parties rests on the idea that it is relatively easy for a small group of politicians to monitor one another. The population at large can then trust promises which represent internal compromises in the party. Ray and Vohra (1997) analyse a general model in which agreements within coalitions are binding, as in our setup.

Let $\mathbf{\Omega}$ be the set of all possible partitions on the set of politicians $\{r_o, p_o, r_y, p_y\}$. Take any partition $\omega \in \mathbf{\Omega}$. For example, $\omega = p_o|p_y|r_o|r_y$ is the partition in which each politician can only run as an individual candidate. Analogously, the partition $p_op_y|r_o|r_y$ denotes that the poor representatives form a party and each of the rich politicians can run as an individual — hence, there are three potential candidates in this situation. Taking the partition of politicians into parties as given, we proceed to the next step which is the process of election.

In an election all candidates in a given partition simultaneously choose whether to offer a platform and if so, *which* platform in their Pareto set to offer. The entire set of citizens then vote for the platform they like most. The election's outcome is the platform which receives the highest number of votes. If there are ties, then each is chosen with equal probability. If no platform is offered by any candidate, a default status quo is implemented. As is standard, we assume that the status quo is a situation which is worse for all players than any other outcome.

2.2.1. *Equilibrium.* Now we are ready to define the equilibrium set platforms for a given partition. A set of platforms given a partition $\omega \in \mathbf{\Omega}$ constitute an equilibrium when given the other platforms, no party can change its action (offering a different platform from within its Pareto set, by withdrawing altogether, or joining the race) and improve the utility of all its members. In effect, the set of platforms constitute mutual best-responses for every party. Given that the platform with the greatest support is the winner, let $\mathbf{q}^*(\omega)$ denote the set of equilibrium winning platforms for the partition ω.

Unlike Levy (2005), we do *not* however assume the following tie-breaking rule: in equilibrium a party does not offer some platform if, given the other platforms that are offered, all party members are indifferent between offering this platform and not running at all.[13]

We characterise *stable* political outcomes — namely, those equilibrium winning platforms and their associated partition which are robust to politicians changing their party membership. Start with a partition $\omega_0 \in \mathbf{\Omega}$ and identify $\mathbf{q}^*(\omega_0)$, i.e., the set of equilibrium winning platforms associated with it. Take any element of $\mathbf{q}^*(\omega_0)$. Next, consider a situation where a politician or group of politicians choose to split from their party, while the rest of the representatives maintain their party membership. In this new induced partition $\omega_1 \in \mathbf{\Omega}$, a new set of equilibrium winning platforms will arise, namely, $\mathbf{q}^*(\omega_1)$. If the deviant splinter group is able to get a (weakly) higher payoff from *any* element in $\mathbf{q}^*(\omega_1)$, then the original equilibrium winning platform associated with the partition ω_0 does *not* constitute a stable political outcome.

[13]This is not because we believe that the tie-breaking rule is implausible. We simply do so for a technical reason — not assuming this tie-breaking rule guarantees that we have pure-strategy equilibrium platforms for every possible partition.

In other words, a stable political outcome is an equilibrium winning platform such that *no* politician (or a group of politicians) can break their party and receive a (weakly) higher utility from some equilibrium winning platform in the newly induced partition. Thus, it is robust to such individual or collective deviations.[14]

Political parties are endogenous in the model in the sense that we identify the structure of coalitions and political outcomes such that no group of politicians wish to quit their party. In such a setup, endogenous parties – namely, stable coalitions of different representatives – always arise in equilibrium. The core prediction of our model is therefore the set of stable political outcomes with endogenous parties. One can easily identify the winning platform in any given stable political outcome. In what follows, we will analyse the dependence of the winning platform on the economic and demographic factors.

3. Main Results

We aim to demonstrate how income inequality and demographic factors affect the pattern of public spending in this model. By demographic factors, we refer to the relative sizes of the young and old agents in the economy. This is captured succinctly by the size of the young $\theta \in (0, 1)$.

What do we mean by income equality? Given our rather parsimonious set of parameters, we focus on the ratio of the incomes of the rich to that of the poor — hence, $\frac{w_r}{w_p}$ while keeping the mean income w constant. In other words, we focus on mean-preserving spreads as our indicator of increased income inequality. One interesting implication of income inequality is the following. When $\frac{w_r}{w_p}$ is sufficiently low, a p_y agent prefers $q^*(r_y)$ over $q^*(p_o)$; otherwise, the ranking of these policies for p_y is reversed. Intuitively, the age-wise alignment of preference over policies dominates the income-wise alignment of the same for the p_y agents for lower levels of inequality. The following lemma states this more explicitly.

LEMMA **2.** *There exists $\rho^* > 1$ such that as long as $\frac{w_r}{w_p} < \rho^*$, a p_y agent prefers $q^*(r_y)$ over $q^*(p_o)$. For $\frac{w_r}{w_p} > \rho^*$, p_y prefers $q^*(p_o)$ over $q^*(r_y)$.*

In what follows, we will use this threshold ρ^* to demarcate the "low" and "high" economic inequality ranges. We begin our analysis with the case where the young agents are a majority in the economy — i.e. $\theta > \frac{1}{2}$.

3.1. **Young majority ($\theta > \frac{1}{2}$).**

When the young outnumber the old, it is the p_y group which is the largest of the four segments in society. To gain an intuition for the set of stable political

[14]The stability requirement used here is the same as the one in Levy (2005).

outcomes in this scenario, first consider the case when *no* coalitions are possible — i.e., each of the four groups must run alone if they decide to. Clearly, in such a situation the set of policies that may be offered are the ideal policies of the groups; hence, $q^*(p_y)$, $q^*(r_y)$, $q^*(p_o)$ and $q^*(r_o)$.

In this partition – i.e., $p_o|p_y|r_o|r_y$ – there is only one possible equilibrium outcome. An agent from the p_y group runs offering its ideal policy $q^*(p_y)$ and wins since the p_o group supports it (in case any of the rich agents run). To see why this is the unique equilibrium outcome for this partition, consider the following arguments.

If p_o also ran with its ideal policy on offer, it would not win as r_y would support p_y over p_o (and thus p_y would win as $\theta > \frac{1}{2}$). This derives from the following:
(a) $q^*(p_y)$ offers the same level of utility to all young agents since it involves $t = 1$ and $k > 0$;
(b) $q^*(p_o)$ offers the same utility to *all* agents since it involves $t = 1$ and $k = 0$; and
(c) the latter payoff is lower than the former for every young agent — i.e., $v_{j_y}(q^*(p_o)) < v_{j_y}(q^*(p_y))$ for $j \in \{p, r\}$.

If any/both of the rich groups ran, it would not affect the outcome as p_o would support p_y over each rich group since the rich offer $h = 0$ and *no* lump sum income transfer.[15]

Now we ask if allowing coalitions to form can change the above equilibrium outcome. As we demonstrate below, the answer is indeed in the affirmative. However, note that any winning coalition *must* have the support of p_o and r_y. If not, either of these two groups may support p_y and thus form the requisite majority needed for the latter's victory. Moreover, as shown in the Appendix, p_y cannot be part of any coalition because this agent has incentives to break the coalition, run alone and thereby win the election. The p_y agent hence cannot credibly commit to cooperate with other groups. As a result, *any* coalition which wins against p_y must have the support of both p_o and r_y.

This begs the question as to whether there exists some feasible policy which both p_o and r_y prefer over the outcome of $p_o|p_y|r_o|r_y$; otherwise, a coalition including both groups would not be possible. The following lemma addresses this specific question.

LEMMA **3.** *There exists a feasible policy $q \in Q$ such that all agents in p_o, r_y and r_o prefer q over $q^*(p_y)$.*

In the proof of Lemma 3, we demonstrate how by one can construct a feasible policy starting from $q^*(p_y)$ by simultaneously lowering t and k while keeping h at $h^*(p_y)$. The reduction in k should be large enough so that the drop in t does not reduce the overall consumption of the numeraire for p_o. The reduction in t is designed to boost the net consumption of the numeraire

[15] Assumption 3 delivers this.

for r_y in spite of the reduction in k. Finally, r_o is in favour of such a policy as the level of spending on h is pegged at the same level (i.e., $h^*(p_y)$) while the lower tax rate enables a greater consumption of the numeraire.

Building on the above lemma, we now state our first main result.

PROPOSITION **1.** *When the young are a majority, then the following obtain:*

(i) An equilibrium always exists.
(ii) Any winning party is composed of the old poor and some rich representatives.
(iii) The equilibrium tax rate is positive but not unity.
(iv) The public provision of the neutral good is positive but no higher than $h^*(p_o)$.

The proof of Proposition 1 involves three steps. First, we characterise the Pareto set of policies for each possible coalition (i.e., party). Next, we characterise the equilibrium platform(s) for each possible partition. Finally, we are able to identify the stable political outcomes based on the various equilibrium payoffs deduced in the preceding step. The details are documented in the Appendix.

To develop the intuition behind the results in Proposition 1, we revert to the discussion about how any winning coalition necessarily needs to secure the support of the p_o and r_y groups. Now, these two sets of agents must get a payoff above what $q^*(p_y)$ offers them. By Lemma 3, we know that at least one such feasible policy does exist. Hence, one possibility is that they form a party – i.e., p_or_y – and offer some policy from their Pareto set which meets this requirement.[16] As long as this policy from their Pareto set is preferred by the r_o agents to $q^*(p_y)$, this meets the requirement for being an equilibrium policy. Clearly, neither p_o nor r_y stand to gain from splitting the party as then we are back in the $p_o|p_y|r_o|r_y$ world where $q^*(p_y)$ is the only possible outcome.

By a similar logic, it may be possible for all the old agents to form a party – i.e., p_or_o – and offer something from their Pareto set which every old agent and r_y prefer over $q^*(p_y)$. Clearly, such a policy involves a positive tax rate (which is less than unity) and $k = 0$.[17] Again, neither p_o nor r_o would want to break this coalition as that would catapult them into the $p_o|p_y|r_o|r_y$ scenario with $q^*(p_y)$ as the (only possible) outcome.

In both these equilibrium partitions – namely, $p_or_y|r_o|p_y$ and $p_or_o|r_y|p_y$ – the provision of h is positive. This is so as the p_o agents value h and the rich prefer spending tax revenue on h rather than face higher tax rates under the $p_o|p_y|r_o|r_y$ scenario. By the preceding discussion,

[16] The policy constructed in the proof of Lemma 3 actually does not belong to the Pareto set of the party p_or_y. The details are available in the Appendix.
[17] The fact that k must be zero follows from the definition of the Pareto set of the old agents.

it is apparent that the multiplicity of equilibria arises not only from the different partitions but also from the variety of policies in the relevant Pareto sets which meet the equilibrium criteria.

3.1.1. *The effect of income inequality.* By Lemma 2, which side of ρ^* the term $\frac{w_r}{w_p}$ lies on, determines p_y preferences as regards $q^*(p_o)$ and $q^*(r_y)$. The ranking of these two ideal policies by p_y is however not crucial in the case of $\theta > \frac{1}{2}$. This is essentially because the party formation process relies on the exclusion – rather than inclusion – of p_y by enlisting the support of p_o and r_y. Hence, regardless of the value of $\frac{w_r}{w_p}$ vis-a-vis ρ^* the results of Proposition 1 apply.

There is one aspect, however, which *does* depend on income inequality — this concerns the equilibrium level of k. One can sharpen the predictions of Proposition 1 in this regard.

PROPOSITION **2.** *When the young are a majority, the level of k offered in equilibrium depends upon $\frac{w_r}{w_p}$. In particular, when this ratio is sufficiently low (while above unity), $k > 0$ in all equilibrium platforms.*

The main idea behind the above result is the following. Consider a policy of positive taxation and provision of h with $k = 0$ which delivers the old poor agents a payoff higher than what $q^*(p_y)$ offers them. For such a policy to appeal to the young rich agents over $q^*(p_y)$, it must leave them with sufficient disposable income to obtain the requisite amounts of the numeraire good and possibly private spending on the neutral good. In other words, the post-redistribution income for r_y from this policy after netting out the private expenditure (i.e., s) must exceed that from $q^*(p_y)$, which is $f(k^*(p_y))[w - h^*(p_y) - k^*(p_y)]$. For this to be possible, w_r needs to be "sufficiently" high relative to the average income w in order to counteract the effect of $f(k^*(p_y))$.[18] In other words, there is a struggle between w_r and $f(k^*(p_y))$ times w. With continued reduction in income inequality, the scales tip in favour of $f(k^*(p_y))$ times w. Hence, in such a scenario, to keep the rich young's payoff above $v_{r_y}(q^*(p_y))$ some positive level of capital spending *has* to be proposed by the old poor to counteract $k^*(p_y)$.

The implications of income inequality are far more substantial in the case when the old agents form a majority in society. This is what we examine in the next section.

3.2. **Old majority ($\theta < \frac{1}{2}$).** In this scenario, the magnitude of $\frac{w_r}{w_p}$ relative to ρ^* is a crucial determinant of the equilibrium outcome. Taking cognisance of this issue, we analyse each case separately.

[18]Recall, $f(k^*(p_y))$ exceeds unity as $k^*(p_y) > 0$.

3.2.1. *Low Inequality ($\frac{w_r}{w_p} < \rho^*$).* Like in the case of $\theta > \frac{1}{2}$, we will begin with the examination of the case where no coalitions are possible — i.e., $p_o|p_y|r_o|r_y$. To keep the analysis tractable, we make one further assumption.

ASSUMPTION 4. The mass of the rich agents taken together (i.e., r_y and r_o) exceeds that of the poor old agents (p_o).

With the above assumption in place, we are able to characterise for $p_o|p_y|r_o|r_y$ a unique pure strategy equilibrium. In this equilibrium, both p_y and r_y run and the policy which wins is $q^*(p_y)$.[19] Note that p_o does *not* run. If p_o did, then $q^*(p_o)$ would not win since $q^*(r_y)$ would defeat $q^*(p_o)$ given Assumption 4 (as r_o prefers $q^*(r_y)$ over $q^*(p_o)$ by Assumption 3). Knowing this, the p_o agent will not run as (s)he prefers $q^*(p_y)$ over $q^*(r_y)$; hence, it is better for p_o to not offer a platform. Given that both p_y and r_y are running, r_o cannot gain by running. By running, r_o would not affect the outcome – i.e., $q^*(p_y)$ – as p_o would vote for $q^*(p_y)$, as would all the p_y agents.

There is no other equilibrium set of platforms for this partition.[20] Any one agent running while the other three do not is not an equilibrium. Take p_y. If p_y decides to run and nobody else does, then p_o can profitably deviate. This is how – by running, p_o wins the election since r_o prefers $q^*(p_o)$ over $q^*(p_y)$ and the old are a majority. Note, p_o running and nobody else doing so is not an equilibrium as r_y can run and win (with r_o and p_y's support). Similarly, r_y running and nobody else doing so is not an equilibrium either — p_y can run and win (with p_o's support). Finally, observe that r_o as the solitary candidate is not an equilibrium as p_y can run and defeat r_o's platform.

In light of the above, much like in the case of $\theta > \frac{1}{2}$, the equilibrium outcome for $p_o|p_y|r_o|r_y$ is p_y's ideal policy. What is noteworthy is that here p_y manages to win despite being *smaller* than p_o. Given the 'no-party' outcome (i.e., $q^*(p_y)$ winning), the equilibria for the $\theta > \frac{1}{2}$ case immediately become candidate equilibria for this scenario. Before examining that more carefully, we briefly discuss what happens when Assumption 4 is violated.

When the mass of the rich is indeed smaller than that of the old poor, then for $p_o|p_y|r_o|r_y$ having p_y and r_y run is no longer an equilibrium. Observe that here if p_o runs too then the winner will be $q^*(p_o)$ as p_o is larger than either p_y or the rich agents. But then this is not an equilibrium either, as p_y can gain by not running. If p_y does not run then r_y would win with r_o and p_y's support — recall, the p_y agent prefers $q^*(r_y)$ over $q^*(p_o)$ in this scenario. In fact, there is no equilibrium in pure strategies for this situation. There is one in mixed strategies since this 'no-party' game is

[19] This is where *not* imposing the tie-breaking rule in Levy (2005) makes a difference. By that rule, r_y would not run and thus nullify this equilibrium.

[20] To be precise, there is no other equilibrium set of platforms in pure strategies.

finite; however, the details of such an equilibrium is quite dependent on parametric assumptions. Therefore, we prefer to impose Assumption 4 for analytical tractability. We would like to emphasise that the *key* demarcation between the rich and the poor in this model is that the mean income lies below the former's income and above the latter's. Hence, Assumption 4 is quite plausible in most settings particularly when one considers that the old agents are in fact richer on average in reality than their younger counterparts.

We now present the main result as regards the stable political outcomes for this scenario.

PROPOSITION **3.** *When the old are a majority and the level of income inequality is such that* $\frac{w_r}{w_p} < \rho^*$*, then the following obtain:*

(i) An equilibrium always exists.
(ii) Any winning party is composed of the old poor and some rich representatives.
(iii) The equilibrium tax rate is positive.
(iv) The public provision of the neutral good is positive but no higher than $h^*(p_o)$*.*
(v) The provision of capital spending is nil, i.e., $k = 0$*, in some equilibrium platforms.*

Like in the case of Proposition 1, we proceed to identify the stable political outcomes for this scenario by first working out all the equilibria for all possible partitions and then eliminating the ones which have profitable deviations by some agents.

There are several similarities between the set of equilibria in this scenario and the one for the young majority case. The main distinction lies in the equilibrium level of capital spending. As noted in Proposition 2, the level of k is positive for sufficiently low levels of income inequality when the young are a majority, while by part (v) of Proposition 3 we have $k = 0$ in some equilibrium platforms when the old constitute a majority. This difference arises from the fact that now the old agents by themselves can win with $k = 0$ as they constitute a majority — hence, there is no need to ensure (by offering $k > 0$) that r_y agents prefer their party's policy over $q^*(p_y)$.

The situation is altogether different in the case of $\frac{w_r}{w_p} > \rho^*$ with the old being the majority.

3.2.2. *High Inequality (*$\frac{w_r}{w_p} > \rho^*$*).* When $\frac{w_r}{w_p} > \rho^*$, we know – by Lemma 2 – that the p_y agents prefer $q^*(p_o)$ over $q^*(r_y)$. This, in conjunction with the fact that the old are a majority, implies that in the $p_o|p_y|r_o|r_y$ partition it is p_o who will win (with p_y's support if any of the rich agents run). The arguments are basically identical to the corresponding case of $\theta > \frac{1}{2}$ and we omit them for the sake of brevity.

We next examine if allowing coalitions to form can change the equilibrium outcome. It is clear that any winning coalition *must* have the support of p_y and r_o. If not, either of these two groups may support p_o and thus form the requisite majority needed for the latter's victory. Moreover, as discussed in the Appendix, p_o cannot be part of any coalition because this agent has incentives to break the coalition, run alone and thereby win the election. The p_o agent hence cannot credibly commit to cooperate with other groups. As a result, *any* coalition which needs to win against p_o must do so with the support of p_y and r_o. But for that to transpire, one needs to ensure that such a winning policy is indeed feasible. The following lemma argues that is indeed the case.

LEMMA **4.** *There exists a feasible policy $\tilde{q} \in Q$ such that all agents in p_y, r_o and r_y prefer $\tilde{q}$ over $q^*(p_o)$.*

In the proof of Lemma 4, we construct a feasible policy starting from $q^*(p_o)$ by suitably choosing t and k while pegging h at $h^*(p_o)$. The key idea is to ensure that p_y and r_o (individually) are guaranteed a level of numeraire consumption *higher* than what $q^*(p_o)$ delivers to them. Our assumptions on the returns from k to the young – particularly, $f'(0) = +\infty$ and $f'' < 0$ – are sufficient to ensure that this is possible. Moreover, such a policy is also more appealing to r_y over $q^*(p_o)$ as the numeraire consumption delivered to this agent exceeds that to r_o (as $k > 0$ and hence $f(k) > 1$) which, in turn, exceeds the one from $q^*(p_o)$.

Using the lemma above, we proceed to identify the stable political outcomes for this scenario by first working out all the equilibria for all possible partitions and then eliminating the ones which have profitable deviations by some agents. The properties of such equilibrium outcomes are stated in more detail below.

PROPOSITION **4.** *When the old are a majority and the level of income inequality is such that $\frac{w_r}{w_p} > \rho^*$, then the following obtain:*

(i) An equilibrium always exists.
(ii) Any winning party is composed of the young poor and some rich representatives.
(iii) The equilibrium tax rate is positive but not unity.
(iv) The public provision of the neutral good is positive but no higher than $h^*(p_y)$.
(v) The provision of capital spending is positive, i.e., $k > 0$ in all equilibria.

As discussed earlier, for a coalition to be stable it has to have the support of the young poor and the old rich agents. One possibility is that these two groups form a party and offer some policy from their Pareto set which each party member *and* the young rich prefer over the ideal policy of p_o. As the young value k more than the old and $f'(0) = +\infty$, setting $k > 0$ is an efficient way to garner the former's support. To ensure that both sets of rich agents enjoy a level of consumption of the numeraire good above what $q^*(p_o)$ offers, the equilibrium tax rate is less than unity. Clearly, neither the young poor nor the old rich agents have any incentive to break this coalition as doing so results in them receiving lower payoffs respectively from $q^*(p_o)$.

Next, we establish the existence of an equilibrium platform where the level of public spending on H is actually in excess of what the poor old agents would ideally want. The following proposition contains the relevant details.

PROPOSITION **5.** *When the old are a majority and the level of income inequality is such that $\frac{w_r}{w_p} > \rho^*$, then there always exists an equilibrium outcome where $h \in (h^*(p_o), h^*(p_y)]$.*

3.3. **Comparisons in terms of public spending.** Our analysis allows for some comparisons in terms of public provision of the two types of goods across different levels of inequality and demographic composition. In all three cases – i.e., $\theta > \frac{1}{2}$, $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, and $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$ – the stable political outcome in equilibrium is generically not unique. The multiplicity arises not only in terms of the possible partitions but also in terms of the platforms offered in equilibrium. This makes a straightforward comparison of public spending across the different scenarios quite challenging.

Nonetheless, some clear distinctions do emerge. We highlight them below.

First, contrast the case of $\theta > \frac{1}{2}$ with that of $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$. These are more "alike" as the party formations in equilibrium are geared towards the avoidance of the emergence of $q^*(p_y)$ as the equilibrium outcome in these two cases. In fact, *all* equilibria in the case of $\theta > \frac{1}{2}$ except those involving $k > 0$ (in the $p_o r_y r_o | p_y$ partition) are also equilibria in the case of $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$. The ones with $k > 0$ in the $p_o r_y r_o | p_y$ partition are *not* equilibria when $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$ since the $p_o r_o$ group can deviate gainfully to induce the $p_o r_o | r_y | p_y$ partition by keeping the same tax rate h but setting $k = 0$ and adjusting h upwards accordingly. Notice, the old agents would win by breaking away and offering this platform as the old are a majority.[21] Hence, the set of equilibrium outcomes for $\theta > \frac{1}{2}$ and those for $\theta < \frac{1}{2}$ with $\frac{w_r}{w_p} < \rho^*$ differ by only those cases.

The following result sheds light in terms of the differences in public spending across these two different scenarios.

PROPOSITION **6.** *Consider the equilibrium winning platforms for two alternative situations: (a) when the young are a majority and (b) when the old are a majority and $\frac{w_r}{w_p} < \rho^*$ ("low" income inequality). For the stable political outcomes that do not overlap for (a) and (b), the following obtain:*

(i) the level of k is positive in such equilibrium winning platforms in scenario (a) and nil in scenario (b); and

[21]Also, both p_o and r_o gain by this deviation.

(ii) for any equilibrium winning platform in (a) there is a corresponding equilibrium winning platform in (b) with the same tax rate where the level of h is (weakly) higher.

The above result clearly indicates that the level of capital spending tends to be higher when the young are a majority as compared to when the old are a majority with "low" income inequality in *all* the cases where the equilibrium winning platforms differ between the two scenarios. As the young prefer k more than the old, it suggests a positive cohort size effect when income inequality is "low" (recall Proposition 2).

Also, part (ii) of Proposition 6 suggests that the set of equilibrium platforms with the $p_o r_o | r_y | p_y$ partition for the for $\theta < \frac{1}{2}$ with $\frac{w_r}{w_p} < \rho^*$ scenario may involve greater public spending on H as compared to the ones with $k > 0$ in the $p_o r_y r_o | p_y$ partition. In this particular sense, one may claim that the case of the old majority with "low" inequality is associated with a higher level of public provision of H relative to the young majority case. Taken together, Proposition 6 suggests a type of substitution across the two publicly provided goods – i.e., k and H – where income inequality is "low" when one compares the young majority and the old majority cases.

Next, we focus on the old majority case and compare between the "low" and "high" inequality scenarios. The comparison here is more complicated than in the previous situation, as the composition of the winning party is quite different in the two cases. As recorded in Proposition 3, in the "low" inequality case it is the *old* poor and some rich representatives while in the case of "high" inequality, it is the *young* poor and some rich representatives (see Proposition 4). In the latter case, the party formations in equilibrium are geared towards the avoidance of the emergence of $q^*(p_o)$ as the equilibrium outcome. Hence, there is no clear way to compare the set of equilibrium winning platforms in one case with those in the other.

Proposition 5 does, however, provide an important insight in this regard. This proposition establishes that there is a set of equilibrium winning platforms in the "high" inequality scenario where h is greater than in *any* equilibrium under the "low" inequality scenario. In other words, *no* equilibrium winning platform in the "low" inequality scenario can match these levels of public provision of H (described in Proposition 5) by part (iv) of Proposition 3. To be sure, given the multiplicity of equilibria in both scenarios there could be some equilibrium outcome in the "high" inequality scenario where h exceeds that under *some* equilibrium outcome in the "low" inequality scenario. However, on the basis of the upper bound of public spending on H in equilibrium, it is fair to claim – for the old majority case – that the "high" inequality scenario has a greater potential to deliver a greater level of public provision of H than the "low" inequality one.

In terms of the level of capital spending, Proposition 4 tells us that the "high" inequality scenario always delivers a positive level of spending in equilibrium although the same does not apply to the "low" inequality scenario (recall, in particular, the cases where the old form a party and win while offering $k = 0$).

In sum, one may thus stake the following claim: the equilibrium level of public spending on the neutral good associated with the young majority scenario is – on average – lower than that under the old majority one. Additionally, within the old majority scenario, the "high" inequality case has a greater potential to deliver a greater level of public provision of H than the "low" inequality one. Thus, income inequality may actually *not* be detrimental for public spending. While this appears to be similar to Levy (2005), there is an important distinction: we have this result in a setting where public spending on *both* the neutral and the age-specific goods is feasible. Recall, public spending on a neutral good is not possible in Levy (2005).

As regards the level of capital spending, we can have a positive cohort size effect when income inequality is "low" but not necessarily so when income inequality is "high". In contrast, Levy (2005) uncovers a negative cohort size with respect to public provision of education *regardless of the level of income inequality*.

Overall, we have that the effect of age (resp., inequality) on equilibrium public spending can go in any direction based on the underlying level of inequality (resp., age). In Levy (2005), the ambiguity is only about the effect of income inequality on public spending, and not for age cohort sizes. Again, the role of the neutral good here is a key factor in bringing about the distinctions between the results in Levy (2005) and ours.

3.4. **Income distribution by age cohorts.** In the baseline model, we assumed that the distribution of incomes among the young agents coincides with that among the old ones. This was done for simplicity and is not strictly necessary for our results. We can allow for the old agents to have a higher proportion of rich individuals relative to the young agents. As long as the poor old agents outnumber the rich old agents, nothing in our analysis is altered. In fact, by allowing this we may ensure that Assumption 4 is more easily satisfied.

When the old are richer than the young on average, it implies that for some values of θ lower than $\frac{1}{2}$ but "close" p_y may still be the largest (sub)group just like in the $\theta > \frac{1}{2}$ case. This possibility, however, does not change the equilibrium outcome for the either the "high" inequality or the "low" inequality scenario. The "low" inequality scenario is perhaps obvious as the $p_y|p_o|r_y|r_o$ partition in that case still leads to $q^*(p_y)$ as the equilibrium outcome. In the case of "high" inequality, the following transpires in the $p_y|p_o|r_y|r_o$ case: the p_o agent runs and wins with the ideal policy $q^*(p_o)$. Although p_y is larger than p_o, the former cannot run and win against the latter as r_o would support $q^*(p_o)$ over $q^*(p_y)$; this guarantees p_o's victory given that the old are a majority. Thus, nothing of substance is altered.

3.5. **Preferences over the neutral good.** In the baseline model, we assumed that the preference over the neutral good did not vary either by age or by income. In other words, the utility from consuming a given amount of H is the same regardless of age or income. While this assumption considerably simplifies the analysis, it can be relaxed to some extent without affecting the main conclusions in any significant way.[22] We demonstrate this by the following change to the utility functions of the different agents.

Let the utility of agent i_j be given by $u(x, \alpha_j^i H)$ for $i \in \{p, r\}$ and $j \in \{o, y\}$, where $\alpha_j^i > 0$, $\alpha_o^p = 1$, and u satisfies Assumption 1 as before.

We show that if α_j^i is sufficiently close to 1, either from above or below, then all the results are substantively unchanged. The formal results are collected in the appendix. Specifically, Lemma 1 is modified as follows.

LEMMA **5.** *The ideal policies of the four subgroups are as in Lemma 1 as long as α_y^p is sufficiently close to* 1.

Lemma 2 is modified as follows.

LEMMA **6.** *The statement in Lemma 2 applies as long as α_y^p is sufficiently close to α_y^r.*

And, Proposition 4 is modified as follows

PROPOSITION **7.** *The statement in Proposition 4 applies as long as α_y^p is sufficiently close to* 1.

The intuition behind the analysis is the following: as long as the differences in value for H are small enough, the ideal policies of the four subgroups are unchanged. In particular, the "income effect" accruing to p_y from $k^*(p_y) > 0$ relative to p_o ensures that $h^*(p_y) > h^*(p_o)$ as before. In other words, the difference between α_o^p and α_y^p has to be small so that the "income effect" is not overturned. Now, the ideal policies remaining the same implies that the Pareto sets are unaffected. This essentially ensures that the equlbrium outcomes described in all the propositions are unchanged.

3.6. **The role of the neutral good.** In the model, the neutral good plays an important role. For one, it is used as compensation to guarantee that the value of a policy is sufficiently high to some particular groups of agents when the level of capital spending is low in order to maintain their

[22]We are grateful to an anonymous referee for suggesting this investigation.

support. However, the neutral good is *not* central to generating the positive cohort size effect for the low inequality case. To see this more clearly, consider the setup without any neutral good. So, the utility for each agent depends *solely* on the numeraire consumption.

Here, the ideal policies of the poor are altered slightly in that there is no spending on H.[23] But that aside, there is no substantial change. Taking this in account, one could pursue the core logic to observe that none of the main results, i.e., Propositions 1 – 4, are substantively altered. The arguments regarding the coalition formation and the political equilibria still apply in this setting. Notice, both Lemma 3 and Lemma 4 rely on pegging the spending on H to a certain level and then adjusting t and k to forge the coalition. The main reason behind this is the lack of any material change in the ideal policies of the four subgroups.

The basic motivation for having the neutral good in this model is to highlight how spending on it is affected by the age and inequality dimensions. In that sense, the neutral good is more of an object than a prime mover.

3.7. **The sizes of the four subgroups.** In the model, we maintain the assumption that none of the four subgroups — namely, p_y, p_o, r_y, and r_o — constitute a majority. This is in line with Levy (2005), and it rules out the possibility that any one of these sections of the population dictate public policy. That said, it is worthwhile exploring the (potentially discontinuous) effects on policy as any of these groups shift from being a majority to a minority.

In the interest of brevity, we only dicuss the case of the young poor (p_y) below. The other cases can be analysed in an analogous fashion. If θ is sufficiently close to 1 then the mass of p_y will exceed $\frac{1}{2}$. In this situation, the equilibrium policy will be $q^*(p_y)$, so that $k = k^*(p_y) >$ and $h = h^*(p_y) > 0$. This is so regardless of the level of inequality. Observe, a shift in population which makes p_y a minority while retaining $\theta > \frac{1}{2}$ will lead us to the equilibrium policy described in Proposition 1. Now suppose inequality is high. Then by Proposition 2, it may be that the equilibrium provision of k is actually nil. Therefore, if income inequality is sufficiently high then a discontinuity in terms of spending on k is possible as the mass of p_y crosses the $\frac{1}{2}$ threshold. If income inequality is sufficiently low, then by Proposition 2 we know that $k > 0$ will result in equilibrium even when p_y a minority as long as $\theta > \frac{1}{2}$. Here too, it is possible that there is a discontinuous drop in k (from $k^*(p_y)$) around the $\frac{1}{2}$ threshold for mass of p_y.

[23]The ideal policies of the rich are unchanged.

4. Conclusion

In this paper, we show how economic inequality and the age composition of the electorate may affect the level and pattern of public spending in democracies. As discussed above, the large body of empirical studies that exists in public economics yields no definitive answer for either factor (age or economic inequality).

Our analysis yields that the effects of one factor on public spending relies quite heavily on the ambient level of the other factor. Hence, to expect an unambiguously positive or negative effect of age or economic inequality on any kind of public spending (neutral or age-specific) would be misleading. Essentially, the presence of two different types of public spending alongside income transfers presents a real trade-off; this is particularly so for the young poor agents. If income inequality is sufficiently low then the young poor agents may prefer to align with the young rich rather than the old poor agents. This makes a critical difference to the equilibrium political alliances. And *that* is the main driving force behind our results.

Our theoretical findings underline the need to simultaneously account for inequality, age *and* their interaction in the determination of public spending not just on age-specific goods (like education) but also neutral goods and services in empirical studies. In a way, our major contribution is to provide a lens through which one can rationalise and reconcile the often conflicting stylised empirical results documenting the cohort size effect and the effect of inequality on education, healthcare and other types of public spending in the extant literature. Note, we generate these predictions from a setup which builds on the well-known frameworks of Epple and Romano (1996) and Levy (2005) — this facilitates our understanding of the mechanisms at play.

While our findings are directly relevant to democracies where party formation is not prohibited (either by law or other socio-economic factors), the core question of the impact of age and inequality on public spending readily extends to non-democracies/weak democracies. The treatment of this issue is, however, beyond the scope of this present work. Similarly, one could envisage other divisions in society – like ethnicity and religion – which could affect public spending patterns in democracies or otherwise. These exciting avenues remain open to be explored in future work.

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APPENDIX

Proof. [LEMMA 1.]

Parts (i) and (ii) have been established in the main body.

For part (iii), first note that the standard two-good utility maximising condition will apply for both $q^*(p_y)$ and $q^*(p_o)$. Specifically, $u_x(x^*(p_y), h^*(p_y)) = u_H(x^*(p_y), h^*(p_y))$ for p_y and $u_x(x^*(p_o), h^*(p_o)) = u_H(x^*(p_o), h^*(p_o))$ for p_o since the price of H equals that of x and the solutions are interior.

Now, as p_y prefers $q^*(p_y)$ over $q^*(p_o)$ (by definition), it follows

$$u(f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)], h^*(p_y)) > u(w - h^*(p_o), h^*(p_o)).$$

Suppose $h^*(p_y) \leq h^*(p_o)$. Then $f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)] > w - h^*(p_o)$. As $u_{xx} < 0$ and $u_{xH} \geq 0$, then given $h^*(p_y) \leq h^*(p_o)$ it must be that

$$u_x(f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)], h^*(p_y)) < u_x(w - h^*(p_o), h^*(p_o)).$$

By the first-order conditions then it follows that

$$u_H(f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)], h^*(p_y)) < u_H(w - h^*(p_o), h^*(p_o)).$$

As $f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)] > w - h^*(p_o)$ and $u_{HH} < 0$, the above relation implies $h^*(p_y) > h^*(p_o)$. This contradicts the initial supposition and completes the proof. ■

Proof. [LEMMA 2.]

Consider $q^*(r_y)$ and $q^*(p_o)$. As r_y strictly prefers the former over the latter, we have

$$u(f(k^*(r_y))[w_r(1 - t^*(r_y)) - s^*(r_y)], s^*(r_y)) > u(w - h^*(p_o), h^*(p_o)).$$

Let $w' = \frac{w_r}{\rho}$ where $\rho > 1$. Now consider $u(f(k^*(r_y))[w'(1 - t^*(r_y)) - s^*(r_y)], s^*(r_y))$ and $u(w - h^*(p_o), h^*(p_o))$. Clearly, for $\rho \to 1$,

$$u(f(k^*(r_y))[w'(1 - t^*(r_y)) - s^*(r_y)], s^*(r_y)) > u(w - h^*(p_o), h^*(p_o)).$$

Now, let s' denote the optimal choice for the maximisation of $u(f(k^*(r_y))[w'(1-t^*(r_y))-s], s)$. Thus, for ρ sufficiently close to 1,

$$u(f(k^*(r_y))[w'(1 - t^*(r_y)) - s'], s') > u(w - h^*(p_o), h^*(p_o)).$$

Note, $u(f(k^*(r_y))[w'(1 - t^*(r_y)) - s'], s')$ is monotonically decreasing in ρ with its value approaching 0 as $\rho \to \infty$. Hence, there exists $\rho^* > 1$ such that

$$u(f(k^*(r_y))[w_r(1 - t^*(r_y))/\rho^* - s'(\rho^*)], s'(\rho^*)) = u(w - h^*(p_o), h^*(p_o)).$$

Hence, $\frac{w_r}{w_p} > (<)\rho^*$ implies $v_{p_y}(q^*(r_y)) < (>)v_{p_y}(q^*(p_o))$. ■

Proof. [LEMMA 3.]

Start with policy $q^*(p_y)$. Consider a policy $q \in Q$ with $t' \in (0,1)$, $k' \in (0, k^*(p_y))$ and $h = h^*(p_y)$ so the numeraire consumption of p_o is higher than $w - k^*(p_y) - h^*(p_y)$. Hence, we need to ensure that

$$(1-t')w_p + t'w - k' - h^*(p_y) > w - k^*(p_y) - h^*(p_y).$$

Let $t'w - k' = w - k^*(p_y)$. Observe that, by construction, p_o prefers this policy over $q^*(p_y)$.

If we can show that for this q, the numeraire consumption of r_y is greater than $f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)]$, then the proof is complete. The numeraire consumption of r_y from q is

$$f(k')[(1-t')w_r + t'w - k' - h^*(p_y)] = f(k')[(1-t')w_r + w - k^*(p_y) - h^*(p_y)].$$

By using $k' = k^*(p_y) - w(1-t')$, we can rewrite the above as

$$f(k^*(p_y) - w(1-t'))[(1-t')w_r + w - k^*(p_y) - h^*(p_y)].$$

Let $Z(t) \equiv f(k^*(p_y) - w(1-t))[(1-t)w_r + w - k^*(p_y) - h^*(p_y)]$.
Observe that $Z(1) = f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)]$.

Consider the problem of choosing t to maximise $Z(t)$. Straightforward differentiation yields:

$$Z'(t) = wf'(k^*(p_y) - w(1-t))[(1-t)w_r + w - k^*(p_y) - h^*(p_y)] - w_r f(k^*(p_y) - w(1-t)).$$

$$Z''(t) = w^2 f''(k^*(p_y) - w(1-t))[(1-t)w_r + w - k^*(p_y) - h^*(p_y)] - 2w_r w f'(k^*(p_y) - w(1-t)).$$

Clearly, $Z'' < 0$ as $f' > 0$ and $f'' < 0$ implying that Z is concave in t. Note that

$$Z'(1) = wf'(k^*(p_y))[w - k^*(p_y) - h^*(p_y)] - w_r f(k^*(p_y)) < 0$$

as $f'(k^*(p_y))[w - k^*(p_y) - h^*(p_y)] = f(k^*(p_y))$ by the definition of $q^*(p_y)$. Hence, by continuity, $\exists \epsilon > 0$ such that $\forall t \in (1-\epsilon, 1)$,

$$Z(t) > Z(1) = f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)].$$

Choosing t' from this $\epsilon-$ interval ensures that r_y prefers q over $q^*(p_y)$.

Finally, note that r_o prefers q over $q^*(p_y)$ as both policies offer the same h while

$$(1-t')w_r + t'w - k' - h^*(p_y) > (1-t')w_p + t'w - k' - h^*(p_y) > w - k^*(p_y) - h^*(p_y)$$

guarantees a higher level of the numeraire good. ■

For the proofs of the main Propositions we go through a set of steps – similar to Levy (2005) – in order to identify the stable political outcomes.

Step 1: Pareto sets of all possible parties.

We will denote the Pareto set of party i by $PS(i)$.

Given the Pareto set of any two groups, the rest (i.e., the Pareto set of three groups) follows from the union of all bilateral Pareto sets.

Consider the party $p_o r_y$.

$$PS(p_o r_y) = \{(t, k, h) \in Q : k \leq k^*(r_y), h \leq h^*(p_o)\}.$$

As p_o prefers $k = 0$ over $k > 0$ and $q^*(r_y)$ has $k = k^*(r_y)$, $PS(p_o r_y)$ cannot have k any higher. Similarly, $h \leq h^*(p_o)$ as both groups are better off switching to $h^*(p_o)$ from $h > h^*(p_o)$. To see why focus on the numeraire consumption of each group. Let $h' > h^*(p_o)$. The numeraire consumption is given by

$$x_{r_y}(t, k, h') = [(1-t)w_r + tw - k - h' - s]f(k)$$

and

$$x_{p_o}(t, k, h') = (1-t)w_p + tw - k - h'.$$

Now consider reducing h to $h^*(p_o)$ while keeping t and k unchanged.

Clearly, $x_{p_o}(t, k, h^*(p_o)) > x_{p_o}(t, k, h')$ and this increment in x is matched by a one-for-one reduction in h. Note, this change leaves p_o better off by Assumption 1 since for p_o

$$\left.\frac{u_x}{u_H}\right|_{(t,k,h')} > \left.\frac{u_x}{u_H}\right|_{(t,k,h^*(p_o))} > \left.\frac{u_x}{u_H}\right|_{q^*(p_o)}.$$

Observe that r_y is indifferent between the two policies as s can be adjusted upwards for the drop in h. This rules out $h > h^*(p_o)$ for $PS(p_o r_y)$.

Consider the party $p_y r_o$.

$$PS(p_y r_o) = \{(t, k, h) \in Q : k \leq k^*(p_y), h \leq h^*(p_y)\}.$$

As r_o prefers $k = 0$ over $k > 0$ and $q^*(p_y)$ has $k = k^*(p_y)$, $PS(p_y r_o)$ cannot have k any higher. Similarly, $h \leq h^*(p_y)$ as both groups are better off switching to $h^*(p_y)$ from $h > h^*(p_y)$. To see why focus on the numeraire consumption of each group. Let $h'' > h^*(p_y)$. The numeraire consumption is given by

$$x_{r_o}(t, k, h'') = (1-t)w_r + tw - k - h'' - s$$

and

$$x_{p_y}(t,k,h'') = [(1-t)w_p + tw - k - h'']f(k).$$

Now consider reducing h to $h^*(p_y)$ while keeping t and k unchanged.

Clearly, $x_{p_y}(t,k,h^*(p_o)) > x_{p_y}(t,k,h'')$ and this increment in x is no less than reduction in h as $f(k) \geq 1$. Note, this change leaves p_y better off by Assumption 1 since for p_y

$$\left.\frac{u_x}{u_H}\right|_{(t,k,h'')} > \left.\frac{u_x}{u_H}\right|_{(t,k,h^*(p_y))} > \left.\frac{u_x}{u_H}\right|_{q^*(p_y)}.$$

Observe that r_o is indifferent between the two policies as s can be adjusted upwards for the drop in h. This rules out $h > h^*(p_y)$ for $PS(p_y r_o)$.

Consider the party $p_y p_o$.

$$PS(p_y p_o) = \{(t,k,h) \in Q : t = 1, k \leq k^*(p_y), h \in [h^*(p_o), h^*(p_y)]\}.$$

As any poor agent prefers t as high as possible, $PS(p_y p_o)$ must have $t = 1$. Given that p_o wants k as low as possible and p_y wants it no higher than $k^*(p_y)$, the level of k in $PS(p_y p_o)$ must be as stated above. By the definition of $q^*(p_o)$ it is clear that h cannot be lower than $h^*(p_o)$. The arguments made for the case of $PS(p_y r_o)$ may be used here to justify the upper bound on h.

Consider the party $r_y r_o$.

$$PS(r_y r_o) = \{(t,k,h) \in Q : t \in [0, t^*(r_y)], k \leq k^*(r_y) \equiv t^*(r_y)w, h = 0\}.$$

Every rich agent prefers t as low as possible and similarly for h. The r_y agent ideally prefers $k^*(r_y) = t^*(r_y)w > 0$ from the definition of $q^*(r_y)$. These considerations define the features of $PS(r_y r_o)$.

Consider the party $p_y r_y$.

$$PS(p_y r_y) = \{(t,k,h) \in Q : t > 0, k \leq max\{k^*(p_y), k^*(r_y)\}, h \leq h^*(p_y)\}.$$

As every young agent ideally prefers $k > 0$, it follows that t and k should be as above. Additionally, as r_y would prefer to keep h as low as possible (so as to keep t down), it follows that $h \leq h^*(p_y)$.

Consider the party $p_o r_o$.

$$PS(p_o r_o) = \{(t,k,h) \in Q : k = 0, h \leq h^*(p_o)\}.$$

As the old agents do not benefit from k, it follows that $k = 0$ in $PS(p_o r_o)$. Additionally, as r_y would prefer to keep h as low as possible (so as to keep t down), it follows that $h \leq h^*(p_o)$.

Step 2: The equilibria for each partition.

We have discussed the case of $p_y|p_o|r_y|r_o$ for different values of θ in the main body. Here we turn to all other possible partitions.

Only one party with two members:

Consider $p_yp_o|r_y|r_o$. The "poor" party wins with those policies in $PS(p_yp_o)$ which each of their members prefer to the ideal policy of either rich group. Such policies always exist. For $\theta > \frac{1}{2}$ and for $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, the policy $q^*(p_y)$ satisfies the requirement. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the policy $q^*(p_o)$ satisfies the requirement.

Consider $r_yr_o|p_y|p_o$. When either $\theta > \frac{1}{2}$ or $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, p_y wins against the "rich" party with its ideal policy $q^*(p_y)$. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the policy $q^*(p_o)$ wins.

Consider $p_yr_y|p_o|r_o$. For $\theta > \frac{1}{2}$, the party p_yr_y wins with all policies in $PS(p_yr_y)$ which r_y agents prefer over $q^*(r_o)$ and p_y agents prefer over $q^*(p_o)$. For $\frac{w_r}{w_p} < \rho^*$, $q^*(r_y)$ is such a policy. If such a policy does not exist when $\frac{w_r}{w_p} > \rho^*$, then p_o runs alone and wins with p_y's support. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, the equilibrium platforms for the $\theta > \frac{1}{2}$ case constitute the equilibria. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the party p_yr_y wins with all policies in $PS(p_yr_y)$ which r_o agents prefer over $q^*(p_o)$. The existence of such a policy is documented in Lemma 4.

Consider $p_or_o|p_y|r_y$. For $\theta > \frac{1}{2}$, the party p_or_o offers a policy in $PS(p_or_o)$ which the r_y and the p_o agents prefer over $q^*(p_y)$. If such a policy does not exist, then p_y runs alone and wins. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, the party p_or_o offers a policy in $PS(p_or_o)$ which the p_o agents prefer over $q^*(p_y)$. Such a policy exists as shown in part (v) of Proposition 3. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the party p_or_o wins with $q^*(p_o)$.

Consider $p_or_y|p_y|r_o$. For $\theta > \frac{1}{2}$, the party p_or_y offers a policy in $PS(p_or_y)$ which the party members and the r_o agents prefer over $q^*(p_y)$. The existence of such a policy is shown in part (iv) of Proposition 1. The same policies are also equilibria for $\theta > \frac{1}{2}$ and for $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the party p_or_y wins with $q^*(p_o)$.

Consider $p_yr_o|p_o|r_y$. For $\theta > \frac{1}{2}$, the party p_yr_o offers $q^*(p_y)$ and wins. Other policies in $PS(p_yr_o)$ which p_y and r_y agents prefer over $q^*(p_o)$ are equilibrium platforms too. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, the party p_yr_o offers $q^*(p_y)$ and wins. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the party p_yr_o wins with all policies in $PS(p_yr_o)$ which r_y agents prefer over $q^*(p_o)$. The existence of such a policy is documented in Lemma 4.

Two parties with two members each:

Consider $p_yp_o|r_yr_o$. The "poor" party always wins with all their policies in $PS(p_yp_o)$.

Consider $p_or_o|p_yr_y$. For $\theta > \frac{1}{2}$, the party p_yr_y must win. In particular, $q^*(p_y)$ is an equilibrium winning platform. For $\theta < \frac{1}{2}$, the party p_or_o must win. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$, the party

$p_o r_o$ offers a policy in $PS(p_o r_o)$ which the p_o agents prefer over $q^*(p_y)$. Such a policy exists as shown in part (v) of Proposition 3. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the party $p_o r_o$ wins by offering the policy $q^*(p_o)$.

Consider $p_y r_o | p_o r_y$. Take any q which lies in $PS(p_y r_o) \cap PS(p_o r_y)$ with $t, k, h > 0$. Any party (or both parties) offering such a q is an equilibrium. To see why, note it is not possible for either party to deviate to a different q' in their Pareto set which will improve the utility of *both* types of members.

Only one party with three members:

Consider $p_o r_o r_y | p_y$. For $\theta > \frac{1}{2}$, the party $p_o r_o r_y$ offers a policy in $PS(p_o r_o r_y)$ which all the party members prefer over $q^*(p_y)$. The existence of such a policy is shown in part (iv) of Proposition 1. The same policies are also equilibria for $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$. For $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$, the party $p_o r_o r_y$ offering $q^*(p_o)$ (and thereby winning) is an equilibrium aside from the ones outlined above.

Consider $p_y r_o r_y | p_o$. For $\theta > \frac{1}{2}$, the party $p_y r_o r_y$ offers a policy in $PS(p_y r_o r_y)$ which all the party members prefer over $q^*(p_o)$. The existence of such a policy is shown in Lemma 4. Additionally, the party $p_y r_o r_y$ offering $q^*(p_y)$ is also an equilibrium. The same policies – except $q^*(p_y)$ – are also equilibria for $\theta < \frac{1}{2}$.

Consider $p_o p_y r_o | r_y$. The party $p_o p_y r_o$ wins with all policies which the poor prefer over $q^*(r_y)$ (e.g., $q^*(p_y)$) or r_y wins with $q^*(r_y)$.

Consider $p_o p_y r_y | r_o$. The party $p_o p_y r_y$ wins with all policies which the poor prefer over $q^*(r_o)$ (e.g., $q^*(p_y)$) or r_o wins with $q^*(r_o)$.

As can be seen, several partitions have multiple equilibrium outcomes. We now examine how many are robust to deviations by one or more members of a party.

Step 3: Stable political outcomes.

Case (1): $\theta > \frac{1}{2}$

Whenever p_y is a member of a party then it is not stable as p_y will break to run alone and win. When the rich agents form the party, then again p_y wins, so this party is not stable either. The partition in which $r_o p_o$ is the only party may be stable provided they can offer a policy from their Pareto set which r_y prefer over $q^*(p_y)$. The partition in which $r_y p_o$ is the only party is stable as they can offer a policy in their Pareto set which they and r_o prefer over $q^*(p_y)$.

Consider the partition $p_y r_o | p_o r_y$. Take any equilibrium policy from that partition — call it q. Now, if $v_{r_o}(q) \leq v_{r_o}(q^*(p_y))$ then r_o breaks away as there is an equilibrium platform for $p_y | r_o | p_o r_y$ which provides to r_o more utility than $v_{r_o}(q^*(p_y))$. In fact, as long as such a policy exists in $PS(p_o r_y)$ which guarantees the $p_o r_y$ members a payoff more than what $q^*(p_y)$ offers, and r_o more than $v_{r_o}(q)$, r_o will choose to break away. Suppose there is actually no such policy in $PS(p_o r_y)$. This implies that any policy which r_o prefers over q delivers lesser utility than $q^*(p_y)$ to either or both of p_o and r_y. W.l.o.g, let p_o be the one getting strictly lower utility. Then p_o can break away and induce the partition $p_y r_o|p_o|r_y$ with $q^*(p_y)$ being offered by $p_y r_o$. Thus, $p_y r_o|p_o r_y$ is not stable.

Finally, $p_o r_o r_y|p_y$ is stable with the same policies as in the case of $r_y p_o|r_o|p_y$.

Case (2): $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$

Here, considerations similar to the case of $\theta > \frac{1}{2}$ apply since in the $r_y|p_o|r_o|p_y$ case, it is p_y who wins. So, $r_y p_o|p_y|r_o$ is stable as the party $r_y p_o$ can offer a policy in their Pareto set which they and r_o prefer over $q^*(p_y)$. The partition in which $r_o p_o$ is the only party is stable as they can offer a policy from their Pareto set which they prefer over $q^*(p_y)$.

Note, $p_y r_o|p_o r_y$ is not stable for exactly the same reasons as in Case (1) above.

Finally, $p_o r_o r_y|p_y$ is stable with the same policies as in the case of $r_y p_o|r_o|p_y$ only if $k = 0$ in those policies. Otherwise, $p_o r_o$ will break away and set $k = 0$ for those same policies and win as the old are a majority.

Case (3): $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} > \rho^*$

Whenever p_o is a member of a party then it is not stable as p_o will break to run alone and win. When the rich agents form the party, then again p_o wins, so this party is not stable either. The partition in which $r_y p_y$ is the only party may be stable provided they can offer a policy from their Pareto set which r_o prefer over $q^*(p_o)$. The partition in which $p_y r_o$ is the only party is stable as they can offer a policy in their Pareto set which they and r_y prefer over $q^*(p_o)$.

Consider the partition $p_y r_o|p_o r_y$. Take any equilibrium policy from that partition — call it q. Now, if $v_{r_y}(q) \leq v_{r_y}(q^*(p_o))$ then r_y breaks away as there is an equilibrium platform for $p_o|r_y|p_y r_o$ which provides r_y more utility than $v_{r_y}(q^*(p_o))$. In fact, as long as such a policy exists in $PS(p_y r_o)$ which guarantees the $p_y r_o$ members a payoff more than what $q^*(p_o)$ offers, and r_y more than $v_{r_y}(q)$, r_y will choose to break away. Suppose there is actually no such policy in $PS(p_y r_o)$. This implies that any policy which r_y prefers over q delivers lesser utility than $q^*(p_o)$ to either or both of p_y and r_o. W.l.o.g, let p_y be the one getting strictly lower utility. Then p_y can break away and induce the partition $p_o r_y|p_y|r_o$ with $q^*(p_o)$ being offered by $p_o r_y$. Thus, $p_y r_o|p_o r_y$ is not stable.

Finally, $p_y r_o r_y|p_o$ is stable with the same policies as in the case of $p_y r_o|r_y|p_o$.

Proof. [PROPOSITION 1.]

Parts (i) — (iii) follow from the arguments in Case (1) under Step 3 above.

(iv) Consider all possible alliances of the old poor and some of the rich, i.e., $p_o r_y$, $p_o r_o$ and $p_o r_y r_o$. The maximum level of h across the Pareto sets of these parties is $h^*(p_o)$. Now we show that there exists a feasible policy q' with $h \leq h^*(p_o)$ such that $v_i(q') > v_i(q^*(p_y))$ for $i \in \{p_o, r_o, r_y\}$.

Start with $q \in Q$ from Lemma 3. Hence, $t \in (0,1)$, $k \in (0, k^*(p_y))$ and $h = h^*(p_y)$. Now consider $q_1 \in Q$ with the same tax rate and k as q but with $h = h^*(p_0)$. Given t is unchanged, q_1 offers p_o more of the numeraire but less of h (by the same amount) than q. Hence, p_o prefers q_1 over q as

$$\left.\frac{u_x}{u_H}\right|_q > \left.\frac{u_x}{u_H}\right|_{q_1} > \left.\frac{u_x}{u_H}\right|_{q^*(p_o)} = 1.$$

Therefore, $\exists t' < t$ such that

$$v_{p_o}(q_1) > v_{p_o}(t', k, h^*(p_0)) = v_{p_o}(q) > v_{p_o}(q^*(p_y)).$$

Denote this policy $(t', k, h^*(p_0))$ by q'.

Now consider r_y. As u is homothetic it follows that $s > 0$ for r_y under q since by Lemma 3

$$f(k)[(1-t)w_r + tw - k - h^*(p_y)] > f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)].$$

As $t' < t$, it means that

$$f(k)[(1-t')w_r + t'w - k - h^*(p_o)] > f(k)[(1-t)w_r + tw - k - h^*(p_y)].$$

Hence, r_y will increase the consumption of s under q' as compared to q. So, r_y gets a combination of lower h and more s where the former is relatively more expensive for the rich than the latter. Hence, r_y prefers q' over q.

Identical arguments apply to r_o and hence we can claim that r_o too prefers q' over q. ∎

Proof. [PROPOSITION 2.]

Denote a candidate policy which every old agent and r_y prefer over $q^*(p_y)$ by $(t' \in (0,1), h', k' = 0)$. By Proposition 1, $h' \leq h^*(p_o) < h^*(p_y)$. Let s denote the private spending on h by r_y. As r_y prefers this over $q^*(p_y)$, it must be that

$$(1-t')w_r + t'w - h' - s > f(k^*(p_y))[w - h^*(p_y) - k^*(p_y)],$$

by the logic in Lemma 1 part (iv). Similarly, $h' + s \geq h^*(p_y)$ implying $s > 0$.

Clearly, as $\frac{w_r}{w_p} \to 1$, it follows that $w_r + t'(w - w_r) \to w$. Moreover,

$$w - h^*(p_y) < f(k^*(p_y))[w - h^*(p_y) - k^*(p_y)]$$

since $h^*(p_o) < h^*(p_y)$ and

$$w - h^*(p_o) < f(k^*(p_y))[w - h^*(p_y) - k^*(p_y)]$$

by the definition of $q^*(p_y)$. This implies

$$w - h' - s < f(k^*(p_y))[w - h^*(p_y) - k^*(p_y)].$$

Hence, for $\frac{w_r}{w_p} > 1$ but sufficiently close to 1

$$w_r + t'(w - w_r) - h' - s < f(k^*(p_y))[w - h^*(p_y) - k^*(p_y)].$$

Thus, $(t' \in (0,1), h', k' = 0)$ cannot simultaneously guarantee every old agent and r_y a payoff over what $q^*(p_y)$ offers for such $\frac{w_r}{w_p}$. Since the choice of t' was arbitrary, we have $k > 0$ for such levels of income inequality. ■

Proof. [PROPOSITION 3.]

Parts (i) — (iii) follow from the arguments in Case (2) under Step 3 above. For part (iii), note that it is possible to have $t = 1$ with the equilibrium winning platform being $q^*(p_o)$ as all the old agents prefer this over $q^*(p_y)$, $q^*(p_o) \in PS(p_o r_o)$ and the old are a majority. Part (iv) comes from Step 1.

Part (v): We will show that $p_o r_o | p_y | r_y$ is a stable equilibrium partition where $p_o r_o$ wins.

Consider $q' \in Q$ from the proof of part (iv) of Proposition 1. Recall $q' = (t', k > 0, h^*(p_0))$ such that $v_i(q') > v_i(q^*(p_y))$ for $i \in \{p_o, r_o, r_y\}$. Consider a policy $q'' \equiv (t', k = 0, h^*(p_0))$. Note, by construction, $q'' \in PS(p_o r_o)$. Moreover, for $i \in \{p_o, r_o\}$, we have

$$v_i(q'') > v_i(q') > v_i(q^*(p_y)).$$

As $\theta < \frac{1}{2}$, it follows that q'' is the winning platform in this partition. This establishes that $k = 0$ in some equilibria. ■

Proof. [LEMMA 4.]

Recall $q^*(p_o)$ delivers the same level of the numeraire (i.e., $w - h^*(p_o)$) and the same level of h (i.e., $h^*(p_o)$) to all agents. We now show that $\exists k > 0$ such that

$$f(k)[w - k - h^*(p_o)] > w - h^*(p_o).$$

Let $\lambda(k) \equiv f(k)[w - k - h^*(p_o)]$. Note, $\lambda(0) = w - h^*(p_o)$ by construction. Straightforward differentiation yields:

$$\lambda'(k) = f'(k)[w - k - h^*(p_o)] - f(k)$$

$$\lambda''(k) = f''(k)[w - k - h^*(p_o)] - 2f'(k) < 0$$

since $f' > 0$ and $f'' < 0$. Moreover, $\lambda'(0) = +\infty$. Hence, $\exists \epsilon > 0$ such that $\forall k \in (0, \epsilon)$ we have $f(k)[w - k - h^*(p_o)] > w - h^*(p_o)$. Pick some k in this interval — call it $\tilde{k}$. By continuity, $\exists \delta > 0$ such that

$$f(\tilde{k})[(1-t)w_p + tw - \tilde{k} - h^*(p_o)] > w - h^*(p_o)$$

whenever $t \in (1-\delta, 1)$. Again, $\exists \sigma > 0$ such that

$$(1-t)w_r + tw - \tilde{k} - h^*(p_o) > w - h^*(p_o)$$

whenever $t \in (0, 1-\sigma)$.

Note that for r_o's case we need $t < 1 - \frac{\tilde{k}}{w_r - w}$. Let $\sigma \equiv \frac{\tilde{k}}{w_r - w}$. Similarly, defining δ as

$$\frac{1}{(w - w_p)}\left[w - h^*(p_o) - \tilde{k} - \frac{w - h^*(p_o)}{f(\tilde{k})}\right]$$

will satisfy p_y's case. Clearly, $\delta = \sigma = 0$ when $\tilde{k} = 0$. Differentiating δ and σ w.r.t. $\tilde{k}$ and using $f'(0) = +\infty$ establishes that for $\tilde{k}$ sufficiently close to 0, it must be that $\delta > \sigma$.

Hence, $\forall t \in (1-\delta, 1-\sigma)$, the following hold:

$$f(\tilde{k})[(1-t)w_p + tw - \tilde{k} - h^*(p_o)] > w - h^*(p_o),$$

and

$$(1-t)w_r + tw - \tilde{k} - h^*(p_o) > w - h^*(p_o).$$

Denote by $\tilde{q}$ a policy with $\tilde{h} = h^*(p_o)$, $t \in (1-\delta, 1-\sigma)$ and $k = \tilde{k}$. By the above two inequalities, p_y and r_o respectively prefer $\tilde{q}$ over $q^*(p_o)$ as the former leaves them with more of the numeraire good while providing the same level of h as the latter.

Finally, r_y also prefers $\tilde{q}$ over $q^*(p_o)$ since the numeraire provision by $\tilde{q}$ is even larger than that for r_o as $\tilde{k} > 0$. ■

Proof. [PROPOSITION 4.]

Parts (i) — (iii) follow from the arguments in Case (3) under Step 3 above. Part (iv) comes from Step 1.

Part (v): Suppose not. Let $q = (t, k = 0, h)$ denote an equilibrium platform. Hence, it follows that $v_i(q) > v_i(q^*(p_o))$ for $i \in \{p_y, r_y, r_o\}$.

Take the case of p_y. Note, $v_{p_y}(q)$ implies a utility of $u((1-t)w_p + tw - h, h)$ for p_y. Given that $w_p < w$ and $t \in (0, 1)$, we have

$$u((1-t)w_p + tw - h, h) < u(w - h, h).$$

By definition,

$$u(w-h,h) \leq u(w-h^*(p_o),h^*(p_o)).$$

Hence,

$$u((1-t)w_p)+tw-h,h) < u(w-h^*(p_o),h^*(p_o))$$

thus implying $v_{p_y}(q) < v_{p_y}(q^*(p_o))$ which leads to a contradiction. ■

Proof. [PROPOSITION 5.]

Start with $\tilde{q} \in Q$ from Lemma 4 and the partition $p_y r_o | r_y | p_o$. Hence, $\tilde{h} = h^*(p_o)$. Also,

$$f(\tilde{k})[(1-\tilde{t})w_p + \tilde{t}w - \tilde{k} - h^*(p_o)] > w - h^*(p_o),$$

and

$$(1-\tilde{t})w_r + \tilde{t}w - \tilde{k} - h^*(p_o) > w - h^*(p_o).$$

By continuity, $\exists\, \bar{t} \in (\tilde{t},1)$ and $\overline{h} > h^*(p_o)$ with $\bar{t}w - \overline{h} = \tilde{t}w - h^*(p_o)$ such that

$$f(\tilde{k})[(1-\bar{t})w_p + \bar{t}w - \tilde{k} - \overline{h}] \geq w - h^*(p_o),$$

and

$$(1-\bar{t})w_r + \bar{t}w - \tilde{k} - \overline{h} \geq w - h^*(p_o).$$

Let $\overline{q} \equiv (\bar{t},\tilde{k},\overline{h})$. We will now show that $\overline{q}$ is an equilibrium platform for $p_y r_o | r_y | p_o$.

The above (weak) inequalities along with $\overline{h} > h^*(p_o)$ ensures that both p_y and r_o prefer $\overline{q}$ over $q^*(p_o)$. Additionally, as

$$f(\tilde{k})[(1-\bar{t})w_r + \bar{t}w - \tilde{k} - \overline{h}] > (1-\bar{t})w_r + \bar{t}w - \tilde{k} - \overline{h} \geq w - h^*(p_o),$$

it follows that r_y also prefers $\overline{q}$ over $q^*(p_o)$. ■

Proof. [PROPOSITION 6.]

As noted in the main text, *all* equilibria in the case of $\theta > \frac{1}{2}$ except those involving $k > 0$ (in the $p_o r_y r_o | p_y$ partition) are also equilibria in the case of $\theta < \frac{1}{2}$ and $\frac{w_r}{w_p} < \rho^*$. Now consider any equilibrium winning platform with $k > 0$ in the $p_o r_y r_o | p_y$ partition — call it $q \equiv (t,h,k)$. Consider $q' \equiv (t,h',0)$ and $h' \geq h$.

Observe, that $v_i(q') > v_i(q) > v_i(q^*(p_y))$ for $i = p_o, r_o$.

Also, $q' \in PS(p_o r_o)$ for a suitable choice of $h' \in [h, h^*(p_o)]$. Suppose not. Hence, $h' < h$ for q' to be in $PS(p_o r_o)$. This implies both p_o and r_o prefer to substitute public spending on h with more T for the tax rate t. Recall, $q \in PS(p_o r_y r_o)$. Here, h was chosen rather than h' even though p_o and r_o prefer otherwise (since T is even lower under q than under q'). This implies r_y must strictly prefer (t,h,k) over (t,h',k). Given that r_y can purchase $s = h - h'$

with the additional T under q', it must be that r_y is indifferent between (t, h, k) and (t, h', k). This contradiction establishes $h' \geq h$.

When $\theta < 1/2$ and $\frac{w_r}{w_p} < \rho^*$, $p_o r_o$ can break away, induce $p_o r_o|r_y|p_y$ and propose q'. By construction, q' is an equilibrium winning platform for $p_o r_o|r_y|p_y$. Such platforms with the $p_o r_o|r_y|p_y$ partition are equilibria for this scenario and not for $\theta > 1/2$. As $k = 0$ in such platforms, part (i) immediately follows.

For (ii), notice that q' involves $h' \geq h$ and since the choice of q was arbitrary, the statement follows. ■

Proof. [LEMMA 5.]

Parts (i) and (ii) are the same as before.

For part (iii), first note that the standard two-good utility maximising condition will apply for both $q^*(p_y)$ and $q^*(p_o)$. Specifically, $u_x(x^*(p_y), \alpha_y^p h^*(p_y)) = \alpha_y^p u_H(x^*(p_y), \alpha_y^p h^*(p_y))$ for p_y and $u_x(x^*(p_o), h^*(p_o)) = u_H(x^*(p_o), h^*(p_o))$ for p_o since the price of H equals that of x and the solutions are interior.

Now, as p_y prefers $q^*(p_y)$ over $q^*(p_o)$ (by definition), it follows

$$u(f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)], \alpha_y^p h^*(p_y)) > u(w - h^*(p_o), \alpha_y^p h^*(p_o)).$$

Suppose $h^*(p_y) \leq h^*(p_o)$. Then $f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)] > w - h^*(p_o)$. As $u_{xx} < 0$ and $u_{xH} \geq 0$, then given $h^*(p_y) \leq h^*(p_o)$ it must be that

$$u_x(f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)], \alpha_y^p h^*(p_y)) < u_x(w - h^*(p_o), \alpha_y^p h^*(p_o)).$$

By the first-order conditions listed above, then it follows for $\alpha_y^p \to 1$ that

$$\alpha_y^p u_h(f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)], \alpha_y^p h^*(p_y)) < \alpha_y^p u_H(w - h^*(p_o), \alpha_y^p h^*(p_o)).$$

As $f(k^*(p_y))[w - k^*(p_y) - h^*(p_y)] > w - h^*(p_o)$ and $u_{HH} < 0$, the above relation implies $h^*(p_y) > h^*(p_o)$. This contradicts the initial supposition and completes the proof. ■

Proof. [LEMMA 6.]

Consider $q^*(r_y)$ and $q^*(p_o)$. As r_y strictly prefers the former over the latter, we have

$$u(f(k^*(r_y))[w_r(1 - t^*(r_y)) - s^*(r_y)], \alpha_y^r s^*(r_y)) > u(w - h^*(p_o), \alpha_y^r h^*(p_o)).$$

Let $w' = \frac{w_r}{\rho}$ where $\rho > 1$. Now consider $u(f(k^*(r_y))[w'(1 - t^*(r_y)) - s^*(r_y)], \alpha_y^r s^*(r_y))$ and $u(w - h^*(p_o), \alpha_y^r h^*(p_o))$. Clearly, for $\rho \to 1$,

$$u(f(k^*(r_y))[w'(1 - t^*(r_y)) - s^*(r_y)], \alpha_y^r s^*(r_y)) > u(w - h^*(p_o), \alpha_y^r h^*(p_o)).$$

Now, let s' denote the optimal choice for the maximisation of $u(f(k^*(r_y))[w'(1-t^*(r_y)) - s], \alpha_y^r s)$. Thus, for ρ sufficiently close to 1,

$$u(f(k^*(r_y))[w'(1-t^*(r_y)) - s'], \alpha_y^r s') > u(w - h^*(p_o), \alpha_y^r h^*(p_o)).$$

Therefore, for ρ sufficiently close to 1 and $\alpha_y^p \to \alpha_y^r$,

$$u(f(k^*(r_y))[w'(1-t^*(r_y)) - s'], \alpha_y^p s') > u(w - h^*(p_o), \alpha_y^p h^*(p_o)).$$

Note, $u(f(k^*(r_y))[w'(1-t^*(r_y)) - s'], \alpha_y^p s')$ is monotonically decreasing in ρ with its value approaching 0 as $\rho \to \infty$. Hence, there exists $\rho^* > 1$ such that

$$u(f(k^*(r_y))[w_r(1-t^*(r_y))/\rho^* - s'(\rho^*)], \alpha_y^p s'(\rho^*)) = u(w - h^*(p_o), \alpha_y^p h^*(p_o)).$$

Hence, $\frac{w_r}{w_p} > (<)\rho^*$ implies $v_{p_y}(q^*(r_y)) < (>) v_{p_y}(q^*(p_o))$. ■

Proof. [PROPOSITION 7.]

Parts (i) — (iv) are the same as before.

Part (v): Suppose not. Let $q = (t, k = 0, h)$ denote an equilibrium platform. Hence, it follows that $v_i(q) > v_i(q^*(p_o))$ for $i \in \{p_y, r_y, r_o\}$.

Take the case of p_y. Note, $v_{p_y}(q)$ implies a utility of $u((1-t)w_p + tw - h, \alpha_y^p h)$ for p_y. Given that $w_p < w$ and $t \in (0,1)$, we have

$$u((1-t)w_p + tw - h, \alpha_y^p h) < u(w - h, \alpha_y^p h).$$

By definition,

$$u(w - h, h) \leq u(w - h^*(p_o), h^*(p_o)).$$

Hence, for $\alpha_y^p \to 1$

$$u((1-t)w_p) + tw - h, \alpha_y^p h) < u(w - h^*(p_o), \alpha_y^p h^*(p_o))$$

thus implying $v_{p_y}(q) < v_{p_y}(q^*(p_o))$ which leads to a contradiction. ■