



## Design and Control of an Innovative Overactuated Thrust Vectoring Six-DoF Quadrotor for Extreme and Challenging Environments

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The wide spectrum of recent applications for UAVs imposes further challenges to their abilities and control. This is especially true when operating in harsh and hostile environments where disturbances are huge and actuators are prone to failure. Conventional systems and traditional control techniques are not sufficient for stability and tracking under these circumstances. This paper proposes a unique innovative overactuated quadrotor system that has six DoFs. The vehicle has four rotors and each rotor can tilt independently in the YZ-plane. It has the ability to correct its position and attitude in a decoupled way which is different from the conventional quadrotor configurations. Sliding-mode control associated with switching control mode is used for the control algorithm of the system. The system shows agility in the face of large disturbances and robustness against actuator failure which makes it a perfect fit for extreme and challenging environments. The concept of the system and its ability are illustrated in simulation with promising results.

**Keywords:** UAVs; sliding-mode control; switching control; quadrotor; thrust vectoring.

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### 1. Introduction

Unmanned Aerial Vehicles (UAVs) have recently gained significant research and commercial attention. They have been deployed in a wide spectrum of civil applications including remote operations, lifting, sensing and communications [1–3]. Quadrotor is a common class of UAVs that is widely used due to its symmetry, which is an advantage for maneuverability, stability and control [4–6].

The wide deployment of UAVs in extended range of applications challenges their ability to fly in various environments and weather conditions. In the context of UAV operation, extreme conditions and challenging environments

are usually associated with factors that seriously affect the UAV performance such as huge disturbances resulting from strong wind, heavy rain and unpredictable weather conditions [7] or the ingress of small particles in the atmosphere from dusts, icing, sea-salt and pollution which might lead to actuator and sensor failure [8, 9]. This necessitates development in the control techniques and aerodynamic design to enable UAVs to work under extreme conditions and challenging environments. This challenge for operating a UAV in severe conditions has been investigated by several researchers (see e.g. [10, 11] and the references therein). To overcome these challenges, the focus is usually either on developing a robust control algorithm that can stabilize the system and ensure good tracking in the presence of these disturbances or on developing a specific system design configuration that increases the ability of the system to respond to and reject the disturbances [12–14].

From the configuration and design perspective, UAVs have seen various developments related to the configuration of the rotors and the airframe design. The main aim of the research in this direction is to enhance the maneuverability and stability of the vehicle, see, for instance, [15–18]. One of the main techniques that is used to enhance

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the performance of UAVs is thrust vectoring [19, 20]. It is used in various UAV configurations including quadrotors. It can be achieved by simply tilting the thrust generating system (the rotors) or designing the system with fixed tilted nonplanar rotors [21–23]. Thrust vectoring technology overcomes the intrinsic limitations of conventional standard coplanar architecture [24]. It enables the system to generate forces and torques in six-degree-of-freedom (DoF) envelope which extends the ability of the vehicle to fly in any direction while maintaining any arbitrary orientation [23, 25]. On the other hand, thrust vectoring for quadrotors creates overactuated systems with more complex and nonlinear control problem. This issue has been addressed explicitly in [26] using pseudo-inverse matrices and by solving a complementary optimization problem, while in [27], thrust-tilting angle limitations are taken into account with prioritization of the position and attitude to solve the overactuation nonlinear control problem.

From the control development perspective, the control of UAV systems is well established in the literature with both linear and nonlinear control techniques [6, 28–30]. Sliding-mode control (SMC) is a nonlinear control tool that is based on Lyapunov stability criteria. It is suitable for quadrotor control due to its advantages of insensitivity to modeling error, parametric uncertainties and other disturbances [6, 31, 32]. This robustness feature of sliding-mode control has been utilized to enhance the UAVs' operations against uncertainties and external disturbances [12, 33, 34], where the sliding-mode control has been used for UAVs of conventional configuration with promising results [14, 35, 36]. For instance, in [37], the authors showed how an adaptive backstepping global sliding-mode control can be used for quadrotors to stabilize the system and ensure finite-time tracking in the presence of input saturation, model uncertainty and wind perturbation. In [38], it was shown that an adaptive PID-sliding-mode control can be used to overcome external disturbances and sensor failure. However, SMC suffers from a chattering phenomenon that affects the controller performance. Several methods and techniques have been proposed in the literature to minimize the chattering effect when utilizing SMC, see e.g. [39–41].

On the other hand, switching control strategy has been used by several projects in the control design of UAVs [42, 43]. Switching-mode scheme depends on mapping the dynamics of the UAV into a set of finite modes in order to improve the efficiency of the controller and overcome various control and computational challenges [44].

This paper approaches the problem of UAVs' flight in hostile environments from both perspectives: the UAV configuration and the control synthesis. The main contribution of the paper is the proposal of an innovative overactuated thrust vectoring six-degree-of-freedom (DOF)

quadrotor which is controlled using an MIMO sliding mode associated with the switching control technique. The ability of the system to overcome large disturbances and tolerate actuator failure is shown in simulation. The proposed system does not exist in the literature and the use of sliding-mode control along with switching-mode technique for such a system has not been reported in published research before.

The designed quadrotor overcomes the well-known inherited disadvantages of underactuated quadrotor systems. The proposed overactuated system with the designed controller enables the vehicle to achieve a decoupled position and orientation trajectory tracking, without prioritization or restricted flight envelope, as well as reject large disturbances and withstand faulty actuators. This is an important and useful advantage especially in harsh and difficult applications which is the main target for the development of this system. The purpose of this paper is to demonstrate the ability of the proposed systems and associated controller and not to compare the performance of the controller with other established techniques, and hence the paper does not contain any comparative analysis between the proposed controller and other controllers.

The configuration of the proposed system is presented in Sec. 2. The dynamic model is derived in Sec. 3 and the control synthesis is introduced in Sec. 4. The simulation study is then detailed in Sec. 5 and the paper concludes with some remarks and conclusions in Sec. 6.

## 2. System Configuration

The proposed system is shown in Fig. 1. It is a symmetrical cross-quadrotor of length  $l$  for each arm with four identical rotors as displayed in Fig. 2. We assume the rotors are spinning by independent highly efficient brushless DC (BLDC) motors. Each opposite pair of rotors rotates in the opposite direction to eliminate the counteracting torque. We randomly select rotors 1 and 3 to rotate in the anti-clockwise direction, while rotors 2 and 4 rotate clockwise, see Fig. 2. Each rotor can be tilted independently in the

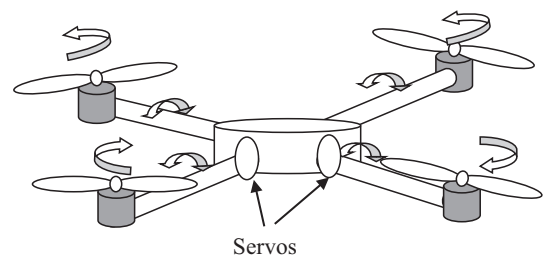


Fig. 1. The design of the proposed six-DoF UAV (3D view).

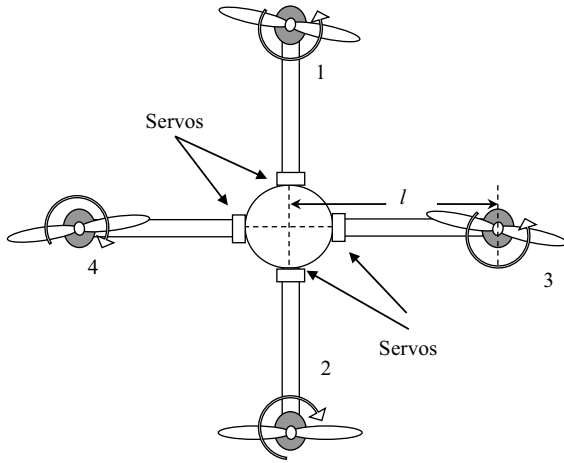


Fig. 2. Top view of the proposed quadrotor.

YZ-plane by an angle  $\alpha_i$  within the range  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ , see Figs. 3 and 4, using a servo motor that is fixed at the mount of the brushless DC motor. The tilting angle is positive in the clockwise direction along the positive direction of the Y-axis as shown in Fig. 3.

### 3. Modeling and Control

The model of the proposed quadrotor is derived using the Euler angles notation and Newton's Second Law. To derive the mathematical model of the system, we use the right-hand coordinate frames shown in Fig. 4. Frame  $e$  represents the earth fixed coordinates and frame  $b$  represents the body coordinates that coincide with the center of gravity of the

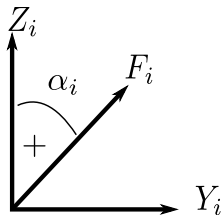
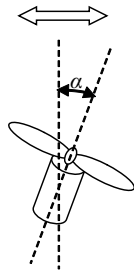


Fig. 3. The thrust vectoring tilting angle.

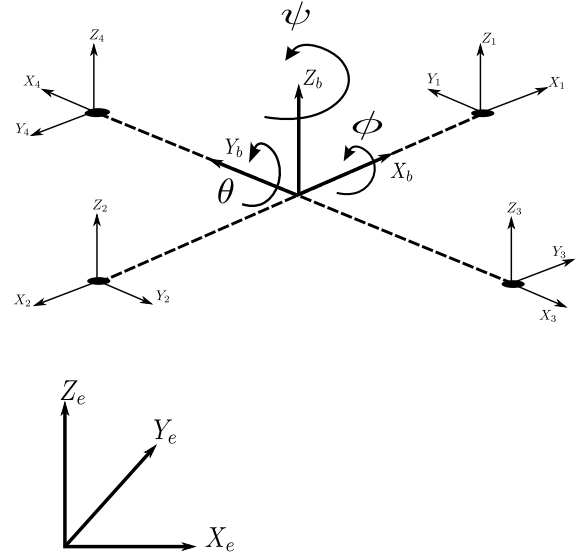


Fig. 4. Frames of the quadrotor system.

UAV. In addition, we use local coordinate frames that are fixed at the center point of each rotor as shown in Fig. 4.

The rotation matrix from the earth frame  $e$  to the body frame  $b$ , using the conventional attitude angles Yaw  $\psi$ , Pitch  $\theta$  and Roll  $\phi$ , as shown in Fig. 4, with the sequence ZYX, is given by

$$R_e^b = \begin{bmatrix} C_\theta C_\psi & C_\theta S_\psi & -S_\theta \\ -C_\phi S_\psi + S_\phi S_\theta C_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & S_\phi C_\theta \\ S_\phi S_\psi + C_\phi S_\theta C_\psi & -S_\phi C_\psi + C_\phi S_\theta S_\psi & C_\phi C_\theta \end{bmatrix}, \quad (1)$$

where  $C = \cos(\cdot)$  and  $S = \sin(\cdot)$ .

Following the general notion of rotation matrices, the rotation matrices from the local coordinates of the rotors to the body coordinate of the quadrotor  $b$  can be derived as

$$R_1^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_2^b = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ R_3^b = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_4^b = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

The forces acting on the system are the propulsive force and gravity force. The Cartesian components of the individual propulsive force from each rotor expressed in the local coordinates are

$$F_i = \begin{bmatrix} 0 \\ k_f \omega_i^2 \sin \alpha_i \\ k_f \omega_i^2 \cos \alpha_i \end{bmatrix}, \quad (3)$$

with  $i = 1, 2, 3, 4$ . The term  $k_f$  is the thrust constant of the propulsion system which depends on the rotor blade characteristics, the air density and the thrust coefficient of the propeller. The rotational speed of each rotor is  $\omega_i$  and  $\alpha_i$  is the tilting angle as defined in Fig. 3.

The propulsive force in the body frame is

$$F_i^b = R_i^b F_i, \quad (4)$$

which gives

$$F_1^b = \begin{bmatrix} 0 \\ k_f \omega_1^2 \sin \alpha_1 \\ k_f \omega_1^2 \cos \alpha_1 \end{bmatrix}, \quad F_2^b = \begin{bmatrix} 0 \\ -k_f \omega_2^2 \sin \alpha_2 \\ k_f \omega_2^2 \cos \alpha_2 \end{bmatrix},$$

$$F_3^b = \begin{bmatrix} k_f \omega_3^2 \sin \alpha_3 \\ 0 \\ k_f \omega_3^2 \cos \alpha_3 \end{bmatrix}, \quad F_4^b = \begin{bmatrix} -k_f \omega_4^2 \sin \alpha_4 \\ 0 \\ k_f \omega_4^2 \cos \alpha_4 \end{bmatrix}. \quad (5)$$

The total propulsive force in the body frame is

$$F_p^b = \Sigma F_i^b \quad (6)$$

$$= k_f H_1 U, \quad (7)$$

with

$$H_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (8)$$

and

$$U = \begin{bmatrix} \omega_1^2 \sin \alpha_1 \\ \omega_1^2 \cos \alpha_1 \\ \omega_2^2 \sin \alpha_2 \\ \omega_2^2 \cos \alpha_2 \\ \omega_3^2 \sin \alpha_3 \\ \omega_3^2 \cos \alpha_3 \\ \omega_4^2 \sin \alpha_4 \\ \omega_4^2 \cos \alpha_4 \end{bmatrix}. \quad (9)$$

The gravity force in the body frame is given by

$$F_g^b = R_e^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \quad (10)$$

$$= mg R_g, \quad (11)$$

with

$$R_g = \begin{bmatrix} -S_\theta \\ S_\phi C_\theta \\ C_\phi C_\theta \end{bmatrix}, \quad (12)$$

where  $m$  is the total weight of the UAV and  $g$  is the gravity acceleration constant.

The total force acting on the UAV becomes

$$F_\Sigma = k_f H_1 U + mg R_g. \quad (13)$$

Two main torques affect the quadrotor which are the propulsive torque resulting from the propulsion force and the drag torque.

The individual propulsive torque from each rotor around the center of gravity can be given as

$$T_{p_i}^b = l_i^b \times F_i^b, \quad (14)$$

where

$$l_i^b = R_i^b \begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix}. \quad (15)$$

This leads to

$$T_{p_1}^b = \begin{bmatrix} 0 \\ -lk_f \omega_1^2 \cos \alpha_1 \\ lk_f \omega_1^2 \sin \alpha_1 \end{bmatrix}, \quad T_{p_2}^b = \begin{bmatrix} 0 \\ lk_f \omega_2^2 \cos \alpha_2 \\ lk_f \omega_2^2 \sin \alpha_2 \end{bmatrix},$$

$$T_{p_3}^b = \begin{bmatrix} -lk_f \omega_3^2 \cos \alpha_3 \\ 0 \\ lk_f \omega_3^2 \sin \alpha_3 \end{bmatrix}, \quad T_{p_4}^b = \begin{bmatrix} lk_f \omega_4^2 \cos \alpha_4 \\ 0 \\ lk_f \omega_4^2 \sin \alpha_4 \end{bmatrix}. \quad (16)$$

The total propulsive torque in the body frame is

$$T_p^b = \Sigma T_{p_i}^b \quad (17)$$

$$= lk_f H_2 U, \quad (18)$$

with

$$H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad (19)$$

and  $U$  being defined as in Eq. (9).

The individual drag torque resulting from the rotation of each propeller acts in the opposite direction to that of rotation and can be derived in the form of a scalar value as

$$T_{d_i} = -k_d \omega_i^2, \quad (20)$$

where  $k_d$  is the drag constant that is related to the propeller design and the flight environment. The negative sign indicates that the torque is in opposite direction to that of rotation.

Similar to the propulsive force analysis, the drag torques in the body coordinate system are given by

$$T_{d_i}^b = R_i^b T_{d_i}, \quad (21)$$

which gives

$$\begin{aligned} T_{d_1}^b &= \begin{bmatrix} 0 \\ -k_d \omega_1^2 \sin \alpha_1 \\ -k_d \omega_1^2 \cos \alpha_1 \end{bmatrix}, & T_{d_2}^b &= \begin{bmatrix} 0 \\ -k_d \omega_2^2 \sin \alpha_2 \\ k_d \omega_2^2 \cos \alpha_2 \end{bmatrix}, \\ T_{d_3}^b &= \begin{bmatrix} -k_d \omega_3^2 \sin \alpha_3 \\ 0 \\ -k_d \omega_3^2 \cos \alpha_3 \end{bmatrix}, & T_{d_4}^b &= \begin{bmatrix} -k_d \omega_4^2 \sin \alpha_4 \\ 0 \\ k_d \omega_4^2 \cos \alpha_4 \end{bmatrix}. \end{aligned} \quad (22)$$

The total drag torque in the body frame can be expressed as

$$T_d^b = \Sigma T_{d_i}^b \quad (23)$$

$$= k_d H_3 U, \quad (24)$$

with

$$H_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \quad (25)$$

and  $U$  being defined as in Eq. (9).

The total torque working on the system is

$$T_\Sigma = (lk_f H_2 + k_d H_3) U \quad (26)$$

$$= H_4 U, \quad (27)$$

with

$$H_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & -k_d & -lk_f & -k_d & lk_f \\ -k_d & -lk_f & -k_d & lk_f & 0 & 0 & 0 & 0 \\ lk_f & -k_d & lk_f & k_d & lk_f & -k_d & lk_f & k_d \end{bmatrix}. \quad (28)$$

In the above analysis of the total forces and torques affecting the system, the gyroscopic force and the gyroscopic inertia are ignored and assumed to be negligible. The gyroscopic effect results from the tilting of the spinning propellers. It is assumed that this effect can be ignored safely in the modeling process by considering it as second-order disturbances to be rejected by the controller. This assumption is valid as long as the inertia of the propeller is very small compared to the inertia of main body of the vehicle.

Following standard Newton–Euler equations in the body frame, the dynamic model of the system is

$$F_\Sigma = m(\dot{\varepsilon} + \zeta \times \varepsilon), \quad (29)$$

$$T_\Sigma = I\dot{\zeta} + \zeta \times (I\zeta). \quad (30)$$

The term  $I$  represents the inertia matrix of the UAV with respect to the fixed body coordinate system. Assuming even distribution of mass across the body of the UAV and given that the vehicle is symmetric, the inertial matrix  $I$  is diagonal with the components of  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  only. The terms  $\varepsilon$  and  $\zeta$  represent the translational and rotational velocities of the UAV. In the body frame, the components of these velocities are given by

$$\varepsilon = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \zeta = \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (31)$$

Substituting the forces and torques into Eqs. (29) and (30) gives the dynamic model of the quadrotor as

$$\dot{\varepsilon} = gR_g - W\varepsilon + k_t H_1 U, \quad (32)$$

$$\dot{\zeta} = I^{-1} H_4 U - I^{-1} W(I\zeta), \quad (33)$$

where the constant  $k_t = \frac{k_f}{m}$  and the matrix  $W$  is the skew matrix of the angular velocity which is given by

$$W = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}. \quad (34)$$

Using the standard notation of rotational matrices between frames, the absolute position and orientation of the vehicle defined in the earth coordinate system as a function of the measured translational and rotational speeds in the body frame can be obtained as

$$\dot{Y}_1 = R_b^e \varepsilon, \quad (35)$$

$$\dot{Y}_2 = \Psi \zeta, \quad (36)$$

with  $R_b^e = (R_e^b)^{-1} = (R_e^b)^T$  and

$$\begin{aligned} Y_1 &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, & Y_2 &= \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \\ \Psi &= \begin{bmatrix} 1 & S_\phi S_\theta / C_\theta & C_\phi S_\theta / C_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi / C_\theta & C_\phi / C_\theta \end{bmatrix}. \end{aligned} \quad (37)$$

Assuming that the output of the system is  $Y = [Y_1, Y_2]^T = [x \ y \ z \ \phi \ \theta \ \psi]^T$ , and the input vector  $U$  is as defined in Eq. (9), the model in Eqs. (32) and (33) with

Eqs. (35) and (36) shows a state space representation of nonlinear overactuated system that has six outputs and eight inputs. This redundancy represents the merits of the proposed quadrotor where it decouples the position and attitude of the vehicle to give better maneuverability and more disturbance rejection. However, at the same time, it imposes a control challenge that will be addressed in the next section.

The dynamic model of the system can be made square by expanding Eqs. (32) and (33) and introducing a new input as

$$\bar{U} = \begin{bmatrix} \bar{U}_1 \\ \bar{U}_2 \end{bmatrix}, \quad (38)$$

with

$$\bar{U}_1 = \begin{bmatrix} U_u \\ U_v \\ U_w \end{bmatrix}, \quad \bar{U}_2 = \begin{bmatrix} U_p \\ U_q \\ U_r \end{bmatrix}, \quad (39)$$

where

$$\bar{U}_u = k_t(\omega_3^2 \sin \alpha_3 - \omega_4^2 \sin \alpha_4), \quad (40)$$

$$\bar{U}_v = k_t(\omega_1^2 \sin \alpha_1 - \omega_2^2 \sin \alpha_2), \quad (41)$$

$$\bar{U}_w = k_t(\omega_1^2 \cos \alpha_1 + \omega_2^2 \cos \alpha_2 + \omega_3^2 \cos \alpha_3 + \omega_4^2 \cos \alpha_4), \quad (42)$$

$$\bar{U}_p = (lk_f/I_{xx})(\omega_4^2 \cos \alpha_4 - \omega_3^2 \cos \alpha_3) - (k_d/I_{xx}) \times (\omega_3^2 \sin \alpha_3 + \omega_4^2 \sin \alpha_4), \quad (43)$$

$$\bar{U}_q = (lk_f/I_{yy})(\omega_2^2 \cos \alpha_2 - \omega_1^2 \cos \alpha_1) - (k_d/I_{yy}) \times (\omega_1^2 \sin \alpha_1 + \omega_2^2 \sin \alpha_2), \quad (44)$$

$$\bar{U}_r = (lk_f/I_{zz})(\omega_1^2 \sin \alpha_1 + \omega_2^2 \sin \alpha_2 + \omega_3^2 \sin \alpha_3 + \omega_4^2 \sin \alpha_4) \quad (45)$$

$$+ (k_d/I_{zz})(-\omega_1^2 \cos \alpha_1 + \omega_2^2 \cos \alpha_2 - \omega_3^2 \cos \alpha_3 + \omega_4^2 \cos \alpha_4), \quad (46)$$

which means the system model can be written in compact matrix form as

$$\dot{\varepsilon} = N_1 + \bar{U}_1, \quad (47)$$

$$\dot{\zeta} = N_2 + \bar{U}_2, \quad (48)$$

$$\dot{Y}_1 = R_b^e \varepsilon, \quad (49)$$

$$\dot{Y}_2 = \Psi \zeta, \quad (50)$$

where

$$N_1 = \begin{bmatrix} -gS_\theta - (qw - rv) \\ gS_\phi C_\theta - (ur - pw) \\ gC_\phi C_\theta - (pv - qu) \end{bmatrix}, \quad N_2 = \begin{bmatrix} I_p qr \\ I_q rp \\ I_r pq \end{bmatrix}, \quad (51)$$

and

$$I_p = \frac{I_{zz} - I_{yy}}{I_{xx}}, \quad (52)$$

$$I_q = \frac{I_{xx} - I_{zz}}{I_{yy}}, \quad (53)$$

$$I_r = \frac{I_{yy} - I_{xx}}{I_{zz}}. \quad (54)$$

This representation makes the dynamic model of the system square having six outputs, the vector  $Y$ , and six inputs, the virtual input vector  $\bar{U}$ , however, the number of physical actuators of the system is eight and hence the system is still overactuated between its real actuators and the required outputs. The full control synthesis of the system will be discussed in detail in the following section.

#### 4. Control Synthesis

The controller of the proposed overactuated quadrotor is designed for the tracking problem which involves two steps. In the first step, it needs to find the required virtual input vector  $\bar{U}$  to drive the system to the desired position and orientation. The second step is to transform the virtual input vector into the physical quantities of motor angular

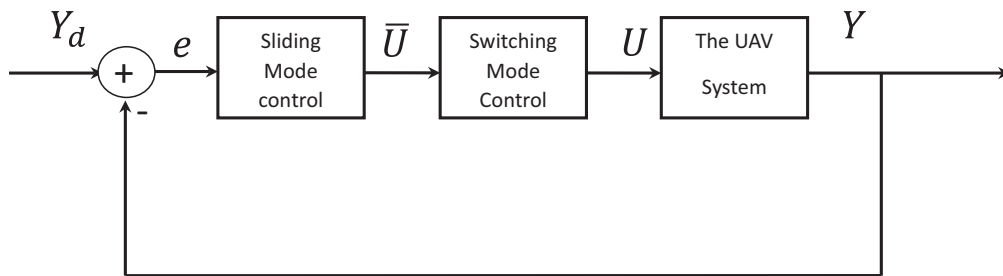


Fig. 5. The block diagram of the control system.

speed and servo tilting angles using Eqs. (40)–(46). We address the first step of the control problem using sliding-mode control algorithm while the determination of the motor angular speeds and servo tilting angles will be made using a switching-mode algorithm. These two steps are shown in the block diagram in Fig. 5 and will be detailed in the following subsections. In this figure,  $Y_d$  is the desired output target,  $e$  is the error between the actual output of the system and the desired output and  $\bar{U}$  and  $U$  are the virtual input and the actual input of the system as defined before in Eqs. (38) and (9) subsequently.

#### 4.1. The relative degree of the system

To design a sliding-mode control, we need to determine the relative degree of the system. Assuming that

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \dots \\ \phi \\ \theta \\ \psi \end{bmatrix}. \quad (55)$$

This means

$$\dot{Y} = \begin{bmatrix} R_b^e & \mathbf{0} \\ \mathbf{0} & \Psi \end{bmatrix} \begin{bmatrix} \varepsilon \\ \zeta \end{bmatrix} \quad (56)$$

and

$$\ddot{Y} = \begin{bmatrix} \dot{R}_b^e \varepsilon + R_b^e \dot{\varepsilon} \\ \dot{\Psi} \zeta + \Psi \dot{\zeta} \end{bmatrix} = f(Y, \varepsilon, \zeta, \bar{U}). \quad (57)$$

This concludes that the relative degree of the system in compact form is  $2 + 2 = 4$  which is equal to the number of state vectors  $\varepsilon$ ,  $\zeta$ ,  $Y_1$  and  $Y_2$ , and there is no hidden dynamics. This makes the synthesis of the sliding-mode control possible with no risk of internal instability.

#### 4.2. Sliding-mode control

To synthesize the sliding-mode control, we define the absolute displacement error as

$$e = Y - Y_d, \quad (58)$$

with  $Y_d$  being the determined target of the absolute position and orientation of the system. It is needless to mention that  $Y$ ,  $Y_d$  and  $e$  are functions of time and the argument  $t$  is omitted for simplicity of representations. The system is nonlinear with fast dynamics and our goal is to design a

state feedback control law such that  $Y$  asymptotically tracks the determined trajectory  $Y_d$  which means  $e$  is bounded and converges to zero.

The absolute velocity error is  $\dot{e}$  and the absolute acceleration error is  $\ddot{e}$ . They can be derived as

$$\dot{e} = \dot{Y} - \dot{Y}_d \quad (59)$$

$$= \begin{bmatrix} R_b^e & \mathbf{0} \\ \mathbf{0} & \Psi \end{bmatrix} \begin{bmatrix} \varepsilon \\ \zeta \end{bmatrix} - \dot{Y}_d \quad (60)$$

and

$$\ddot{e} = \ddot{Y} - \ddot{Y}_d \quad (61)$$

$$= \begin{bmatrix} \dot{R}_b^e \varepsilon + R_b^e \dot{\varepsilon} \\ \dot{\Psi} \zeta + \Psi \dot{\zeta} \end{bmatrix} - \ddot{Y}_d \quad (62)$$

$$= \begin{bmatrix} \dot{R}_b^e \varepsilon + R_b^e (N_1 + \bar{U}_1) \\ \dot{\Psi} \zeta + \Psi (N_2 + \bar{U}_2) \end{bmatrix} - \ddot{Y}_d \quad (63)$$

$$= \begin{bmatrix} \dot{R}_b^e \varepsilon + R_b^e N_1 \\ \dot{\Psi} \zeta + \Psi N_2 \end{bmatrix} + \begin{bmatrix} R_b^e & \mathbf{0} \\ \mathbf{0} & \Psi \end{bmatrix} \begin{bmatrix} \bar{U}_1 \\ \bar{U}_2 \end{bmatrix} - \ddot{Y}_d \quad (64)$$

$$= M_1 + M_2 \bar{U} - \ddot{Y}_d, \quad (65)$$

where

$$M_1 = \begin{bmatrix} \dot{R}_b^e \varepsilon + R_b^e N_1 \\ \dot{\Psi} \zeta + \Psi N_2 \end{bmatrix}, \quad M_2 = \begin{bmatrix} R_b^e & \mathbf{0} \\ \mathbf{0} & \Psi \end{bmatrix} \quad (66)$$

and  $\bar{U}$  is as defined in Eq. (38). We assume that  $Y_d$ ,  $\dot{Y}_d$  and  $\ddot{Y}_d$  are bounded for all  $t > 0$  and they are available online.

We define a set of sliding-mode surfaces as  $s = ke + \dot{e}$  with  $k > 0$ . This gives

$$\dot{s} = k\dot{e} + \ddot{e} \quad (67)$$

$$= k\dot{e} + M_1 + M_2 \bar{U} - \ddot{Y}_d. \quad (68)$$

It is worth mentioning here that  $s$  is a vector with the dimension of  $6 \times 1$ . Hence,  $s$  is a group of six sliding-mode manifolds with each sliding-mode surface being denoted as  $s_i$ ,  $i \in \{1, 2, \dots, 6\}$ .

To analyze the performance of the designed sliding-mode controller, we examine the reaching phase by using a Lyapunov function defined as  $V(s) = \frac{1}{2} s^T s$ . We have

$$\dot{V} = s^T \dot{s}. \quad (69)$$

We put

$$\dot{s} = v, \quad (70)$$

where  $v$  is the artificial control input, or the pure switching component, that we choose to define as

$$v = -\beta_1 s - \beta_2 \text{sign}(s) \quad \text{with } \beta_1, \beta_2 > 0. \quad (71)$$

Here,  $\text{sgn}(s)$  represents the sign function of the sliding surfaces and is defined as

$$\text{sgn}(s) = \begin{bmatrix} \text{sgn}(s_1) \\ \text{sgn}(s_2) \\ \vdots \\ \text{sgn}(s_6) \end{bmatrix}, \quad \text{where } \text{sgn}(s_i) = \begin{cases} +1 & \text{if } s_i > 0, \\ 0 & \text{if } s_i = 0, \\ -1 & \text{if } s_i < 0. \end{cases} \quad (72)$$

This leads to

$$\dot{V} = s^T(-\beta_1 s - \beta_2 \text{sgn}(s)) \quad (73)$$

$$= -\beta_1 s^T s - \beta_2 |s| < 0 \quad \text{for all } s \neq 0, \quad (74)$$

where  $|s|$  is equal to the sum of all elements of the sliding surface  $|s| = \sum_1^6 |s_i|$ . As  $\dot{V} < 0$  for all  $s \neq 0$ , this ensures that all trajectories starting off the manifold  $s = 0$  reach it in finite time and those on the manifold cannot leave it.

To compensate for the chattering effect and avoid control discontinuities in the control loop, we define the artificial control input  $v$  using the boundary layer solution [45] such as

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_6 \end{bmatrix}, \quad \text{where } v_i = -\beta_1 s_i - \begin{cases} \beta_2 \text{sgn}(s_i) & \text{when } |s_i| > \epsilon, \\ \frac{\beta_2}{\epsilon} \text{sgn}(s_i) & \text{when } |s_i| \leq \epsilon, \end{cases} \quad (75)$$

for all  $i \in \{1, 2, \dots, 6\}$  with  $\epsilon$  being a small positive value to be chosen by the designer. The value of  $\epsilon$  represents the range of the boundary layer that is imposed to prevent chattering of the controller. Now, to obtain  $\bar{U}$ , we can use Eqs. (68) and (70) to get

$$\bar{U} = M_2^{-1}(-M_1 - k\dot{e} + \ddot{Y}_d + v). \quad (76)$$

The stability analysis of the designed sliding-mode controller can be established by considering the error dynamics. We define  $\xi_1 = e$  and  $\xi_2 = \dot{e}$ . This leads to

$$\dot{\xi}_1 = \xi_2, \quad (77)$$

$$\dot{\xi}_2 = M_1 + M_2 \bar{U} - \ddot{Y}_d = -k\xi_2 + v, \quad (78)$$

with the term  $v$  defined as in Eq. (75) and  $\beta_1, \beta_2 > 0$ , so it is straightforward to conclude that there exists a finite time  $T_1$  such that the error is bounded for all  $t \geq T_1$ . This shows that the derived sliding mode is stable.

### 4.3. Determining the speed of BLDC motors and the angles of the servo motors

After obtaining the control input  $\bar{U}$ , we can solve the equations set (40)–(46) to obtain the physical quantities of the servo angles and the rotational speed of the brushless

DC motors. However, these equations are nonlinear and are computationally expensive to solve. The equations can be rewritten as

$$\bar{U} = AU, \quad (79)$$

with

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & k_t & 0 & -k_t & 0 \\ k_t & 0 & -k_t & 0 & 0 & 0 & 0 & 0 \\ 0 & k_t & 0 & k_t & 0 & k_t & 0 & k_t \\ 0 & 0 & 0 & 0 & -k_d & -lk_f & -k_d & lk_f \\ -k_d & -lk_f & -k_d & lk_f & 0 & 0 & 0 & 0 \\ lk_f & -k_d & lk_f & k_d & lk_f & -k_d & lk_f & k_d \end{bmatrix}. \quad (80)$$

If we manage to solve Eq. (79) to find  $U$ , it will be straightforward to determine the rotational speed of the rotors and tilting angles of the servos from  $U$ . However, we have  $\bar{U} \in R^{6 \times 1}$ ,  $U \in R^{8 \times 1}$  and the matrix  $A \in R^{6 \times 8}$  is not square which presents a computational challenge for any onboard controller. This control problem can be solved by choosing an operating point  $P_i$  that results in  $U_{P_i} \in R^{6 \times 1} \subset U$  and makes, based on the analysis of the forces and torques, the matrix  $A_{P_i} \in R^{6 \times 6} \subset A$  square while guaranteeing that  $\text{rank}(A_{P_i}) = 6$ . For instance, the operating point of  $(\alpha_1 = 0, \alpha_2 = \frac{\pi}{2})$  can satisfy this condition and make the solution for  $U$  possible. Another operating point that can be considered is  $(\alpha_3 = 0, \alpha_4 = \frac{\pi}{2})$  or any other reasonable operational assumption. This is a switching control technique that enables the controller to determine a solution for the equation  $U = A^{-1}\bar{U}$  which leads to the calculation of the rotor speeds and tilting angles from Eq. (9). The only condition that needs to be enforced for any chosen operating point is that  $\text{rank}(A_{P_i}) = 6$ . For instance, the point  $(\alpha_1 = \alpha_2, \alpha_3 = \alpha_3)$  or  $(\omega_1 = \omega_2, \omega_3 = \omega_4)$  gives an operation point with  $\text{rank}(A_{P_i}) = 4$  which makes the matrix  $A_{P_i}$  ill-posed and then the solution of Eq. (79) becomes nonunique. Also, for the special case of  $(\alpha_1 = -\alpha_2, \alpha_3 = -\alpha_4, \omega_1 = \omega_2, \omega_3 = \omega_4)$ , we have  $U_p = U_q = U_r = 0$ . This makes the attitude of the quadrotor uncontrollable and these operating points should be avoided.

Without loss of generality, we can select a limited number of predetermined operating points where the controller can switch between these operating points based on the flight conditions. In this paper, we consider the following four operating points:

$$P_1 : \alpha_3 = \frac{\pi}{2}, \quad \alpha_4 = 0, \quad (81)$$

$$P_2 : \alpha_3 = 0, \quad \alpha_4 = \frac{\pi}{2}, \quad (82)$$

$$P_3 : \alpha_1 = \frac{\pi}{2}, \quad \alpha_2 = 0, \quad (83)$$



$$P_4 : \alpha_1 = 0, \quad \alpha_2 = \frac{\pi}{2}, \quad (84)$$

where each operating point will result in a new nonsingular matrix  $A_{p_i} \subset A$  with  $\text{rank}(A_{p_i}) = 6$  for  $i = 1, \dots, 4$ . The tilting angle  $\alpha_i = \frac{\pi}{2}$  will mainly have impact on the  $XY$ -plane. Therefore, the switching between these operating points can be determined by the direction of flight on the  $XY$ -plane. This can be regulated by the following algorithm:

$$\begin{aligned} &\text{if } |u| \geq |v| \text{ and } u \geq 0, \\ &\quad \text{the vehicle operates at } P_1 \text{ and we have } U_{P_1} = A_{P_1}^{-1} \bar{U}; \end{aligned} \quad (85)$$

$$\begin{aligned} &\text{if } |u| \geq |v| \text{ and } u < 0, \\ &\quad \text{the vehicle operates at } P_2 \text{ and we have } U_{P_2} = A_{P_2}^{-1} \bar{U}; \end{aligned} \quad (86)$$

$$\begin{aligned} &\text{if } |v| > |u| \text{ and } v > 0, \\ &\quad \text{the vehicle operates at } P_3 \text{ and we have } U_{P_3} = A_{P_3}^{-1} \bar{U}; \end{aligned} \quad (87)$$

$$\begin{aligned} &\text{if } |v| > |u| \text{ and } v < 0, \\ &\quad \text{the vehicle operates at } P_4 \text{ and we have } U_{P_4} = A_{P_4}^{-1} \bar{U}. \end{aligned} \quad (88)$$

The physical interpretation of the above algorithm is that the selection of the operating point will be such as to generate more thrust in the direction of flight in the  $XY$ -plane. This will enhance the ability of the UAV to overcome any wind opposing its movement.

For the purpose of illustration, we will consider the operating point  $P_4$  where  $\alpha_1 = 0$ ,  $\alpha_2 = \frac{\pi}{2}$ . In this case, the physical input vector  $U_{P_4}$  can be obtained as

$$U_{P_4} = A_{P_4}^{-1} \bar{U}, \quad (89)$$

where

$$U_{P_4} = \begin{bmatrix} U_1 \\ \vdots \\ U_6 \end{bmatrix} \quad \text{and} \quad A_{P_4} = \begin{bmatrix} 0 & 0 & k_t & 0 & -k_t & 0 \\ 0 & -k_t & 0 & 0 & 0 & 0 \\ k_t & 0 & 0 & k_t & 0 & k_t \\ 0 & 0 & -k_d & -lk_f & -k_d & lk_f \\ -lk_f & -k_d & 0 & 0 & 0 & 0 \\ -k_d & lk_f & lk_f & -k_d & lk_f & k_d \end{bmatrix}. \quad (90)$$

The matrix  $A_{P_4}$  is square and nonsingular. It is independent of the position and attitude of the vehicle and therefore its inverse exists always. Then, from Eq. (9), the rotational speed of the rotors can be determined as

$$\omega_1 = \sqrt[4]{U_1^2}, \quad (91)$$

$$\omega_2 = \sqrt[4]{U_2^2}, \quad (92)$$

$$\omega_3 = \sqrt[4]{U_3^2 + U_4^2}, \quad (93)$$

$$\omega_4 = \sqrt[4]{U_5^2 + U_6^2} \quad (94)$$

and the servo angles can be determined as

$$\alpha_3 = \tan^{-1} \left( \frac{U_3}{U_4} \right), \quad (95)$$

$$\alpha_4 = \tan^{-1} \left( \frac{U_5}{U_6} \right). \quad (96)$$

Similar derivation for the other operating points can be obtained easily.

It is important to state here that while the results of this paper are based on the selection of four operating points for the switching-mode control, the designer has the full freedom of choosing any number of operating points. As long as the condition of any selected point, i.e.  $\text{rank}(A_{p_i}) = 6$ , is fulfilled, the chosen number of operating points should not affect the results and it is just a matter of design preferences based on the specific situation.

## 5. Simulation Results

In this section, we show some simulation results to demonstrate the proposed control design and the ability of the system. The simulation is conducted in Matlab 2019b and the parameters used to simulate the system are presented in Table 1. These parameters represent realistic values of a small-size lab-based UAV with some payload. The thrust constant and drag constant are calculated for off-the-shelf propellers of suitable size that have been used by the authors in other projects.

The controller is implemented using the tuned parameters of  $k = 1$ ,  $\beta_1 = 1.1$ ,  $\beta_2 = 1.1$  and  $\epsilon = 0.1$ . The selection of these parameters is based on the observation of the actuator load where higher values of  $k$ ,  $\beta_1$  and  $\beta_2$  will create tighter control performances with higher load on the actuators. On the contrary, small values of  $\beta_1$  and  $\beta_2$  will

Table 1. Simulation parameters for the UAV system.

Description	Symbol	Value	Unit
Total mass of the UAV	$m$	2	kg
Arm length	$l$	0.25	m
Inertia around the X-axis	$I_{xx}$	0.008	kg·m <sup>2</sup>
Inertia around the Y-axis	$I_{yy}$	0.008	kg·m <sup>2</sup>
Inertia around the Z-axis	$I_{zz}$	0.013	kg·m <sup>2</sup>
Thrust constant	$k_f$	$3.13 \times 10^{-5}$	kg·m
Drag constant	$k_d$	$7.5 \times 10^{-7}$	kg·m <sup>2</sup>

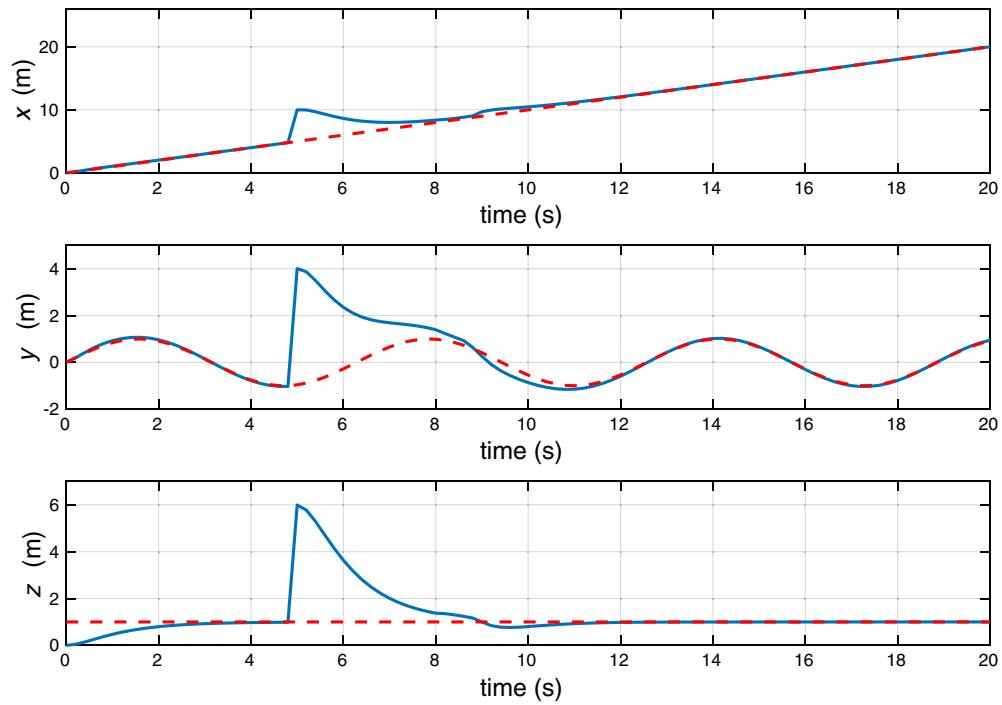


Fig. 6. Position of the UAV for the scenario of flying in a sinusoidal pattern at a fixed altitude under the disturbances. The reference trajectory is in red-dashed line and the actual UAV position is in blue-solid line.

relax the control performance with less load on the actuators. The value of  $\epsilon$  controls the chattering phenomenon. We need to choose  $\epsilon$  as small as possible, but a too small value will induce chattering in the presence of unmodeled dynamics or disturbances. Full discussion about the tuning of

these parameters and their impact on the sliding control performance is well established in the literature and it is beyond the scope of this paper. More information related to the tuning of these control parameters can be found in [46].

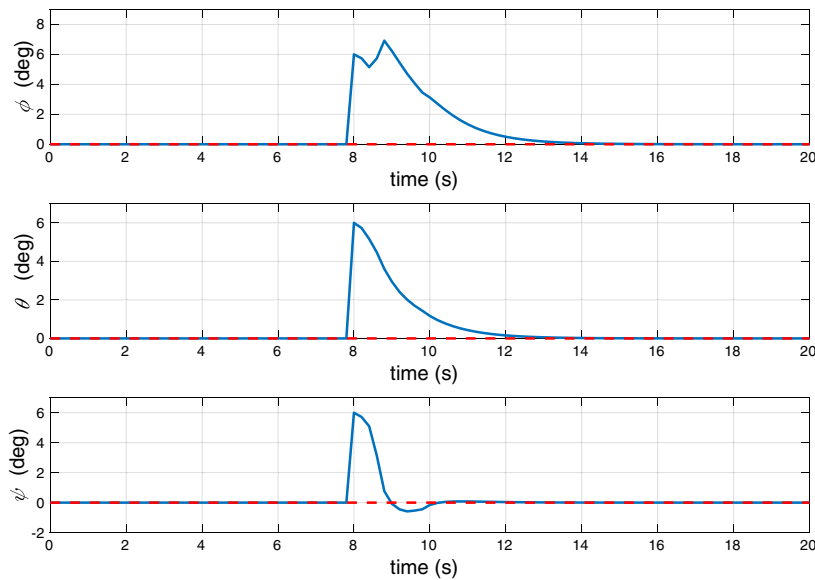


Fig. 7. Orientation of the UAV for the scenario of flying in a sinusoidal pattern at a fixed altitude under the disturbances. The reference trajectory is in red-dashed line and the actual UAV orientation is in blue-solid line.

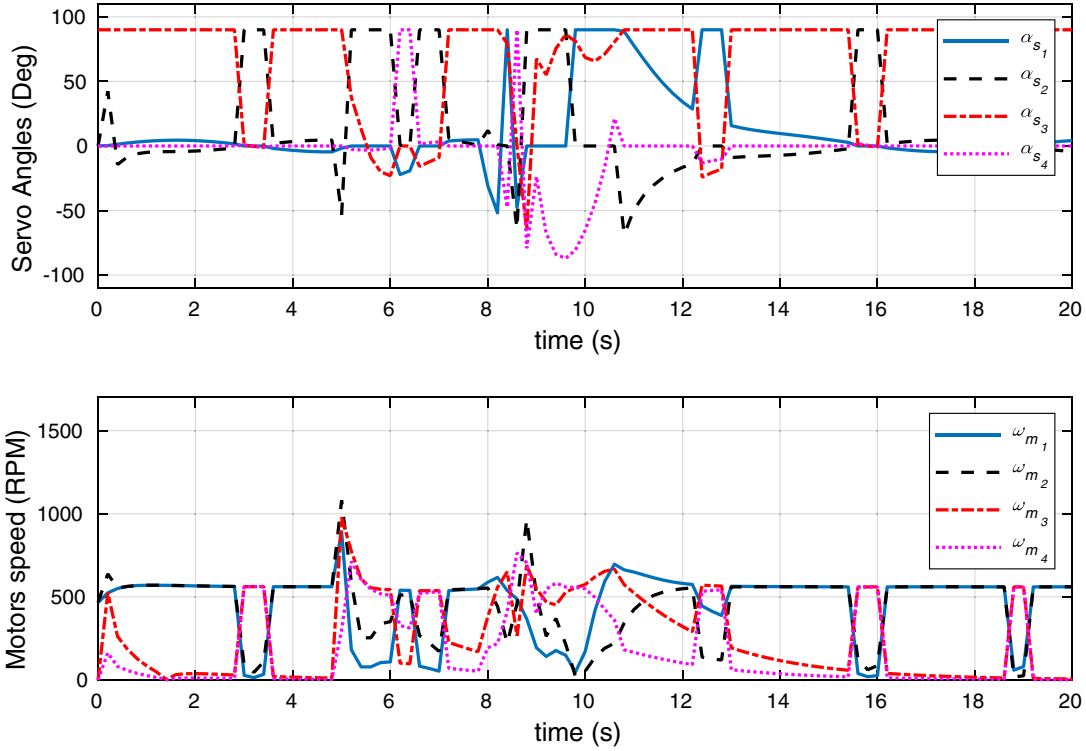


Fig. 8. Actuators' load (angles of servo motors and speed of brushless DC motors) for the scenario of flying in a sinusoidal pattern at a fixed altitude under the disturbances.

We run the simulation for the case of flying in a sinusoidal pattern at a fixed altitude. The flight will be examined under the scenario of normal operation and under the scenario of actuator fault including a faulty rotor or a faulty servo. All flight scenarios are conducted in the presence of disturbances that result from winds which start at a certain point of time during the flight. Attention will be given to the ability of the system to reject disturbances and get back to track while maintaining original orientation of the system even with a faulty actuator. The wind disturbances will take the form of a constant blow that affects the orientation and the position alike. The wind effect can be simulated by a step function of the form

$$D(t) = D_m d(t - t_0), \quad (97)$$

with  $D_m \in \mathbb{R}^{3 \times 1}$  and  $d(t)$  being the unity function. The magnitude of the disturbances is  $D_m = [5, 5, 5]^T$  on the position vector and is  $D_m = [6, 6, 6]^T$  on the orientation vector. The magnitude of the wind impact is chosen based on the simulation scenario where a disturbance of 5 m on the position vector represents a 500% disturbance on the reference trajectory as the UAV flies at an altitude of 1 m with a sine wave pattern of 1-m magnitude. This represents a huge disturbance impact which highlights the case of harsh and challenging environment. Also, a disturbance

magnitude of  $6^\circ$  on the orientation vector represents a strong shift from the set point and it is the limit of the small angle convention that is used in UAV systems. The wind disturbances are simulated to affect the position and orientation of the UAV at different times of  $t_0 = 5$  s and

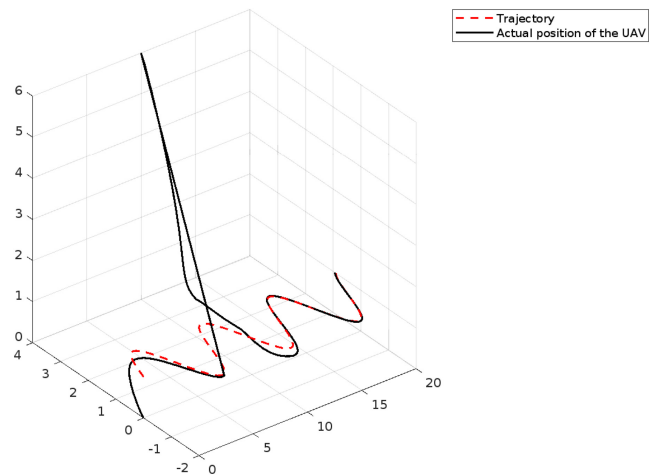


Fig. 9. The 3D flight track for the scenario of flying in a sinusoidal pattern at a fixed altitude under the disturbances. The reference track is in red-dashed line and the actual UAV trajectory is in black-solid line.

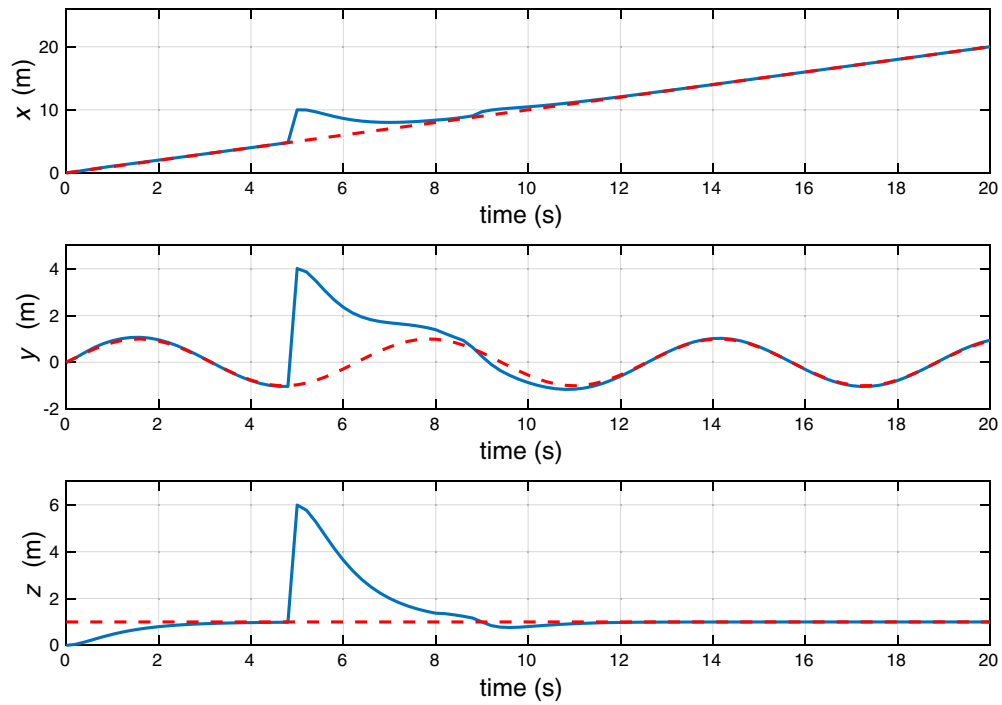


Fig. 10. Position of the UAV for flying in a sinusoidal pattern at a fixed altitude when rotor 1 is faulty and with the disturbances. The reference trajectory is in red-dashed line and the actual UAV position is in blue-solid line.

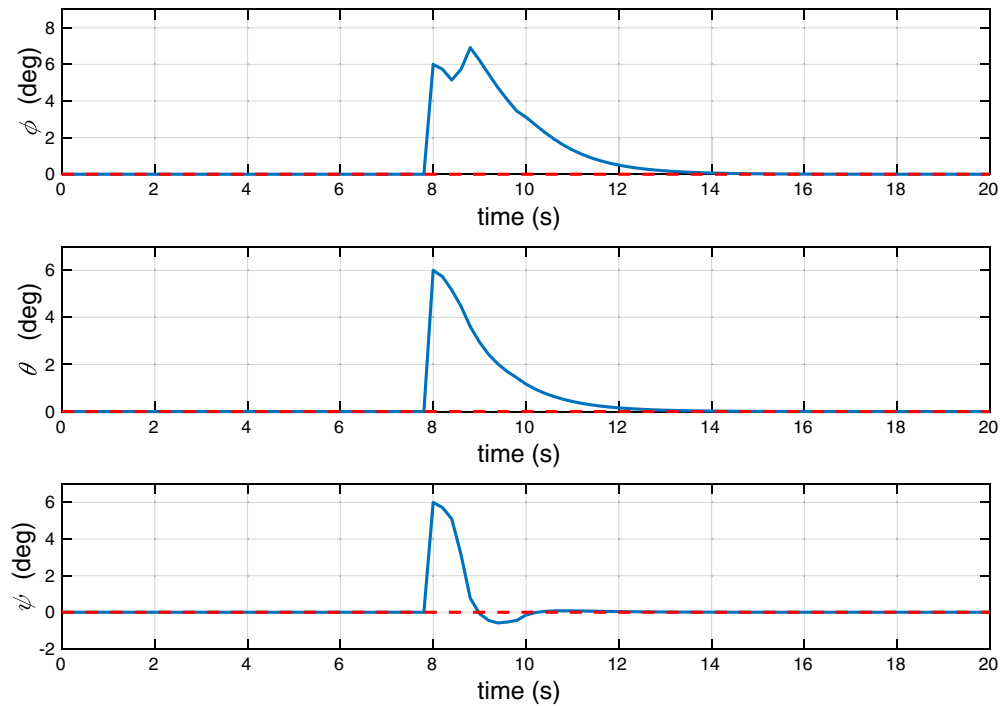


Fig. 11. Orientation of the UAV for flying in a sinusoidal pattern at a fixed altitude when rotor 1 is faulty and with the disturbances. The reference trajectory is in red-dashed line and the actual UAV orientation is in blue-solid line.

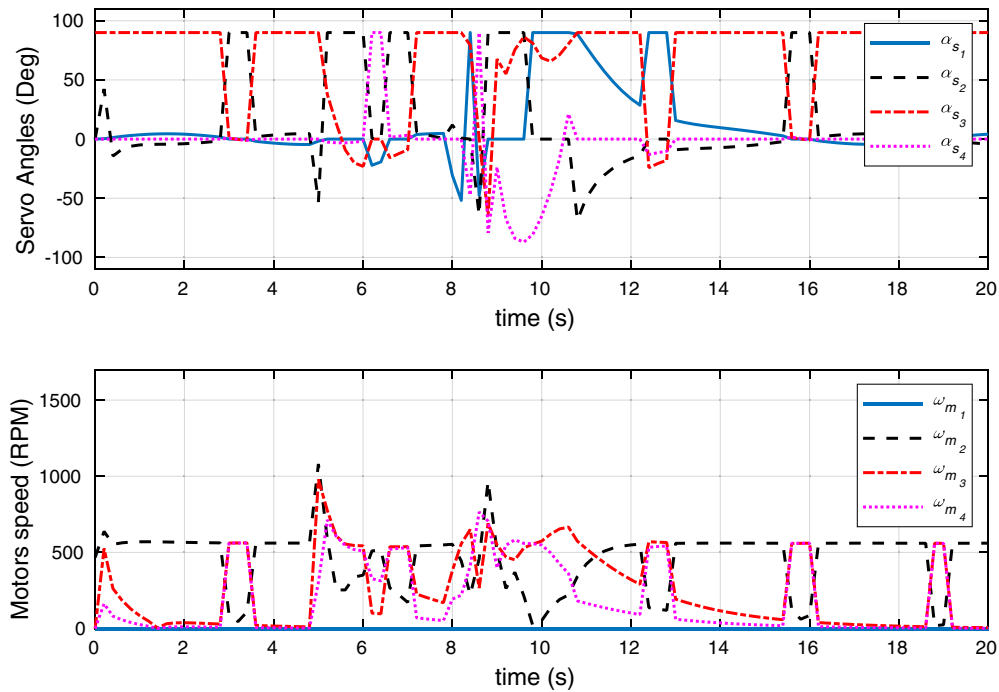


Fig. 12. Actuators' load (angles of servo motors and speed of brushless DC motors) for flying in a sinusoidal pattern at a fixed altitude when rotor 1 is faulty and with the disturbances.

$t_0 = 8$  s, respectively, in order to demonstrate the impact on the actuators for each case separately.

To demonstrate the advantages of the thrust vectoring technique, the flight scenario will be simulated for a reference of fixed orientation at  $(0, 0, 0)$ . This is to prove that the vehicle can travel to any point in any pattern while maintaining a fixed orientation which is one of the main advantages of the proposed overactuated system. It increases the maneuverability of the vehicle without affecting its orientation which is useful to balance any load the system carries such as a camera.

The simulation of the nominal scenario, under no actuator fault, is demonstrated in Figs. 6 and 7, while the actuators' load is shown in Fig. 8. The 3D flight path is demonstrated in Fig. 9. As it can be seen from these figures, the overactuation design of the system enables the position of the vehicle to change without any tilting which is a unique feature of the system compared with traditional quadrotor configuration. We can see that the controller guarantees tracking, manages to overcome the wind and brings the system back to track. The simulation results show clearly the advantages of the proposed overactuated system with thrust vectoring techniques where the disturbances to the position are rejected without affecting the attitude. Similarly, the disturbances to the attitude are rejected without affecting the position of the vehicle. In the

conventional quadrotor systems, this is impossible where the position and attitude of the system are coupled and both will be affected by any wind or external disturbances. The profile of the actuator load indicates that in an ideal case with no disturbances mainly two motors are rotating to generate the required thrust and these are the motors of the same pair that rotate in opposite direction. This mode of

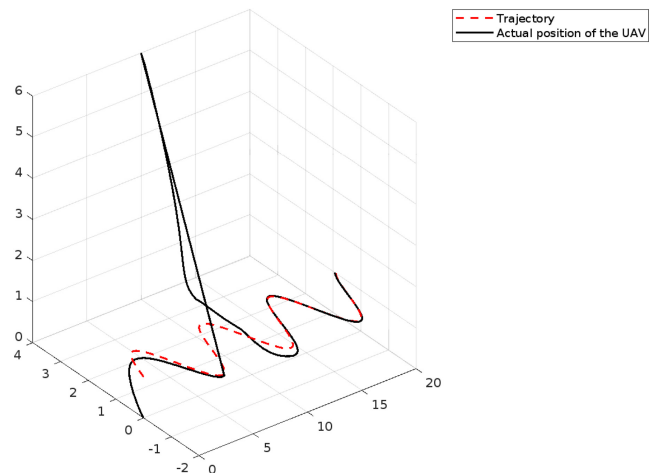


Fig. 13. The 3D flight track for flying in a sinusoidal pattern at a fixed altitude when rotor 1 is faulty and with the disturbances. The reference track is in red-dashed line and the actual UAV trajectory is in black-solid line.

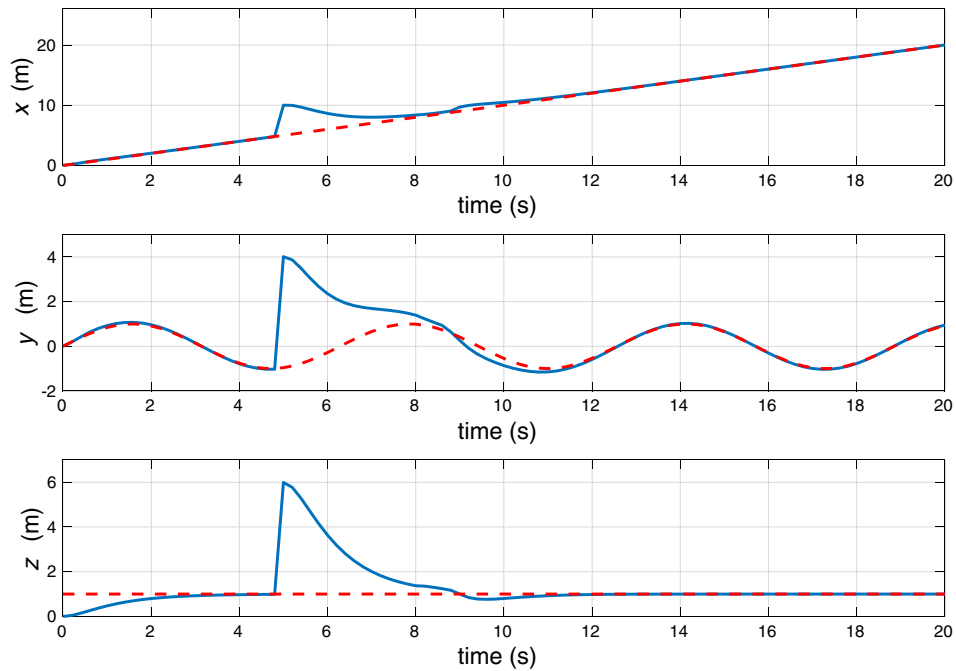


Fig. 14. Position of the UAV for flying in a sinusoidal pattern at a fixed altitude when servo 2 is faulty and with the disturbances. The reference trajectory is in red-dashed line and the actual UAV position is in blue-solid line.

operation can help in extending the time of flight where less energy is required from the battery.

Figures 10–13 show how the vehicle can withstand a faulty rotor and achieve trajectory tracking even under the

impact of disturbances. We operate the vehicle under the same flight pattern while rotor 1 is switched off. The results show clearly that even with one of the rotors not working the UAV can still follow the trajectory and reject disturbances.

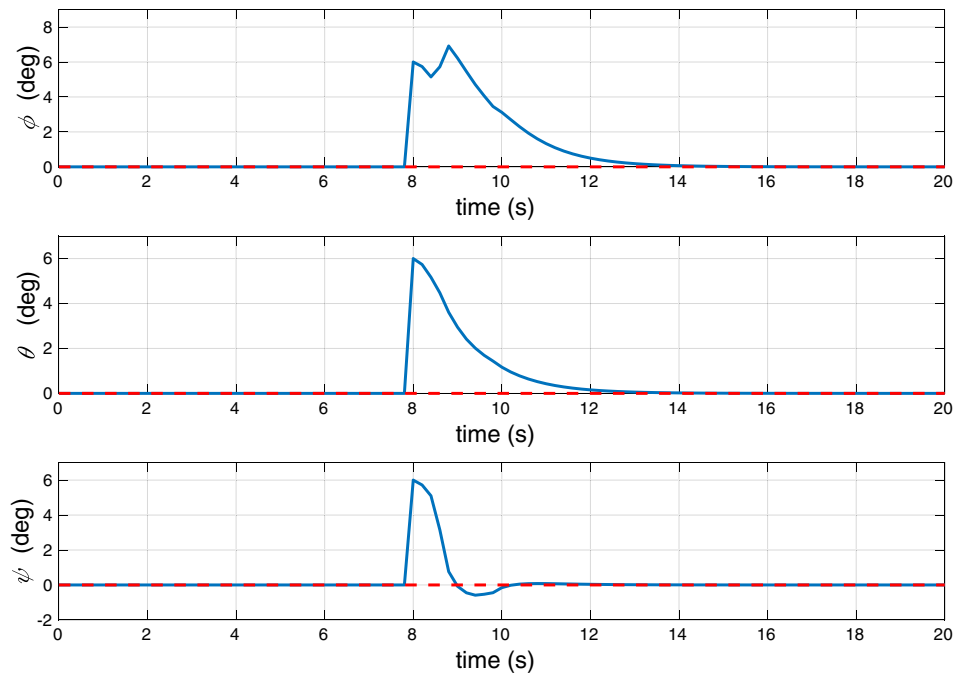


Fig. 15. Orientation of the UAV for flying in a sinusoidal pattern at a fixed altitude when servo 2 is faulty and with the disturbances. The reference trajectory is in red-dashed line and the actual UAV orientation is in blue-solid line.

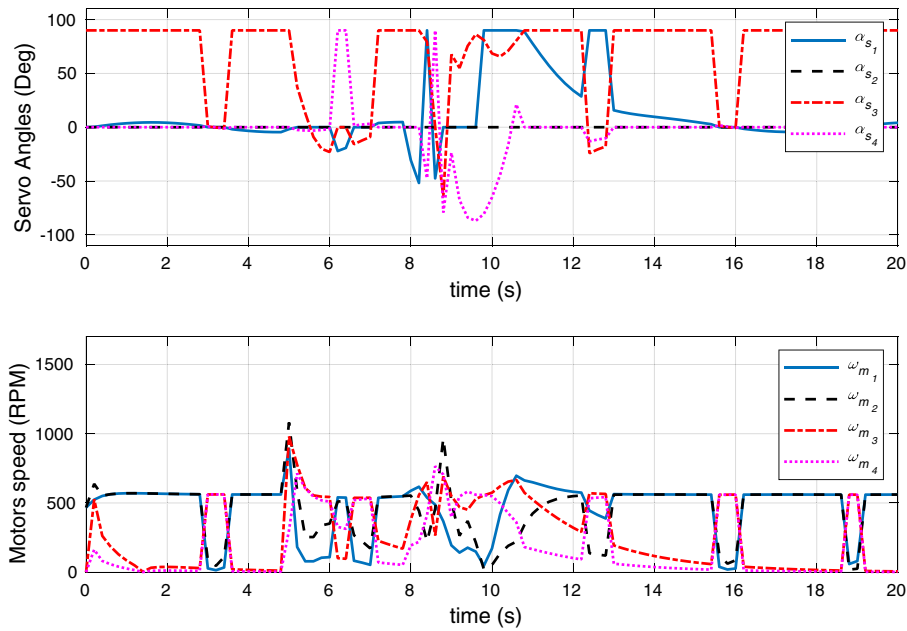


Fig. 16. Actuators' load (angles of servo motors and speed of brushless DC motors) for flying in a sinusoidal pattern at a fixed altitude when servo 2 is faulty and with the disturbances.

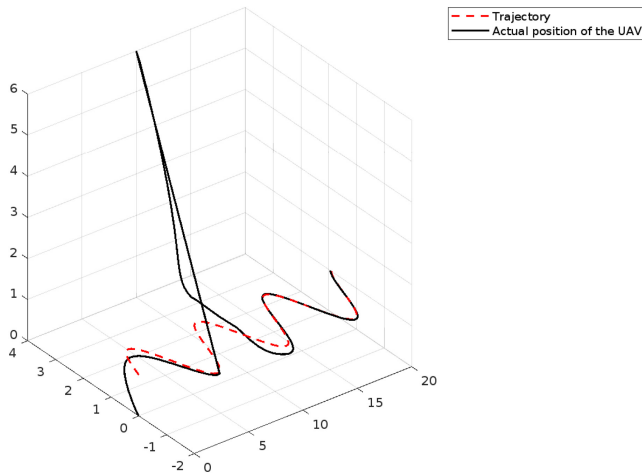


Fig. 17. The 3D flight track for flying in a sinusoidal pattern at a fixed altitude when servo 2 is faulty and with the disturbances. The reference track is in red-dashed line and the actual UAV trajectory is in black-solid line.

Figures 14–17 show how the vehicle can withstand a faulty servo and achieve trajectory tracking even under the impact of disturbances. In this case, servo 2 is turned off and the vehicle is tested for the same flight pattern as above. The results show clearly that even with one of the servos not working the UAV can still follow the trajectory and reject disturbances. The choice of the faulty rotor and servo is



random and the results are rigorous regardless of the chosen rotor or servo for the fault tolerance test.

## 6. Conclusion

This paper proposes an innovative overactuated quadrotor system controlled by a sliding mode associated with switching control algorithm. The UAV has six DoFs with eight actuators where each rotor can tilt independently using a servo motor. The configuration of the system and its proposed control algorithm guarantee decoupling between attitude and position of the vehicle which helps to achieve independent position and orientation references. The system has also strong ability to reject huge disturbances and has good tolerance toward any faulty actuator during the operation which is very advantageous for operating within extreme and challenging environments. Results of this paper are verified in simulation for various scenarios to demonstrate the concept of the vehicle, its ability and the robustness of the proposed control algorithm against large disturbances and actuator fault. Future research will focus on testing the proposed configuration and control algorithm for different UAV sizes with hardware test results.

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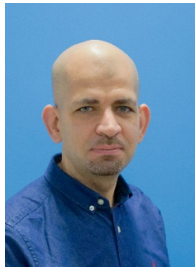
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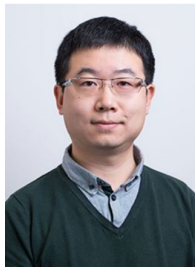
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