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Citation Networks and the Emergence of Knowledge Core

Yang Zhang and Chi Zhang

Abstract—Observations on the citation networks often confirm a *core-periphery* structure: A clustered group of artifacts possess the *core* knowledge to the field, which is widely cited by artifacts at *periphery*. We explain this as an outcome resulted from decentralized knowledge contributions from individuals who maximize their own utilities. Our model sheds insights on how knowledge creation, knowledge citation, and knowledge heterogeneity affect the emergence of knowledge core, in both cases of direct and indirect citations. We find through simulations that the core-periphery architecture of citation networks is robust to generalizations on knowledge heterogeneity and knowledge creativity. By studying the incentive rationale that underlies the growth of citation networks, our research has potential implications on the design and administration of intellectual communities.

Index Terms—Complex network, multi-agent, citation network, optimization

1 Introduction and Literature Background

An enduring empirical observation on the connectivity among intellectual artifacts (e.g., patents, academic articles, etc.) is that, described as networks, they often hold a core-periphery structure—A group of densely interconnected artifacts contains the *core* knowledge to the field, which is commonly cited by *peripheral* artifacts [5], [13], [21], [23], [26].

This article is primarily interested in explaining how the core-periphery structure emerges over time in citation networks. A citation network is viewed as a collection of artifacts created by individual authors in a decentralizefd manner. In creating an artifact, each author both makes citations to existing artifacts and creates new knowledge in the present artifact, in order to maximize her own utility. Both citation and knowledge creation are costly activities to the author.

We furthermore incorporate the heterogeneity of artifacts: Knowledge stored in an existing artifact is only partially useful for creating a new artifact. In this line, our analysis reveals an explicit relation between artifact heterogeneity and the characteristics of knowledge core.

After all, our research delivers two streams of insights on the growth of citation networks:

- *Network structure*: What is the architecture of the citation network formed by decentralized, self-interested knowledge contributions?
- *Knowledge creation*: How much knowledge does one create and how does the level of knowledge creation change over time?

As an answer to the first question, our model produces a *core-periphery* landscape for the citation network. As for the second question, we characterize the optimal individual knowledge creation as a decreasing function of time. Compared to conventional statistics-based modeling of networks (e.g., preferential attachment [1], [2], or more recent mechanism by [20]), our approach is novel in being incentive-based, which allows us to describe both the configuration of citation networks and knowledge production and transition, owing to utility optimization.

We view knowledge in citation networks as public goods, attributed to its open accessibility and convenient transmission. The public goods representation of knowledge is typical in the economics literature, and has been widely studied in network contexts. [3] and [4] investigated public goods games upon an exogenously given social networks, while [16] inspected endogenous network structure. On the other direction, [18] studied a problem of network formation, but considering public goods as exogenously endowed to agents in the network. [10] modeled networked public goods supply under a bargaining framework. In evaluating the author's utility regarding knowledge, we technically examine a payoff scheme similar to the works in the above literature, especially [16]. However, unlike [16], our model is dynamic, nonstrategic, and incorporates discount of knowledge flow that is sensitive to the citation network structure. In this line, we also inspect different spatial discount structures (namely, indirect citation and direct citation) on the knowledge flow.

Our paper is broadly related to the literatures on knowledge creation and diffusion. [11] and [12] discussed network-based models on creating and spreading knowledge. [9] and [14] examined diffusions of R&D knowledge in industry and pointed to significant incentive for free ride in the presence of knowledge spill-over, which supports our view of knowledge as public goods. [7] analyzed learning in social networks and, like our paper, viewed knowledge citing as a conscious and costly decision of individuals. In this article, we adopt a multi-agent view in the evolution of the citation network. This, as discussed in [27], allows us to stress the decentralized

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aspects of knowledge diffusion, and examine the impact of individual behaviors on the system performance.

We are also aware of studies on core-periphery network structures in other relevant contexts. [8] studied characteristics and measures of core-periphery topology in general networks. [28] presented statistical evidence for knowledge clustering in technological development networks. [25] reported that core-periphery social networks are efficient institutions for spreading creation. In their paper, the authors assumed random network interaction (except for Appendix B—see footnote 1), whereas our paper views networks as outcomes of rational choices made by individual players.[1] Also, the focus of [25] was given to the efficient diffusion of exogenous initial knowledge, while our paper further incorporates the knowledge supply into the scope.

The rest of the paper is organized as follows. Section 2 establishes the benchmark model, analytical results, and comparative statics. Section 3 conducts robustness tests. Section 4 proposes a model of direct citations. Section 5 concludes and discusses this research. Before proceeding, we define some terms that will be repeatedly referred to in the rest of this article.

Definitions

1) Knowledge heterogeneity: The extent to which the knowledge across artifacts is different, measured by the proportion of knowledge contained in one artifact that can be meaningfully transcribed into another artifact via citation.
2) Knowledge creativity: The capability of an author to create new knowledge, calibrated by the unit cost of knowledge creation.

2 The Baseline Model

2.1 The Citation Network

Individual authors arrive to the field and create their artifacts in a discrete sequence. The specific arrival process is not important so long as to keep arrivals one at a time (e.g., Bernoulli arrival process). After contributing a single artifact the author leaves but her artifact remains in the system. This way creates a one-to-one mappings between authors, artifacts, and system time: At period i (i=1,2,3...), author i composes artifact i.[2] In producing her artifact, the author both creates new knowledge and cites knowledge via citations to existing artifacts. In the latter case, a connection is made between the artifact in question and an existing one from which the knowledge is cited. We use $N := \{1, 2 \ldots\}$ as the set of artifacts, and $E := \{g_{jk} \in \{0, 1\} : j, k \in N\}$ as the set of connections between artifacts. If artifact k cites artifact j , then $g_{jk} = 1$, and we say there is a link pointing from j to k.[3]

Throughout the paper we refer to an *author* who creates an artifact as a *player*. Artifact i's *out-neighborhood* on citation network g, denoted as $N_i^o(g)$, is the set of artifacts that quote i, i.e. $N_i^o(g) := \{j \in N : g_{ij} = 1\}$. Each artifact in one's out-neighborhood is referred as her *out-neighbor*. $D_i^o(g) := |N_i^o(g)|$[4] is the number of out-neighbors, or *out-degree* of player i. The *in-neighbor(-hood)* and *in-degree* are similarly defined: $N_i^I(g) := \{j \in N : g_{ji} = 1\}$, and $D_i^I(g) := |N_i^I(g)|$. A *path* from artifact j to i on network g is induced by a set of intermediate, interconnected artifacts $k_1, k_2 \ldots k_{P_{ji}(g)-1}$ such that $g_{j,k_1} = g_{k_1,k_2} = \ldots g_{k_{P_{ji}(g)-1},i} = 1$, where $P_{ji}(g)$ is the *length* of the path. If there exists a path from j to i, we say i is *reachable* to j.[5] We denote a *reachable set* of artifact i by $\vec{\mathcal{N}}_i(g)$, i.e., the set of artifacts to whom i is reachable. The length of the shortest path between two artifacts is referred to as their *distance*. A *distance-d neighborhood* of artifact i, $\vec{\mathcal{N}}_i^d(g)$, is the collection of foregoing artifacts that are of distance-d on network g to artifact i.

2.2 Individual Author's Problem

Individual player i *maximize*s her payoff given as (1), by choosing a nonnegative level of *knowledge creation*, x_i, and the set of preceding artifacts to cite, g_i.

$$\pi_i(s_{i-1}; x_i, g_i) = f\left(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(G)} \delta^d x_j\right) - cx_i - kD_i^I(G), \tag{1}$$

In the rest of this section we shall explain the expression of (1) by terms. $s_{i-1} := (\mathbf{x}_{i-1}, G_{i-1})$ is the state of the system faced by player i, constituted by the collection of knowledge creations up to period $i-1$, $\mathbf{x}_{i-1} := (x_t)_{t=1\ldots i-1}$, and the network structure up to period $i-1$, G_{i-1}. Let $G := G_{i-1} \cup g_i$, that is, G is the network G_{i-1} added with player i's choice of links, g_i.

The player reaps a revenue that is increasing and concave with the total knowledge stored in her artifact, which includes both the new knowledge she created (called the *original* knowledge of that artifact) and the knowledge she cited from extant artifacts.[6] This setting is captured in (1) by an arbitrary concave and increasing function $f(\cdot)$. The marginal cost for creating one unit of knowledge is denoted by c. Each citation incurs a cost k to the player who seeks knowledge,[7] which summarizes the costs for locating the targeted artifact, digesting its content, and transcribing the knowledge into one's own work, etc. The concavity of knowledge benefits and linearity in costs have been used in literature such as [16] and [3], and reflect the economic notion that the marginal cost of knowledge acquisition will exceed its marginal benefit after the knowledge piles up to a certain amount, so that it

1. Appendix B of [25] considered individual-controlled (non-random) interactions, however the control policy was learning-based and is hence different from our model which is optimization-based.

2. Our model allows repeated arrivals of the same author to the system, if the author makes decisions each time to maximize identical payoff given by (1) (shown later in Section 2.2). This way, a repeated arriving author is treated as a new author every time. In case of repeated arrivals, the robustness analyses of Section 3 can be thought as varying the personal attributes of an author at each of her arrivals.

3. The direction of links indicates the direction of knowledge flow between artifacts.

4. $A := B$ means A is defined as B. $|S|$ denotes the number of elements in the set S, or cardinality of S.

5. Notice that reachability is defined in a reverse direction of the knowledge flow, for the convenience of analysis.

6. In this paper we consider citation not as a simple "copy-and-paste" of existing knowledge, but rather a transformation and remanufacturing of existing knowledge into one's own artifact. Thereby, one is also credited by, and benefits from, her cited knowledge (in addition to her innovated knowledge).

7. Put in network terms, this means that connecting each in-neighbor incurs a cost of k to the player.

is no longer rewarding to continue collecting knowledge. We make the following assumptions in the model.

Assumption 1. $f' > 0, f'' < 0, f(0) = 0, f'(0) > c, f'(\infty) < c$.

Assumption 2. *The player has complete information on the knowledge in existing artifacts and how these artifacts are connected in the citation network.*

Assumption 1 is made for the model to well-behave. Assumption 2 is an approximate for many epistemic communities where intellectual products and citation lists are transparent and synchronized, e.g., patent systems, academic journals. Underlying the complete information setup, there is also an associate assumption that, once completed, the knowledge artifact will be made public, with explicit references to existing works. This assumption is compatible with many field practices.[8] We relax Assumption 2 in the Electronic Companion,[9] in order to examine the effect of imperfect network information.

We define the ability that each author creates original knowledge as creativity, and measure it via knowledge creation cost, *c*. For the time being, we focus on homogeneous knowledge creativity of players but will show in Sections 3.2 and 4 that the main insight is robust to differentiated knowledge capabilities.[10]

In the development of a knowledge field, people who contribute their knowledge might hold diverse perceptions and comprehension on knowledge. As a result, a player cannot extract knowledge indiscriminately from any distinct artifact. Consider the example of scientific writing. When an author references an existing paper, she might find only part of knowledge in the cited paper relevant to her own research. In order to have our model capable in handling this situation, we suppose that each player can derive payoff from every unit of knowledge she herself creates, but can only benefit from a fraction, $\delta \in (0, 1)$, of knowledge stored in a distinct artifact. A lower value of δ indicates less compatible knowledge, or higher heterogeneity, across artifacts. We will relate the citation network structure and knowledge creation to δ in closed-form expressions. While the value of δ represents the *system-wise* heterogeneity of artifacts, we also introduce *pairwise* artifact heterogeneity—later in Section 3.1.

Via a citation link the player accesses the original knowledge stored in the cited artifact (discounted by δ), as well as the knowledge that the cited artifact cites elsewhere, but with heavier discount. In that spirit, a connected citation path transfers knowledge to the player from every artifact that lies on the path, but the transmitted knowledge gets more discounted from more distant artifacts in the citation network. In case that there are more than one paths linking two artifacts, we assume that the knowledge flow will only occur via the shortest path. This captures the perspective that a longer path results more loss of original knowledge due to knowledge incompatibility, and hence is dominated by a shorter path in terms of effective knowledge transition. Specifically, a player absorbs δ^d portion of the knowledge from an upstream artifact that is d-distance away.

At her decision epoch, player i maximizes her payoff given by (1), by choosing creation level x_i, and a set of preceding artifacts g_i to cite. The latter decision determines the player's position in the newly formed network G and thus affects the amount of knowledge that player i seeks from elsewhere in the network. After one's decision is made, the following state transition occurs: The profile of knowledge creation is updated to include the optimal knowledge created by player i, $\mathbf{x}_i = (\mathbf{x}_{i-1}, x_i^*)$, and the citation network updated to encompass artifact i and its optimal connections with previous artifacts, g_i^*, i.e., $G_i = G_{i-1} \cup g_i^*$.

Notice that, although the total amount of knowledge (original knowledge + cited knowledge) in one's artifact, $x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(G)} \delta^d x_j$, is settled by one's decisions in a single period, this body of knowledge will continuously generate benefit for the author throughout the entire horizon, and the total benefit generated is captured by the first term in (1), $f(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(G)} \delta^d x_j)$.[11] This setup captures the normal situation where the reward for one's publication is not immediately realized, but will be gradually collected in future (Say, one's work is valued by the reviews it receives over time).

Assumption 3. *There is an infinitesimal reduction of linking cost when citing a higher out-degree artifact.*

Assumption 3 is motivated by some common practice: For instance, Google's page-rank mechanism places heavily-cited records at the silent entries, which may *slightly* lower the search cost to users. The assumption is also a deterministic analogy to the protocol of preferential attachment commonly employed in the complex network literature (e.g., [1], [2]). That said, Assumption 3 is not as aggressive as it may first appear, since the citation cost difference by degree is negligible in all cases wherever payoff preference is strict.[12] Moreover, Assumption 3 can be removed from a model with direct citations while all results remain untouched (see Section 4). Neither necessary is Assumption 3 for a model with "two-way" citation links.[13]

8. In many scenarios like scientific publishing, the benefit to the author is only received after the artifact is made public. Therefore, it is reasonable to assume that the knowledge artifacts are publicly visible and thus quotable to later community members.

9. The URL of the Electronic Companion is provided at the end of this paper.

10. In Section 3.2 and Proposition 4 of Section 4 we modify the cost of knowledge creation to be individual-specific, and therefore differentiates knowledge capabilities.

11. Since the benefit of knowledge is gained over time, one may explicitly consider the time factor in our model. Denote by $\sigma \in (0, 1)$ the time discount on cash flow, and by $b(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(G)} \delta^d x_j)$ the benefit that one receives from her artifact at a single period, where $b(\cdot)$ is an increasing and concave function. Then re-denoting $f(\cdot) := \Sigma_{t=1}^{\infty} \sigma^{t-1} b(\cdot) = \frac{1}{1-\sigma} b(\cdot)$ will pass the increasing and concave properties to f. In this case, the player at period i solves $max_{x_i \geq 0, g_i}\, \sigma^{i-1} \pi_i(s_{i-1}; x_i, g_i))$, where $\pi_i(\cdot)$ is given by (1). Obviously, all the results from our model will survive the above complications to incorporate the time factor.

12. That means, artifacts with lower out-degrees will be cited if they yield strictly more payoff to the player than do higher out-degree artifacts.

13. Two-way citation means that the knowledge flows both ways through the citation link (from the cited one to the one cites *and* vice versa). It is the case when both the cited one and the one that cites receive knowledge from each other. Although not explicitly discussed in this paper, straightforward changes in the proofs of Propositions 1 and 2 suggest that the two propositions hold, without Assumption 3, for the case of two-way citations.

The Electronic Companion provides a numerical example for the optimization problem faced by individual authors.

2.3 Analytical Results

Our model delivers the following insights: First, it yields a layout of the citation network as accumulatively shaped by individual authors. Second, it generates a sequence of knowledge creations by individual authors. All the proofs are found in the appendix. For the convenience of analysis, let $y := argmax_x\{f(x) - cx\}$.

Proposition 1 (Homogeneous Artifacts). *When* $\delta = 1$,

a) *If* $cy > k$, *the citation network is structured as a star with Artifact 1 the core. Artifact 1 possesses a knowledge amount of* y. *Other artifacts in the network cite Artifact 1 but do not contain any original knowledge on their own.*

b) *If* $cy < k$, *each artifact possesses* y *units of knowledge but does not cite any other artifact.*[14]

When knowledge is fully compatible across individuals ($\delta = 1$), incentive to create own knowledge vanishes whenever knowledge citing is possible. As the consequence, the knowledge creation falls in two extremes depending on the costs: If citation is less expensive than producing the same amount of cited knowledge ($cy > k$), everyone solely seeks knowledge from a singleton knowledge core; otherwise ($cy < k$) each one produces her own knowledge but takes no inheritance of knowledge.

Proposition 2 (Heterogeneous Artifacts). *Suppose* $\delta \in (0, 1)$. *If* $c\delta y < k$, *each artifact holds* y *units of knowledge but does not cite any other artifact; otherwise there exists a threshold* $I \geq 1$, *determined as the largest artifact index* i *satisfying* $c\delta(1-\delta)^{i-1}y > k$, *such that*

a) *[citation network structure] Each artifact from 1 to* I *cites all the artifacts prior to it. From artifact* $I+1$ *onwards, each artifact cites artifacts from 1 to* I, *but not any artifact created later than artifact* I. *We refer to artifacts from 1 to* I *as the knowledge core, and artifacts* $I+1$ *and afterwards as knowledge periphery.*

b) *[knowledge creation] If artifact* t *belongs to the core, it has original knowledge at an amount of* $(1-\delta)^{t-1}y$. *If the artifact falls in periphery, the amount of its original knowledge is* $(1-\delta)^{I}y$.

When artifacts are homogeneous ($\delta = 1$), a singleton core provides adequate knowledge to serve the community-wide knowledge demands (Proposition 1). As the knowledge becomes partially communicable due to heterogeneity ($0 < \delta < 1$), players seek knowledge from not one but rather a variety of artifacts which collectively define the core (Proposition 2). The core size I is fully determined by the system parameters ($c, \delta, f(\cdot), k$) and is easily computable through a straightforward loop on these parameters (refer to Proposition 2). Fig. 1 gives two examples of the citation network structures resulted from our model, with core size

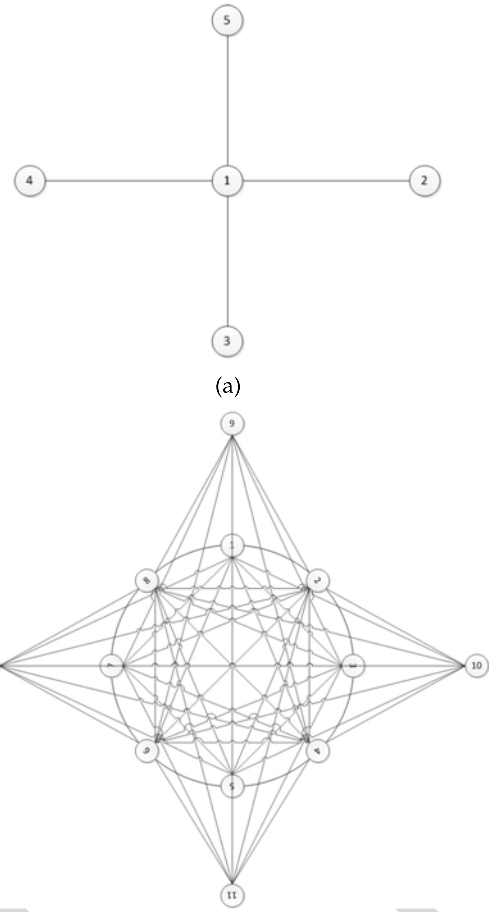


Fig. 1. Examples of core-periphery citation networks generated from our model.

being 1 (Fig. 1a, for Proposition 1) and 8 (Fig. 1b, for Proposition 2) respectively. The directions of knowledge flows are determined from the artifact indexes and omitted for brevity.

To understand the intuition of Proposition 2, notice the main drive of knowledge core emergence in our model is the decreasing marginal return of citations. Under Proposition 2, as a maximally connected citation network grows from size j to size $j+1$, the knowledge introduced to an incoming player by connecting all the existing artifacts increases by $\delta(1-\delta)^j y$ (which is the extra knowledge acquired from artifact $j+1$), which declines with the network size. We first ask: Is the player better off creating the same amount of knowledge by her own instead of citing it via a connection? At early phase of network evolution (j is small), the marginal knowledge return of citations are sufficiently high (i.e., $\delta(1-\delta)^j y$ is high when j is small) so that the answer to our first question is no. After the size of citation network exceeds a certain threshold such that $c\delta(1-\delta)^j y < k$, a successive player will find less costly creating by herself the knowledge that was formerly acquired by an additional citation. At this point the answer to the previous question turns yes. The number of citations stop increasing with the size of network. Subsequent players extract knowledge from a constant subset of predecessors (artifacts 1...j). Then, our second question

14. In this circumstance, like the many other cases in this paper, equality ($cy = k$) represents the indifference case and can technically join either side of the statement.

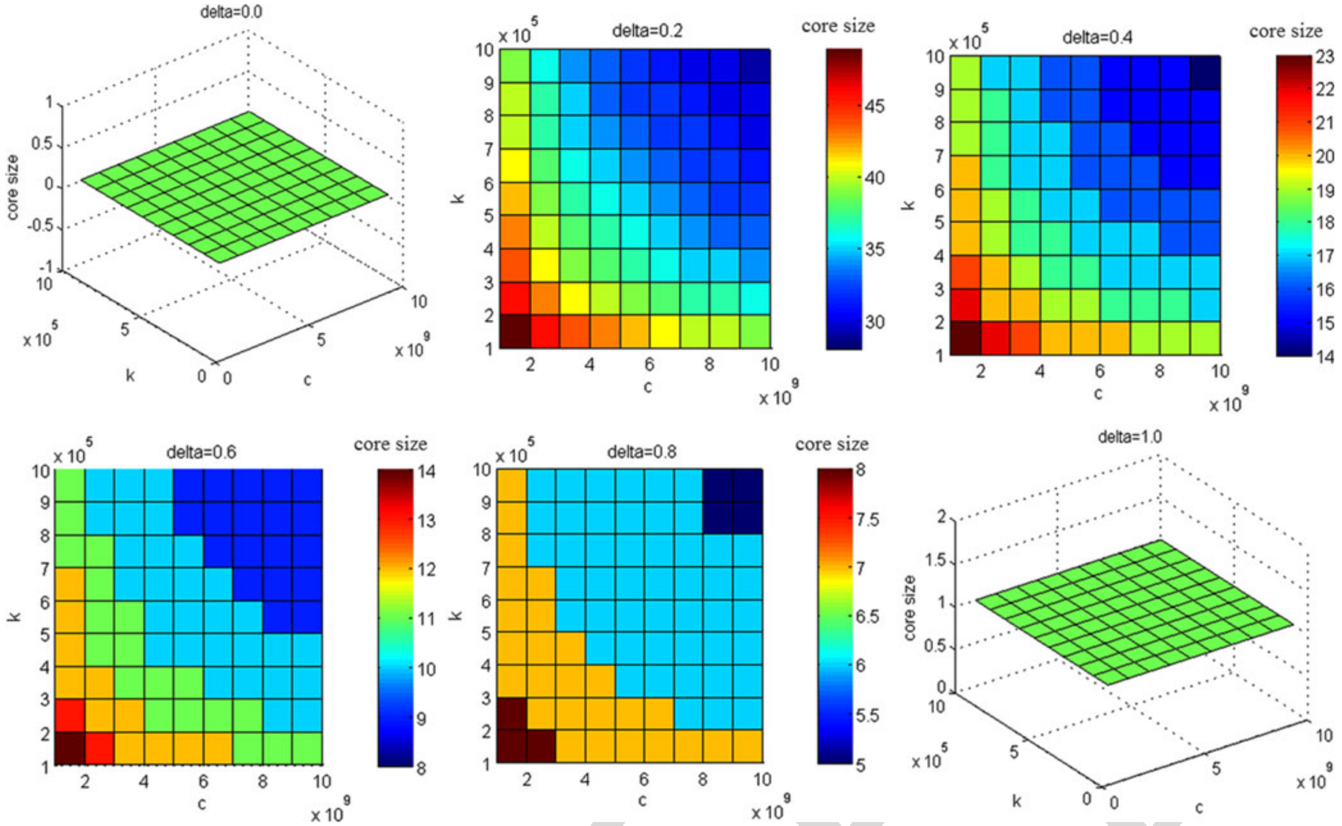


Fig. 2. Comparative statics: core size versus c, δ, k.

follows: Given the amount of cited knowledge, is it still worth the cost to create any new knowledge by oneself?

The answer to this question is yes, because the knowledge gathered from connections to artifacts 1 through j totals to $(1-(1-\delta)^j)y$ under Proposition 2, which is less than y for all finite j. The marginal benefit of self-producing knowledge hence remains above its marginal cost (since $f'(x) > c$ for all $x < y$). Furthermore, the player should produce knowledge up to the point where her total knowledge derived from creation and citation equals y. This leads her optimal amount of knowledge creation to $(1-\delta)^j y$, which reinforces the statement of Proposition 2.

2.4 Implications

The growth of the citation network is completely captured by the interaction of knowledge creation, citation, and artifact heterogeneity, i.e., c, k and δ. In order to shed more light on the citation network formation and knowledge provision, we shall next investigate comparative statics with respect to the above ingredients of the model, and the associated policy implications. We will begin with a graphical illustration on the impacts of c, k and δ on the size of knowledge core and (asymptotic) average knowledge creation, which are important metrics of the system performance. We choose $f(x) = 10^{10}\sqrt{x}$ as the benefit function, for the purpose of illustration.

Fig. 2 depicts the change of core size in response to the variation of c and k, under different levels of δ. We observe that the core size non-increasing with k, which confirms a prediction from Proposition 2. If $\delta = 0$ the citation network will sustain no connection. On the contrary, the core only includes one artifact when $\delta = 1$—a result of our Proposition 1. The core size overall decreases with c as well.[15] Next we study how the level of (asymptotic) average knowledge creation, $(1-\delta)^I y$,[16] is affected by the model parameters. As shown in Fig. 3, average knowledge creation generally rises with k—Knowledge citing being expensive encourages self supply of knowledge. If $\delta = 1$, the incentive for knowledge creation is eliminated, as players can exhaust the benefit of existing knowledge. On the other extreme, $\delta = 0$ forces players each supplying y units of knowledge independently.[17] Given the way c affects y and I, average knowledge creation generally declines with c, but in a non-smooth way due to the discrete change in core size.

A probably more direct goal of system planning would be to improve the *utility* of its users. In that line, one can make useful recommendations from the following corollary.

Corollary 1. *At optimum, the utility of a contributing author increases with δ, and decreases with c and k. The utilities of authors in the core rise with their artifact indexes. All*

15. To see that analytically, note $y = \frac{10^{20}}{4c^2}$ in our example. Thus $cy = \frac{10^{20}}{4c}$, which declines in c. Then the definition of I in Proposition 2 implies that it overall decreases with c. We thank an anonymous referee for raising this argument to us.

16. Notice that the asymptotic average knowledge creation amount is $(1-\delta)^I y$ in our present model, because the artifact population will be dominated by peripheral ones as the number of artifacts approaches infinity.

17. Notice y is lowered with increasing c but independent of k. That explains the curvature of graph for the $\delta = 0$ case in Fig. 3.

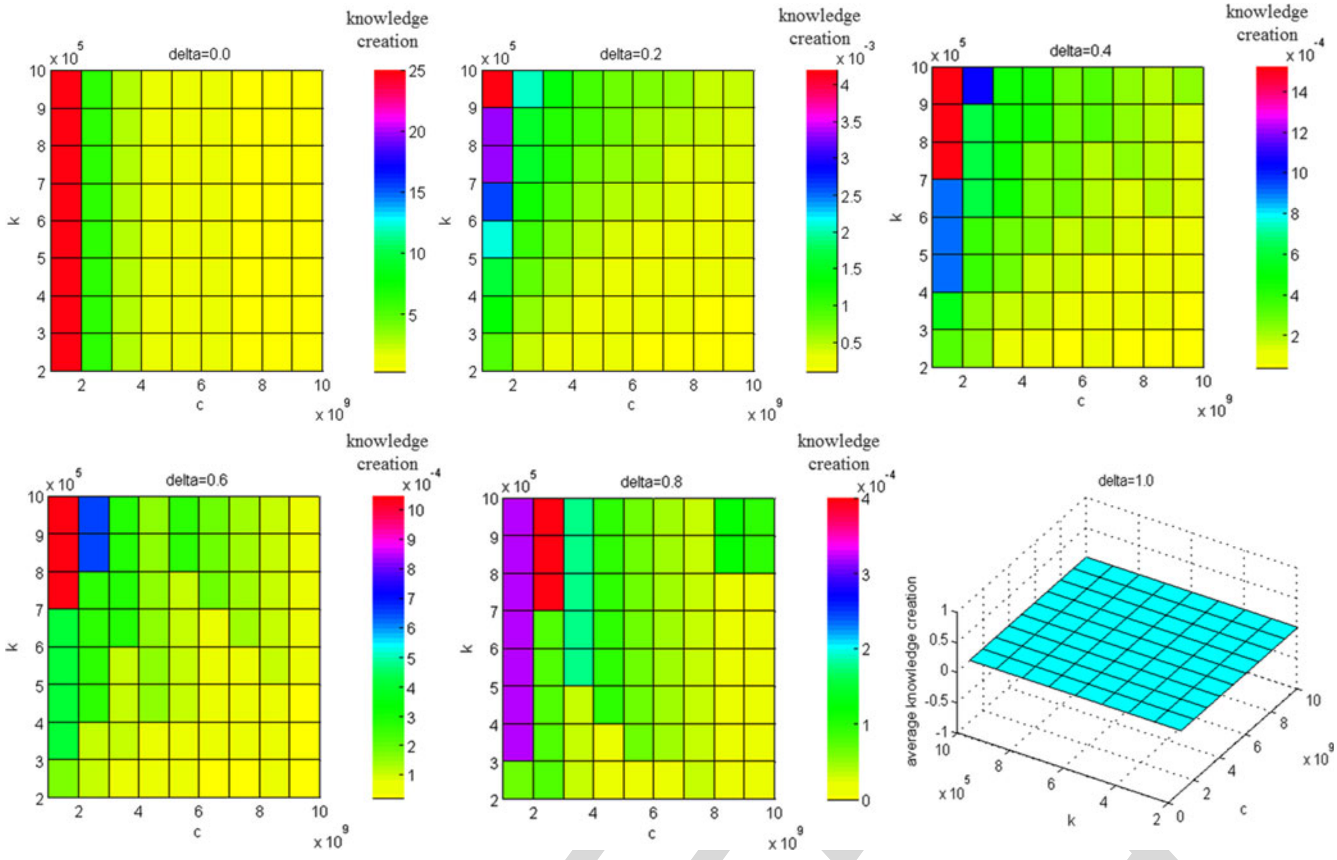


Fig. 3. Comparative statics: asymptotic average knowledge creation versus c, δ, k.

peripheral authors earn the same utility, which is less than the utility of Author I.

As Corollary 1 suggests, the designer or policy maker of the intellectual community should seek to reduce the costs of citation and knowledge creation, provide higher digest of existing knowledge, in order to raise the social welfare of knowledge contributors. These can be accomplished by means of maintaining up-to-date references, providing more efficient search engines, and offering professional training (e.g., courses, workshops) to the authors. As for an individual player, the timing of entry to the field also matters: While he or she should certainly attempt to take up a core position, a contribution made "too early" would be accompanied with the difficulty of citing extant knowledge, which incurs disutility to the author. In fact, a rational individual should aim to be the Ith one to make the contribution.

3 Robustness of the Model

This section examines the robustness of our model in different aspects, including artifact heterogeneity (Section 3.1) and knowledge creativity (Section 3.2). In electronic companion we continue to perform robustness tests on information incompleteness and initial system states . In this section we measure the core identity of an artifact by its Bonacich centrality in the citation network with linking directions removed (see footnote 19 for more details). Bonacich centrality ([6]) assigns one's centrality score as proportional to the sum of centrality scores of her neighbors, so that groups of highly connected nodes receive high marks.[18,19] In addition, we also identify the average path length, diameter, density, and average clustering coefficient for each network, in order to provide further recommendation on its core-periphery-ness.

3.1 Pairwise Heterogeneity

In previous sections we have established the artifact heterogeneity as an important factor in shaping the knowledge core. We shall further the investigation with a finer grained inspection on the concept of heterogeneity: Let δ_{ij} be the fraction of artifact i's knowledge that is useful to a

18. The exact statistical measurement of core-periphery structure is still generally understudied, and developing one of such would be out of the scope of this paper. See [8] for some discussion on the proximity between centrality and core-periphery-ness. In context of this paper, Bonacich centrality as an indicator of core-periphery-ness is adequate to reveal the insights of the model, and creates ease for numerical computations.

19. We base the structural evaluation on the network that has all linking directions removed, since Bonacich centrality is defined on undirected networks. Thus a citation connection contributes to the centrality of both the cited artifact and the citing one. This may seem to contradict the idea that core-ness should be evaluated by the number of times being cited (out-degree). Notice that, however, a core-periphery-ness measure based only on out-degrees will favor pioneering artifacts in the system. Therefore, if under our measure one detects a core-periphery division where the core appears to be the early-created artifacts (which will later be shown as what we primarily observed), a measure based exclusively on out-degrees would give the same result.

successive artifact j, and refer to δ_{ij} as *pairwise* artifact heterogeneity.[20] To differentiate, we refer to the case with all δ_{ij}, c, k the same as the *benchmark* case, and the resulting knowledge as the *benchmark* core.

By incorporating variation across δ_{ij}, the enriched heterogeneity notion leads to differentiated player preferences over prior artifacts (That is, for example, an artifact with high volume of original knowledge may be unattractive to a subsequent author because the latter's demand happens to be particularly misaligned with the knowledge stock of the former). Therefore, each player possesses a different perspective of what artifacts to be in the knowledge core.

To proceed, we simulate the formation of a citation network of 35 artifacts with 30 independent runs, with δ_{ij} following $U[15/32, 17/32]$. We set $f(x) = \sqrt{x}, c = .3$ and $k = .01$, which leads to a benchmark core size of 6 with $\delta = 0.5$ (corresponding to the mean of δ_{ij}). Bonacich centrality and knowledge creation levels of the artifacts within each run are shown in Fig. 4 (with box plots showing the minimum, and the first, second, third quartiles, and the maximum of the data for each artifact). For comparison purposes, we mark in Figs. 4a and 4b by the red dotted line the division of core and periphery in the benchmark case with $\delta = 0.5$. For the purpose of illustration, we visualize one network randomly selected from our simulation (Fig. 4c), with some relevant statistics included in the display. In (Fig. 4c, the centrality of a node is reflected in its color: Darker nodes have higher centrality. The top centrality score is normalized to 1 in Fig. 4a.

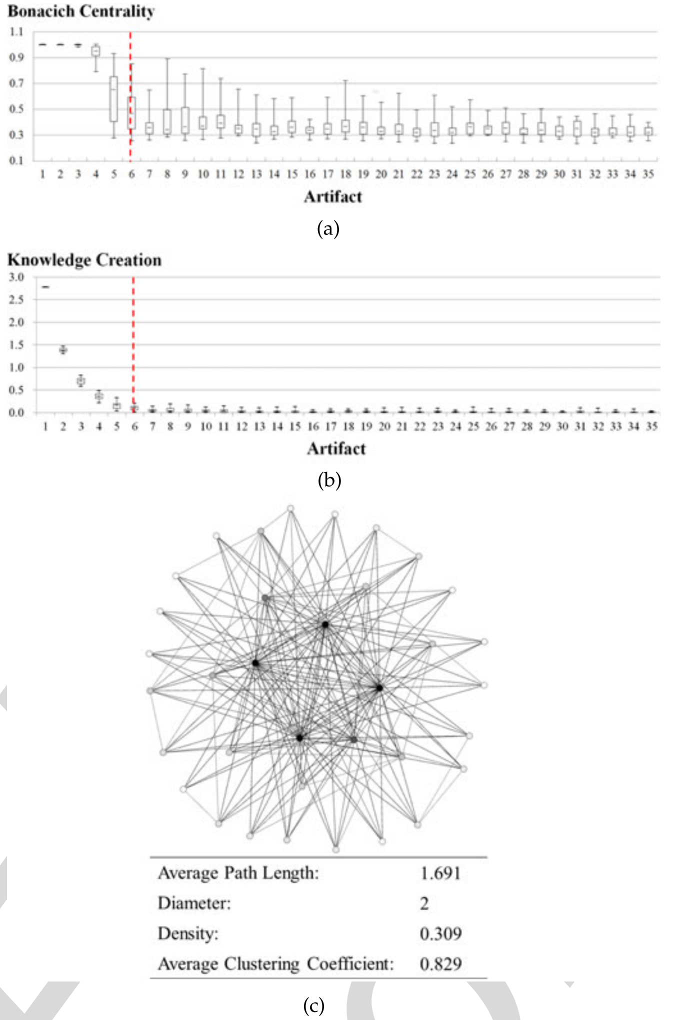


Fig. 4. Knowledge creation and core identity of players under pairwise heterogeneity.

We have three remarks regarding Fig. 4. First, on average there is a significant gap of centrality between early- and late-created artifacts in the system. Artifacts invented prior to others enjoy high citations and assume the central locations in the network. This observation is supported by the overall decreasing trend of centrality scores over simulation runs, as shown in Fig. 4a. Second, we observe some variation of the centrality scores of later-created artifacts across simulation runs, whereas the variation tends to be reducing over time. The stabilized centrality over time implies that later players tend to place citations to a constant group of prior artifacts—that is a sign of formation of the knowledge core. Third, the volume of original knowledge contained in artifacts dwindles with time (Fig. 4b). These occur since the total knowledge in the system keeps accumulating in amount and the connectivity of citation network continuously being enriched, thereby creating potentially more opportunities for citations and reducing the incentive for creating new knowledge.

Altogether, our simulation suggests the emergence of a core-periphery type of architecture in citation networks under pairwise artifact heterogeneity. In such networks, forerunning players are the main workforce to supply knowledge, and build their centrality to the system by large out-degrees of their artifacts. Subsequent entrants mostly place citations to prior artifacts, rather than creating their own knowledge. The limited original knowledge in later artifacts combined with their non-attractive network positions daunt future citations directed to them, and hence enclose the core. The levels of centrality of subsequent artifacts are mainly contributed by their in-degrees.

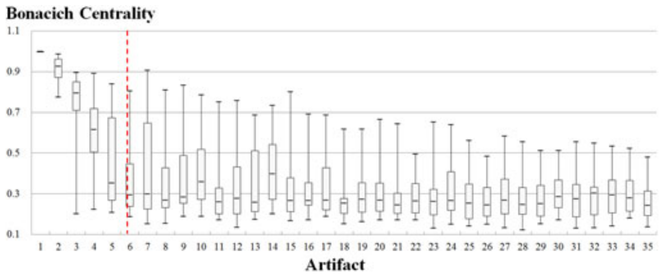
However, since players are granted with individual-specific preferences over existing artifacts, each of them might cite a different portion of core knowledge, and may even seek peripheral knowledge occasionally. This blurs the absolute boundary between knowledge core and periphery. We conclude that the core-periphery structure of citation networks demonstrate a degree of robustness with respect to the generalized knowledge heterogeneity (δ_{ij}), whereas a proper classification of core/peripheral knowledge needs to be based on the *relative* frequency of citations.

3.2 Heterogeneous Knowledge Creativity

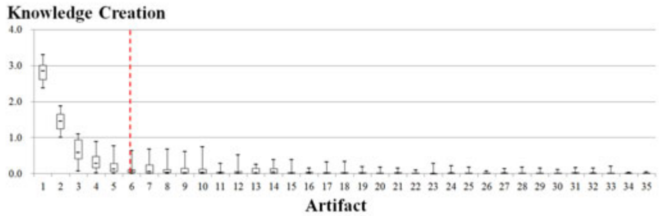
We have focused on the knowledge heterogeneity across artifacts while viewing authors as homogeneous in the capability of creating new knowledge. As a consequence, the credits of producing the core knowledge are strictly given to the pioneers of the field.[21] This fits in the

20. If there are multiple paths between two artifacts, knowledge is transferred on the path that carries the largest amount of discounted knowledge, i.e., the path $k_1 \rightarrow k_2 \rightarrow ...k_m$ that has the greatest value $\Pi_{i=1}^{m-1} \delta_{k_i k_{i+1}}$. Such path coincides the shortest-length path in the baseline model (Section 2) where δ_{ij}s are equal across i and j.

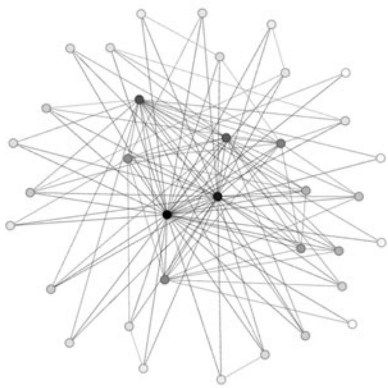
21. Empirically, [24] presents evidence that early created patents generally have higher citations, which supports our theoretical insight.



(a)



(b)



Average Path Length:	1.802
Diameter:	2
Density:	0.198
Average Clustering Coefficient:	0.836

(c)

Fig. 5. Knowledge creation and core identity of artifacts under heterogeneous knowledge creativity.

circumstances where authors do not significantly differ in their skills and education, or those who are apparently defected in ability have been pre-screened out of the field. Nonetheless, it is certainly worthwhile investigating situations where differentiated knowledge creativity should be assumed, e.g., cases where some knowledge contributions are considered "enormous" compared to others, because of the superior ability of their authors. That said, this section studies the resilience of core-periphery structure to heterogeneous knowledge creativity.

Denote by c_i the unit cost of knowledge creation of player i. We simulate a 35-artifact system with 30 independent runs with c_i drawn randomly from a uniform distribution between .275 and .325. We fix $f(x) = \sqrt{x}, \delta = .5, k = .01$, which results in a 6-artifacts benchmark core with $c = 0.3$ (corresponding to the mean of c_i). The Bonacich centrality of artifacts and the volume of created knowledge are presented in Fig. 5 (where box plots show the quartiles of the data). For comparison, we mark in Figs. 5a and 5b by the red dotted line the closure of benchmark core in the corresponding case ($c = 0.3$). Also included is a demonstration of a particular network generated from our simulation, together with its statistical features (Fig. 5c). In Fig. 5c, nodes with darker color have higher centrality (with the highest centrality score normalized to 1 in Fig. 5a.). Fig. 5 exhibits features similar to those of Fig. 4 (c.f. centrality gap and diminishing variation of centrality over time (Fig. 5a), overall decreasing knowledge creation (Fig. 5b)). Arguments analogous to those in Section 3.1 suggest that a core-periphery type of network topology appears. On average, the pioneering artifacts keep high original knowledge and undertake core locations. However there are exceptions in some individual runs. For example in Fig. 5a, the top centrality scores earned by artifacts 8-12 (which are outside the benchmark core) are all greater than those at 75th percentile of artifacts 4-6 (which are core artifacts in the benchmark case). Therefore, while the core-periphery architecture survives some degree of variation of knowledge creativity, there are chances for belated authors of sufficiently high knowledge creativity to still place their artifacts into the knowledge core.

4 Direct Citation

In some cases, one may benefit only from the original knowledge of the cited artifact, but not the knowledge that the cited artifact cites elsewhere, i.e., citations are "direct". This section analyzes a direct citation model. We drop Assumption 3, while maintaining the rest of assumptions from Section 2.

Proposition 3. *Proposition 2 carries exactly to the model with direct citations.*

Proposition 3 shows that the *exact* core-periphery structure carries over to direct citation networks. To understand it, notice in a maximally connected network, the amount of one's cited knowledge does not differ by scenarios of direct/indirect citations. Therefore, one can utilize the same intuition that supports Proposition 2 to establish the threshold at which players stop citing all preceding artifacts (but rather citing a subset of them referred as the knowledge core). Noticeably, the formal proof of Proposition 3 does not require Assumption 3. Next, we inspect the robustness of results of the direct-citation model to a disturbance of homogeneous knowledge creation cost, c.

We suppose, while others possess homogeneous benefit and cost structure, there is a player indexed with n who is intrinsically advantageous in creating new knowledge. Denote by $\underline{c}$ her cost of knowledge creation and $\bar{f}(\cdot)$ her benefit function, while those of other players remain c and $f(\cdot)$. $\underline{c} \leq c$. Let $\bar{y} := argmax_x\{\bar{f}(x) - \underline{c}x\}$ and assume $\bar{y} > y$. We refer to the *original core (periphery)* as the core (periphery) formed by players having homogeneous c and $f(\cdot)$, and denote by I the size of the original core. Recall that x_i^* is the optimal knowledge creation level of player i.

Obviously the citation network shaped prior to artifact n will remain unaltered. We will investigate the change afterwards—this is summarized in Proposition 4 below.

Proposition 4. *For the direct citation model, suppose player n is mutated to have $\bar{y} > y$ and $\underline{c} \leq c$. In this case, the* *core-periphery network structure remains to hold. However, compared to the case with original core,*

a) *[If artifact n was within the original knowledge core, i.e., $n \leq I$] Artifact n remains in the knowledge core, but the core size is reduced. The knowledge creation levels of all players succeeding player n are reduced.*

b) *[If artifact n was outside the original knowledge core, i.e., $n \geq I+1$] There exists a threshold $\underline{Y}$. If $\bar{y} > \underline{Y}$, artifact n is quoted by all the succeeding artifacts and thus becomes a member of the core. The knowledge creation levels of artifacts afterwards are all reduced. If $\bar{y} < \underline{Y}$, artifact n will not be cited and remain peripheral. The knowledge creation levels of subsequent artifacts are unaffected.*

If the knowledge creation cost of the author of an original core artifact is lowered, the author would offer more knowledge for subsequent core artifacts to seek, thereby shrinking their supply of original knowledge. This reduces the number of artifacts in a knowledge core (part a, Proposition 4). A more interesting question is what happens if the knowledge creativity of an author of an originally peripheral artifact is increased. In this case, if $\bar{y}$ surpasses a certain level $\underline{Y}$ ($\underline{Y}$ is specified in Appendix A6), the artifact produced by this author will be admitted into the knowledge core (part b, Proposition 4). Hence, $\underline{Y}$ represents the threshold amount of knowledge required for a player to produce a core artifact, overcoming the disadvantage of a late entry to the field.

5 Conclusion and Limitation

Through a simple multi-agent model, we explain the emergence of core-periphery architecture in citation networks as an accumulative process, driven by voluntary knowledge contribution from self interested individual authors. We characterize the relationship between knowledge creation, citation, knowledge heterogeneity and the properties of knowledge core, in both cases of direct and indirect citations. Our simulations show that the core-periphery network topology survives extensions on knowledge heterogeneity and knowledge creativity (cost of knowledge creation).

Citation network is often regarded as a "window on the knowledge economy" (a quote from [21]). Our model differs from conventional descriptive analysis of citation networks in its incentive-based approach, which yields some new insights that were not available from descriptive modeling. First, the citation network configuration in our model is jointly determined with the supply of new knowledge. Second, our model reveals factors that steer the growth of citation networks, and in particular, shape its core-periphery landscape: knowledge creation, knowledge citation, and knowledge heterogeneity. For these factors, we provide further insights on their influences on knowledge contribution as well as on the utility of knowledge contributors. These findings may potentially assist a policy maker with the design and development of knowledge-oriented or scientific communities to sustain contributions of original knowledge and improve the social welfare.

Our model can be generalized on the cost structure. The total cost could, for example, be defined as a multivariate function of created knowledge and cited knowledge, and exhibit convexity on each cost term. This reflects the scenario where knowledge acquisition and creation become increasingly difficult as more knowledge has been incorporated into one's artifact. Based on the formulation above, one may also specify the super-/sub-modularity of the total cost function as appropriate.

We consider knowledge creation and citation as simultaneous choices from an individual author. It remains an interesting question whether the core-periphery network layout is robust to sequential, and potentially correlated decisions on knowledge creation and citation. For instance, an author may first cite existing artifacts, then create her own knowledge at a cost negatively affected by the volume of cited knowledge.

Given the dynamic nature of citation network development, strategic behavior might be salient in its own right. For instance, an author who makes repeated entries might expect forthcoming activities of other authors and best-respond to those. Under those circumstances, a sequential game model should be formulated and analyzed with appropriate equilibrium notions.

Appendix
Proofs

A1 Proof of Proposition 1

Player 1 solves $max_{x_1}\{f(x_1) - cx_1\}$ since as first arriver he has no artifact to quote and hence only decides on knowledge creation. Recall that y is his optimal knowledge creation level.

Player 2 solves

$$\begin{cases} max_{x_2}\{f(x_2) - cx_2\} & if\, g_{12} = 0 \\ max_{x_2}\{f(x_2 + y) - cx_2 - k\} & if\, g_{12} = 1. \end{cases}$$

The solution for player 2 is

$$\begin{cases} cy > k & x_2^* = 0 \,\&\, g_{12} = 1 \\ cy < k & x_2^* = y \,\&\, g_{12} = 0. \end{cases}$$

For player 2 through player i, set up the following *hypothesis* on their optimal decision rule:

$$\begin{cases} \text{If } cy > k, & x_i^* = 0, g_{1i} = 1 \text{ and } g_{ji} = 0\, \forall j \in \{2, \ldots i-1\} \\ \text{If } cy < k, & x_i^* = y,\, g_{ji} = 0\, \forall j \in \{1, \ldots i-1\} \end{cases}$$
$$(cy = k \text{ can be put on either side}).$$

We shall verify that the hypothesis holds for player $i+1$. Then by induction the proposition is proved.

When $cy < k$, if the player cites a given number of n artifacts, then $\pi_{i+1}(s) = f(x_{i+1} + ny) - cx_{i+1} - nk$, which can be rewritten as $\pi_{i+1}(s) = f(x_{i+1} + ny) - c(x_{i+1} + ny) + n(cy - k) < f(x_{i+1} + ny) - c(x_{i+1} + ny)$. Note that the ultimate right-hand side of the inequality gives the player at most the payoff as he would optimally acquire with no link. So it is concluded that linking any n artifacts is payoff-inferior to having no links. Therefore $x_{i+1}^* = y, g_{j,i+1} = 0\, \forall j$.

When $cy > k$, first we notice that artifact $i+1$ is reachable to artifact 1 if player $i+1$ ever cites any preceding artifact. Now, if player $i+1$ ever chooses to quote any artifact, he would only link to *exactly one* artifact, since 1) one link is enough for him to absorb knowledge created by player 1; 2) by hypothesis any more links will not give him more benefit (All other players except player 1 create zero knowledge), but rather more cost. In addition, if the player implements only one link, he will elect to link Artifact 1; because Artifact 1 has an out-degree higher than others, and thus incurs an infinitesimally lower cost of citation to the player than does any other artifact (recall Assumption 3). So, we only need to compare player $i+1$'s optimal payoffs when he has no link and when he has one link quoting knowledge from Artifact 1. If he has no link, his payoff is $f(x_{i+1}) - cx_{i+1}$; with linking Artifact 1 the payoff is $f(x_{i+1}+y) - cx_{i+1} - k = f(x_{i+1}+y) - c(x_{i+1}+y) + cy - k \underbrace{>}_{cy>k} f(x_{i+1}+y) - c(x_{i+1}+y)$. Again, the right end of the above inequality yields the player equal payoff as he would optimally acquire with no link (assuming the resulting $x^*_{i+1} \geq 0$, which is shown true). Thus player $i+1$ should quote Artifact 1 and produce no knowledge on his own.

Since player $i+1$ satisfies the hypothesis, then by induction Proposition 1 is proved.

A2 Proof of Proposition 2

The proposition holds for player 2 (since player 1 simply produces y). Suppose it holds for player $t-1$; we will show it is true for player t. Then by induction the proof is finished. We begin with specifying the structure of the decision problem faced by an individual player. Denote $v_m(g) := c \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j - km$. Then player i's payoff function (1) can be rewritten as $\pi_i(s) = f(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j) - c(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j) + v_{|\vec{\mathcal{N}}_i^1(g)|}(g)$.Thus the optimization problem of a player i can be described as follows. First the player chooses citation g_i to maximize the second part of his utility function, $v_{|\vec{\mathcal{N}}_i^1(g)|}(g)$. Then he selects a level of knowledge creation x_i to maximize the first part, $f(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j) - c(x_i + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j)$. This is (by the concavity of $f(\cdot)$) equivalent to choosing x_i^* such that $f'(x_i^* + \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j) = c$, or equivalently (by the definition of y) $x_i^* = y - \sum_d \sum_{j \in \vec{\mathcal{N}}_i^d(g)} \delta^d x_j$. If the resulting x_i^* is nonnegative, $\pi_i(s)$ is globally maximized (It is *sufficient* to maximize the sum of the two terms, if the two achieve their own maximums simultaneously at *one single choice* of decision variable.). That provided, we are able to show the following lemma, as it is useful for the remainder of the proof.

Lemma 1. *A single citation of an existing artifact with positive original knowledge yields a knowledge amount of δy to the author.*

Proof. Since the lemma presumes the cited artifact has nonnegative original knowledge, then by foregoing analysis it must possess exactly a total of y units of knowledge in it (original + cited knowledge). Citing that single artifact then brings in δy knowledge. Lemma 1 is proved. □

We first prove for the case $c\delta y < k$, as the proof parallels the $cy < k$ part of Proposition 1: Artifact 1 has y units of knowledge. If player 2 cites it, she faces $v_{|\vec{\mathcal{N}}_i^1(g)|}(g) = c\delta x_1 - k = c\delta y - k$. To maximize player 2 should not cite artifact 1, and consequently, produce y units of knowledge in her artifact (by solving the reduced problem $max_{x_2} \{f(x_2) - cx_2\}$). The subsequent player finds only isolated artifacts in the system. Thus the reason that prevents citing one single artifact also negates citations of any arbitrary set of artifacts. Therefore each artifact after that holds y units of knowledge but does not connect to any other artifact.

The case $c\delta y > k$ is less straightforward. To proceed, we decompose the problem of maximizing $v_{|\vec{\mathcal{N}}_i^1(g)|}(g)$ to *inner decision* and *outer decision*. After determining the optimal citation (g_i) and computing the maximal $v_{|\vec{\mathcal{N}}_i^1(g)|}(g)$, we will find the corresponding optimal knowledge creation and verify it is indeed nonnegative.

Inner decision. Inner decision addresses that, *suppose the player t will execute a fixed number citations, say m citations, which set of artifacts he should cite to maximize the amount of cited knowledge.*[22] Label the m artifacts cited by t by $j_1, j_2 ... j_m$ such that $j_1 > j_2 > ... j_m$. Note that by definition $j_1 \geq m$, and $m \leq t-1$. In cases without confusion, we will call an artifact simply by its index (e.g., Instead of saying "artifact j is connected" we will simply say "j is connected").

[Case $t \leq I+2$]: Since $t \leq I+2$, all the cited artifacts should belong to the set $\{1 \dots I+1\}$, which by Hypothesis has been maximally connected. Now we compute $v_m(g)$ in two steps: First show how much knowledge player t obtains from citing j_1; then calculate the knowledge brought-in by the rest of $m-1$ citations (citations with $j_2 \dots j_m$). The sum of the two will be the total knowledge introduced via the m citations. By Lemma 1 linking with *any* j_1 brings δy to player t. Notice citing j_1 would make the player stays distance-2 to $j_2 \dots j_m$, since $j_2 \dots j_m$ are cited by j_1 (by Hypothesis and the labelling $j_1 > j_2 > \dots j_m$). Therefore, the distances between player t and artifacts $j_2 \dots j_m$ change from 2 to 1 by the citations. This way brings in additional knowledge by means of reducing the knowledge discounts. Specifically, the amount of additional knowledge introduced by link to any$j$ ($j \in \{j_2 \dots j_m\}$) is $(\delta - \delta^2)x_j^*$. Note that by Hypothesis $x_1^* > x_2^* > \dots x_m^*$. So $j_2, j_3 \dots j_m$ should be chosen as artifacts with the smallest indexes (artifacts $m-1, m-2 \dots 1$), so that the amount of knowledge sought from citations is maximized. Therefore, with a fixed number m of links, the player should link to $\{1 \dots m-1\} \cup \{j_1\}$ where $j_1 \geq m$. That leads to $v_m(g) = (\delta + (\delta - \delta^2) \sum_{j=1}^{m-1} (1-\delta)^{j-1}) cy - km = (1 - (1-\delta)^m) cy - km$.

[Case $t \geq I+3$]: Suppose player t has cited j_1. Let j be a artifact from the set $\{j_2 \dots j_m\}$ and Δ_j the marginal knowledge introduced via the link with j. By Hypothesis,

22. Notice by definition, $v_m(g) :=$ $c*$amount of cited knowledge $- km$. The amount of cited knowledge depends on the number of citations, m, and how these citations are chosen. The inner decision maximizes the amount of cited knowledge for any given m, by determining which m artifacts to cite. The outer decision then determines the optimal value of m to maximize $v_m(g)$, based on the optimal inner decision.

- If $j_1 \leq I+1$, then we have $\Delta_j = (\delta - \delta^2) x_j^* = \delta(1-\delta)^j y$.
- If $j_1 \geq I+2$, then
  - for $j \in \{1...I\}$, $\Delta_j = (\delta-\delta^2)x_j^* = \delta(1-\delta)^j y$;
  - for $j \in \{I+1...j_1-1\}$, $\Delta_j = \delta x_j^* = \delta(1-\delta)^I y$.[23]

Notice $\Delta_j \geq \delta(1-\delta)^I y$ for any $j \in \{1\dots I\}$. Hence the player should prefer quoting the set $\{1\dots I\}$ over $\{I+1\dots t-1\}$ (in addition to the link with j_1).[24] It is now easy to see:

- If $m \leq I$, the player t should link $\{1\dots m-1\} \cup \{j_1\}$ (with $j_1 \geq m$).
- If $m \geq I+1$, artifacts $1\dots I$ should be cited. Player t is then indifferent in placing his remaining links to whichever artifacts among $I+1\dots t-1$. So he will form connections with a randomly selected subset of $m-I$ artifacts from the set $\{I+1\dots t-1\}$ (in addition to quoting the set $\{1\dots I\}$).

Outer decision. We have studied player t's inner decision: which artifacts he should cite given m links, to maximize the amount of cited knowledge. Next we will examine his *outer decision*: *determining the optimal number* (m^*) *of citation links that player t should place to maximize* $v_m(g)$. After that, we shall compute the resultant x_t^* and check its nonnegativity, so as to complete the maximization of $\pi_i(s)$.

[$t \leq I+1$]: In this case $m \leq I$. If we increase the number of citations from $m-1$ to m, we have $v_m(g) - v_{m-1}(g) = cy\delta(1-\delta)^{m-1} - k \geq cy\delta(1-\delta)^{I-1} - k > 0$ (recall the derivation of $v_m(g)$ for inner decision when $t \leq I+2$). Therefore, m should be increased to its upper bound, i.e., $m^* = t-1$. That means, player t should cite all prior artifacts. The total amount of cited knowledge is $\delta\Sigma_{j=1}^{t-1} x_j^* = \delta\Sigma_{j=1}^{t-1}(1-\delta)^{j-1}y = (1-(1-\delta)^{t-1})y$. Consequently $x_t^* = y -$ amount of cited knowledge $= (1-\delta)^{t-1}y > 0$.

[$t = I+2$]: Obviously we should increase the number of links until I. After that, because $v_{I+1}(g) - v_I(g) = cy\delta(1-\delta)^I - k < 0$. m should stop increasing and $m^* = I$. According to the optimal inner decision, player t will be indifferent between citing $\{1\dots I-1, I\}$ and citing $\{1\dots I-1, I+1\}$. In this situation the player will prefer connecting artifacts with higher out-degrees for a infinitesimally lower cost (Assumption 3), and thereby cite $\{1\dots I\}$. Her optimal knowledge creation is therefore $x_t^* = (1-\delta)^I y > 0$.

[$t \geq I+3$]: Again, definitely m should be raised to I. For $m \geq I+1$, $v_m(g) - v_{m-1}(g) = cy\delta(1-\delta)^I - k < 0$.[25] Thus, $m^* = I$. Under Assumption 3, in this case player t cites artifacts $\{1\dots I\}$ and produces $x_t^* = (1-\delta)^I y > 0$.

Joining the results from inner decision and outer decision, it is straightforward to verify that the proposition holds for player t. Then by induction the proof is complete.

A3 Proof of Corollary 1

First, we need to derive the expression of the *optimal* profit of individual authors, π_t^*, given the system conditions c, δ, k. Recall player i's payoff function (1) can be written as $\pi_i(s) = f(x_i + \sum_d \sum_{j\in\vec{\mathcal{N}}_i^d(g)} \delta^d x_j) - c(x_i + \sum_d \sum_{j\in\vec{\mathcal{N}}_i^d(g)} \delta^d x_j) + v_{|\vec{\mathcal{N}}_i^1(g)|}(g)$, where $v_m(g) := c\sum_d \sum_{j\in\vec{\mathcal{N}}_i^d(g)} \delta^d x_j - km$. Per Proposition 2,

$$\pi_t^*(c,\delta,k) = \begin{cases} f(y) - cy + c\delta\sum_{i=1}^{t-1}(1-\delta)^{i-1}y & \\ -k(t-1) & \text{if } t \leq I \\ & \\ f(y) - cy + c\delta\sum_{i=1}^{I}(1-\delta)^{i-1}y & \\ -kI & \text{if } t > I \end{cases} \quad (2)$$

Rearranging, we have

$$\pi_t^*(c,\delta,k) = \begin{cases} f(y) - c(1-\delta)^{t-1}y - k(t-1) & \text{if } t \leq I \\ f(y) - c(1-\delta)^I y - kI & \text{if } t > I. \end{cases} \quad (3)$$

[To show $\frac{\partial \pi_t^*}{\partial c} < 0$]: For any range c_I, *where the core size I stays invariant with* c, we have

$$\frac{\partial \pi_t^*}{\partial c} = \begin{cases} f'(y)\frac{\partial y}{\partial c} & \\ \quad -(1-\delta)^{t-1}\left(y + c\frac{\partial y}{\partial c}\right) & \text{if } t \leq I \\ & \\ f'(y)\frac{\partial y}{\partial c} & \\ -(1-\delta)^I\left(y + c\frac{\partial y}{\partial c}\right) & \text{if } t > I, \end{cases} \quad (4)$$

$$\underbrace{=}_{f'(y)=c} \begin{cases} (1-(1-\delta)^{t-1})c\frac{\partial y}{\partial c} & \\ -(1-\delta)^{t-1}y & \text{if } t \leq I \\ (1-(1-\delta)^I)c\frac{\partial y}{\partial c} & \\ -(1-\delta)^I y & \text{if } t \leq I. \end{cases} \quad (5)$$

$$\underbrace{<}_{\frac{\partial y}{\partial c}<0} \quad 0 \quad (6)$$

That is, π_t^* decreases with c within the interior of c_I. It remains to show the same holds true at the boundary points of c_I. To proceed, notice at one such point where $c\delta(1-\delta)^{I-1}y = k$ holds, the core size is shrinking from I to $I-1$.[26] Then the change in the optimal payoff is as below.

$$\pi_t^*\big|_{I-1} - \pi_t^*\big|_I = c(1-\delta)^I y - c(1-\delta)^{I-1}y + k \quad (7)$$

$$= c(1-\delta)^{I-1}y(-\delta) + k \quad (8)$$

$$= 0 \text{ if } t \geq I+1 \quad (9)$$

23. By hypothesis, j would be peripheral artifact if $j \in \{I+1\dots j_1-1\}$. Since the network positions of j and j_1 are symmetric, citing j will not reduce any discount on the knowledge obtained from existing core and peripheral artifacts. Therefore the additional knowledge brought in by citing j is simply $\Delta_j = \delta x_j^*$.

24. By the hypothesized network structure and the way we define Δ_j, the knowledge gained from the set $\{j_2\dots j_m\}$ equals to the sum of Δ_j over $j \in \{j_2\dots j_m\}$. Thus we are able to establish the set preference relation (i.e., $\{1\dots I\}$ preferred over $\{I+1\dots t-1\}$) by comparing knowledge amounts derived from a typical artifact in each set.

25. To understand that, recall the optimal inner decision when $t \geq I+3$ and $m \geq I+1$ is citing $\{1\dots I\}$ plus a randomly selected subset of $m-I$ artifacts from the set $\{I+1\dots t-1\}$. Therefore when the number of citations increases by one, the additionally introduced knowledge amount is δx_j^*, where j is the peripheral artifact newly involved into citations of player t. Thus, $v_m(g) - v_{m-1}(g) = c\delta x_j^* - k = cy\delta(1-\delta)^I - k < 0$.

26. Note that in general, the core size I varies with c nonmonotonically. The case under current examination is that I decreases with c. The other case with increasing core size will be addressed later.

and

$$\pi_t^*\big|_{I-1} - \pi_t^*\big|_I = 0 \text{ if } t \leq I, \tag{10}$$

since in this case artifact t remains in the core before and after the change.

As for the other endpoint where $c\delta(1-\delta)^I y = k$ holds and the core size shifting from I to $I+1$,[27]

$$\pi_t^*\big|_{I+1} - \pi_t^*\big|_I = -(\pi_t^*\big|_I - \pi_t^*\big|_{I+1}) \tag{11}$$

$$= c(1-\delta)^I y(-\delta) + k \tag{12}$$

$$= 0 \text{ if } t \geq I+1 \tag{13}$$

$$\pi_t^*\big|_{I+1} - \pi_t^*\big|_I = 0 \text{ if } t \leq I. \tag{14}$$

Therefore, the change of π_t^* is continuous at boundaries of all intervals $c_I, \forall I$. Thus the piecewise monotonicity (6) applies globally. We conclude that π_t^* declines with c.

[To show $\frac{\partial \pi_t^*}{\partial k} < 0$]. Within any interval of k that holds the core size constant, obviously we have $\frac{\partial \pi_t^*}{\partial k} < 0$ for both core and peripheral artifacts. Then at the thresholds of k where the core size steps down[28] (i.e., $c\delta(1-\delta)^{I-1}y = k$), we have

$$\pi_t^*\big|_{I-1} - \pi_t^*\big|_I = c(1-\delta)^I y - c(1-\delta)^{I-1} y + k \tag{15}$$

$$= c(1-\delta)^{I-1} y(-\delta) + k \tag{16}$$

$$= 0 \text{ if } t \geq I+1 \tag{17}$$

and

$$\pi_t^*\big|_{I-1} - \pi_t^*\big|_I = 0 \text{ if } t \leq I. \tag{18}$$

Hence, the changes of optimal payoff across the indifferentiable boundary points are smooth. π_t^* falls down with the citation cost k, for both core and peripheral authors.

[To show $\frac{\partial \pi_t^*}{\partial \delta} > 0$]. For fixed core sizes, it immediately follows that $\frac{\partial \pi_t^*}{\partial \delta} > 0$. This conclusion can be expanded onto the entire range of δ by similar approach as above.

Finally, we show how the player utility changes over time. Based on (3), if $t \in$ core ($t \leq I$),

$$\pi_{t+1}^* - \pi_t^* = c(1-\delta)^{t-1} y - c(1-\delta)^t y + k(t-1) - kt \tag{19}$$

$$= c\delta(1-\delta)^{t-1} y - k \tag{20}$$

$$> c\delta(1-\delta)^{I-1} y - k \tag{21}$$

$$> 0. \tag{22}$$

That means the largest payoff within the core is obtained at artifact I. Moreover,

$$\pi_{I+1}^* - \pi_I^* = c(1-\delta)^{I-1} y - c(1-\delta)^I y + k(I-1) - kI \tag{23}$$

$$= c\delta(1-\delta)^I y - k \tag{24}$$

$$< 0 \tag{25}$$

and for $t \geq I+1$,

$$\pi_{t+1}^* - \pi_t^* = 0. \tag{26}$$

Summarizing the above, author I earns the highest payoff in the community.

A4 Proof of Proposition 3

With direct citation the individual player i maximizes payoff as (27).

$$\pi_i(s_{i-1}) = f\left(x_i + \delta \sum_{j \in N_i^I(g_{i-1})} x_j\right) - cx_i - kD_i^I(g_{i-1}). \tag{27}$$

Similar to the proof of Proposition 2, the proof undergoes objective function decomposition and inner / outer decision. We denote $v_m(g) := c\delta \sum_{j \in N_i^I(g)} x_j - km$, <nbw> i.e., </nbw > $v_m(g) := c *$ amount of cited knowledge $- km$. Then, as "inner decision", for each given m we work out the optimal g (i.e., which artifacts to cite given a total number of citations) to maximize the amount of cited knowledge. As "outer decision" we find the optimal m to maximize $v_m(g)$, and verify the non-negativity of the resultant knowledge creation (y−the amount of cited knowledge) so that the payoff function is globally maximized.

One can continue the proof in parallel to that of Proposition 2, but note the following differences: Since one only absorbs knowledge from immediate in-neighbors in the network, Lemma 1 no longer holds. Therefore we do not need to break ties by assuming degree-related cost differentiation (Assumption 3 is no longer needed). The fact that creation declines with time (artifact index) will sustain preference on earlier artifacts and enclose the knowledge core.

A5 Proof of Proposition 4

We first show a lemma that is necessary for the proof of the proposition.

Lemma 2. *Player n cites artifacts $1, 2 \ldots d$ where d is the largest artifact index satisfying $\underline{c}\delta(1-\delta)^{d-1} y > k$, and creates knowledge at amount of $x_n^* = (1-\delta)^d y + \bar{y} - y$.*

Proof. The proof of Lemma 2 can be easily done in analogy with the case in which player n has regular $f(\cdot)$ and c. Player n keeps citing foregoing artifacts until the marginal return of citation declines to a level such that it is more cost-saving to produce by herself the otherwise cited knowledge. That leads to citing artifacts $1, 2 \ldots d$ where d is the largest artifact index satisfying $\underline{c}\delta(1-\delta)^{d-1} y > k$. As a result, the player produces knowledge $x_n^* = (1-\delta)^d y + \bar{y} - y$, where $(1-\delta)^d y$ is the amount that she should provide if she had normal benefit and cost functions, and $\bar{y} - y$ the increment in her knowledge creation due to her higher knowledge creativity. □

Define $\Delta := -(1-\delta)^{n-1} y + (1-\delta)^d y + \bar{y} - y$, and $\bar{\Delta} := -(1-\delta)^I y + (1-\delta)^d y + \bar{y} - y$. Then let $\Delta_i := \delta(1-\delta)^{i-n-1}\Delta$, and $\Delta_{\underline{I}} := \delta(1-\delta)^{\underline{I}-n}\Delta$. While Proposition 4 present in the

27. See footnote 26 for analysis of the other case where I decreases with c.

28. I declines with k in general.

main text was curtailed for better focus on main insights, we now provide and prove its full version.

Proposition 4 (Complete Version). *In a direct citation network defined as in Section 4, suppose player n is mutated to have $\bar{y} > y$ and $\underline{c} \leq c$. There exist $Y := \frac{(1-\delta)^n}{\delta} y + (1-\delta)^{n-1} y - (1-\delta)^d y + y$, $\underline{Y} := \frac{k}{c\delta} + y - (1-\delta)^d y$, and $\underline{\underline{Y}} := \frac{(1-\delta)^I}{\delta} y - (1-\delta)^d y + y$ such that*

a) [If artifact n was within the original knowledge core] $n \leq I$ *: The core-periphery structure of the citation network holds with the core size reduced from I to $\underline{I}$.*

If $\bar{y} < Y$, artifacts $\{1, 2 \ldots \underline{I}\}$ constitutes the core, and $n \in \{1, 2 \ldots \underline{I}\}$. $\underline{I}$ is defined as the largest artifact index i that sustains $c\delta x_i^* > k$. $x_i^* = (1-\delta)^{i-1} y [x_i^* = (1-\delta)^{i-1} y - \Delta_i]$ for all artifact i in the core that precedes[succeeds] artifact n, and $x_i^* = (1-\delta)^{\underline{I}} y - \Delta_{\underline{I}}$ for all artifact i at the periphery.

If $\bar{y} > Y$, the growth of the knowledge core is terminated at artifact n, and if $\bar{y}$ is high enough some of preceding artifacts may not remain in the core (i.e. $\underline{I} \leq n$). The artifacts after $n+1$ cite all core artifacts but does not contain any knowledge on their own. That is, $x_i^* = 0$ for all $i \geq n+1$.

b) **[If artifact n was outside the original knowledge core]** $n \geq I+1$ *: If $\bar{y} > \underline{Y}$, artifact n is quoted by all the succeeding artifacts and thus becomes a member of the core. Specifically, if $\underline{Y} < \bar{y} < \underline{\underline{Y}}$, The artifacts afterwards each contains created knowledge at an amount of $(1-\delta)^I y - \delta x_n^*$. If $\bar{y} \geq max\{\underline{\underline{Y}}, \underline{Y}\}$, the players afterwards will produce zero knowledge. if $\bar{y}$ is high enough some of originally core artifacts may not remain in the core. If $\bar{y} < \underline{Y}$, artifact n will not be cited and remain peripheral. Subsequent artifacts each holds created knowledge at an amount of $(1-\delta)^I y$.*

To prove Proposition 4 (complete version), we still use the techniques in the proof of Proposition 2 to address individual optimization—Partition the payoff function so that the utility maximization can be done in two steps: First maximize $v_{D^I_{n+i}(g_{n+i-1})}(g_{n+i-1})$, and then compute the optimal creation amount (while checking for its non-negativity). To avoid redundant presentation we do not explicitly address inner and outer decisions but rather highlight the difference from previous proofs.

The proof of Proposition 4 is done by induction. Since the proposition holds for player $n+1$, suppose it is satisfied for players $n+i-1$ $(i = 2, 3 \ldots)$; then we shall prove it for player $n+i$.

Part a, $n \leq I$. In this case, notice if $\bar{y} < Y$ we have a decreasing order of core knowledge creations since $x^*_{n+i-1} - x^*_{n+i-2} < 0$. Earlier artifact is more preferred in citation.

If $n + i \leq \underline{I} + 1$, all the quoted artifacts are in the core. By hypothesis their self knowledge amount are all greater than $\frac{k}{c\delta}$, thus all of them should be cited (Citing knowledge from them is better than self-producing the same amount of knowledge). Given the hypothesized knowledge creation pattern, the total knowledge that player $n+i$ collects from extant artifacts is $(1 - (1-\delta)^{n+i-1}) y + \delta (1-\delta)^{i-1} \Delta$. Therefore, player $n+i$ should create $x^*_{n+i} = (1-\delta)^{n+i-1} y - \delta (1-\delta)^{i-1} \Delta$ by herself. That $\bar{y} < Y$ ensures the resulting creation quantity is not negative, i.e., $x^*_{n+i} > 0$.

If $n + i \geq \underline{I} + 2$, the player should cite artifacts $1, 2 \ldots \underline{I}$ but not subsequent ones since by hypothesis they each contains less than $\frac{k}{c\delta}$ units of original knowledge. That leads to $x^*_{n+i} = x^*_{\underline{I}+1} = (1-\delta)^{\underline{I}} y - \delta (1-\delta)^{\underline{I}-n} \Delta$, which is guaranteed nonnegative ($x^*_{n+i} > 0$) since $\bar{y} < Y$.

If $\bar{y} > Y$, by hypothesis the core ends at the artifact n. Notice the hypothesis is satisfied for player $n+1$: If player $n+1$ continues to develop the core and cite all previous artifacts, then her self knowledge creation is $x^*_{n+1} = (1-\delta)^n y - \delta \Delta < 0$. Since the non-negativity constraint binds, the player should produce zero knowledge but (potentially) withdraw some citations of low knowledge return. The following players face the same problem as player $n+1$ does, and will quote the same group of prior artifacts (core) and create no knowledge by themselves. To judge which citation should remain in the core, notice that the marginal benefit of that citation has to exceed the cost, provided no knowledge creation by oneself. Combining with the fact that the amount of original knowledge declines in artifact index, we conclude that the core now consists of $\{1, 2 \ldots h, n\}$, where h is the largest artifact index that satisfies $f(\delta(\sum_{j=1}^{h} x_j^* + x_n^*)) - f(\delta(\sum_{j=1}^{h-1} x_j^* + x_n^*)) > k$ and $\delta(\sum_{j=1}^{h} x_j^* + x_n^*) \geq y$. Substituting the expression of x^*-s into the foregoing conditions for h, those conditions become $f((1-\delta)y - (1-\delta)^h y + \delta(1-\delta)^d y + \delta\bar{y}) - f((1-\delta)y - (1-\delta)^{h-1} y + \delta(1-\delta)^d y + \delta\bar{y}) > k$ and $(1-\delta)y - (1-\delta)^h y + \delta(1-\delta)^d y + \delta\bar{y} \geq y$.

Part b, $n \geq I+1$. Note if $\frac{k}{c} \leq (1-\delta)^I y$ then $\underline{Y} \leq \underline{\underline{Y}}$. First consider $\bar{y} \geq \underline{Y}$. If $\underline{\underline{Y}} > \bar{y} \geq \underline{Y}$, $x_n^* > \frac{k}{c\delta}$. Then artifact n should be cited by all later artifacts, whose own creation levels are reduced (since they now execute one more citation): $x^*_{n+i} = (1-\delta)^I y - \delta x_n^* > 0$ for $i = 1, 2 \ldots$. If $\bar{y} \geq \underline{\underline{Y}} \geq \underline{Y}$ or $\bar{y} \geq \underline{Y} > \underline{\underline{Y}}$, the succeeding players will produce zero knowledge while citing artifact n. High enough $\bar{y}$ will cause some artifacts prior to n ruled out from the core due to low knowledge return (in the same way illustrated in proof of Part a). Artifacts after n will remain at periphery, since their knowledge supplies are reduced compared to the original case with homogeneous c and y, in which they are already peripheral. If $\bar{y} < \underline{Y}$, player n provides less knowledge than the minimal cite-worthy level and her artifact will not be quoted. The decisions of the rest of players are not affected: They still innovate at amount of $(1-\delta)^I y$ and seek knowledge only from the original core.

The Electronic Companion to this paper is found online at http://www.ie.tsinghua.edu.cn/yangzhanguser/Appendix-KnowledgeCoreIEEETKDE.pdf

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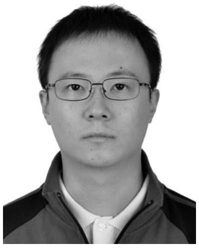

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