Solving a Hub Location-Routing Problem with a Queue System under Social Responsibility by a Fuzzy Meta-Heuristic Algorithm

Pardis Pourmohammadi[a], **Reza Tavakkoli-Moghaddam**[a], **Yaser Rahimi**[a], **Chefi Triki** [b,c,*]

[a] *School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

[b] *Division of Engineering Management and Decision Sciences, College of Sciences and Engineering, Hamad Bin Khalifa University, Doha-Qatar*

[c] *Department of Engineering for Innovation, University of Salento, Lecce, Italy*

Abstract: This paper presents a new multi-objective mathematical model for the hub location and routing problem under uncertainty in flows, costs, times and number of job opportunities. This model aims at minimizing the total transportation cost consisting of routing and fixed cost, and maximizing the employment and regional development as social responsibility. An $M/M/c/K$ queue system is applied to estimate the waiting time at hub nodes and maximize the responsiveness. In addition, a fuzzy queuing method is applied to model the uncertainties in this network. A powerful evolutionary meta-heuristic algorithm based on the fuzzy invasive weed optimization (FIWO), variable neighborhood search (VNS) and game theory is developed to solve the introduced model and obtain near-optimal Pareto solutions. Many experiments as well as a real transportation case-study show the superiority of the proposed approaches compared to the state-of-the-art algorithm.

Keywords: Hub location-routing problem; Queue system, Responsiveness; Social responsibility; Fuzzy meta-heuristic algorithm.

1. Introduction

Hub Location Problems (HLP) are nowadays attracting a lot of attention in the field of location problems. Hubs are special facilities, that are being used for transportation, telecommunication, cargo delivery and many-to-many flow distribution systems. In other words, hub networks are involved in delivering people, commodities or information between different origin-destination (O-D) nodes (Zhalechian et al., 2017a).

The hub location-routing problem is concerned with the location of hub facilities, the allocation of non-hub nodes and the establishment of local tours among the nodes allocated to them. When the nodes do not have sufficient demand, it is not economically feasible to set a direct connection between them. In this situation, consolidation must be performed from origins to hubs and hubs to destinations (Mohammadi et al., 2016). An HLP is called a p-hub location problem when the number of hub nodes is pre-determined to be equal to p. In the literature, several types of HLPs are being developed: p-hub center, p-hub median and hub covering location problems. There are two primary assumptions characterizing most of the HLPs: (i) there is no direct connection between each non-hub nodes so all the flow must pass by at least one hub on its route and (ii) the network between hub nodes is a complete graph, which means there is a link between each pair of hub nodes (Eiselt and Marianov, 2009). There are two main types of allocation in a hub and spoke networks, namely single- and multiple-allocation.

* Corresponding Author: ctriki@hbku.edu.qa. (C. Triki). ORCID: 0000-0002-8750-2470.

In single-allocation, each non-hub node must be allocated to just one hub node while in multiple-allocation, each non-hub node can be allocated to more than one hub node.

In a classical HLP, the objective is to find the location of hubs and non-hub nodes in order to minimize the total costs. Nevertheless, the massive arrival rate of the flows at hub nodes ends in a queue and long waiting time that often affects the customer satisfaction. The responsiveness of a hub network should be considered as one of the main factors in designing a hub and spoke network (Van Woensel et al., 2006). A hub-and-spoke network problem is considered with crowdedness or congestion in the system. The transportation time and the rate of arrived flows to each hub are random variables. In addition, a hub cannot serve all the demands simultaneously since it has some restrictions, such as capacity and service time limitations. Hubs, which are the most crowded parts of the network, are usually modeled as a $M/M/c/K$ queuing system. In this paper, several efforts have been done to calculate the waiting time and improve the responsiveness in the network.

In the context of hub network design, Social Responsibility (SR) is becoming one of the noticeable topics of interest in recent years. It is concerned with the organization's impact on the social system. In general, all the individuals and companies have a duty to act in the best interests of their environment and society as a whole. It follows the logic of sustainable development, with the company being responsible for the impact of its decisions and activities on society and the environment (Zhalechian et al., 2017b). Nowadays organizations and companies are attempting to highlight SR elements in their strategies, visions and development. Also, from the companies' managers viewpoint, SR can improve the social image and the brand of corporate in addition to risk reduction. To provide a complete framework for social matters, an international standards organization (ISO) has represented a new standard (i.e., International Guidance Standard on Social Responsibility-ISO 26000), which categorize the social issues into the following major groups: corporate governance; human rights; labor practices; the environment; correct business conduct, questions; relative to consumers; and commitment to society (Pishvaee et al., 2012).

Due to the multi-stakeholder nature of SR, it is hard to measure all aspects of SR. In this paper, the aspect of SR related to the strategic decision will be considered in designing a hub-and-spoke network. In both GRI guidelines (www.globalreporting.org) and ISO 26000, the importance of improving the community and environment around the workplace has been recommended. Moreover, there is a considerable concern for the job opportunity and economic development as strategic decisions. Hence, implementing SR in a hub-and-spoke networks can represent a valuable contribution towards addressing the concern about employment opportunities and economic development (Mota et al., 2015; Pishvaee et al., 2014).

The first goal of this paper is to develop a new multi-objective mixed-integer mathematical programming model with a quadratic objective function to (1) minimize the total investment and transportation costs, (2) maximize the employment and economic development and (3) maximize the responsiveness by minimizing the transportation time between each pair of O-D nodes.

The rest of this paper is organized as follows. Section 2 presents a brief review of the related literature. In Section 3, the problem description and mathematical formulation are described. The proposed solution method is presented in Section 4. Our computational experiments are summarized in Section 5 and, finally, some conclusions are drawn in Section 6.

2. Literature review

In this section, the literature review of HLPs is presented. O'kelly (1987) presented the first known quadratic integer programming formulation for the single allocation p-hub median problem. Campbell (1994) simplified this model by introducing a linear integer programming version of the suggested model. Ernst et al. (1998) presented a more practical LP formulation for single-allocation p-hub problem with fewer variables and constraints than those used in previous works. de Camargo et al. (2008) developed a new mixed-integer formulation that decided on the location of hubs and the allocations of tours to the hubs. They also assumed a time limitation for each tour and defined a set of possible arcs that can form a local tour to decreases the number of variables. They proposed a Benders decomposition algorithm in order to obtain an exact solution. Meier (2017) defined a new model for the uncapacitated single allocation p-hub median problem that minimizes the number of used vehicles, instead of considering the transport costs as a linear function of the volume. A new mixed integer program formulation with fewer variables but more constraints is introduced.

Due to the importance of responsiveness in a hub-and-spoke network, the queuing theory has been used for managing the congestion. The first study that addressed this important aspect was due to Grove et al. (1986). Elhedhli et al. (2005) considered a single-allocation p-hub model with congestion and extended their model by adding a non-linear cost term in the objective function. Marianov and Serra (2003) developed a single-allocation model as an $M/\mathrm{D}/c$ queuing system for an airport system. They used a capacity constraint for limiting the waiting time at each hub node to a predefined value and considered two objectives: (i) the facilities number that is treated by limiting the capacity, and (ii) the percentage of demand that may be lost because of the model's limitation. Rahmati et al. (2014) developed a bi-objective model for the facility location–allocation problem, which is immobile service and stochastic demands with the $M/M/1/K$ queue system. The objectives of the model were to minimize the total cost of server providers and minimize the total time of serving customers. Ishfaq et al. (2012) modeled a hub operation as a $GI/G/1$ and studied the effect of limited hub sources on the design of multiple job classes with deterministic routing. Rodríguez et al. (2007) presented a hub network for a cargo transportation, in which the trucks should wait in a queue if unloading services are busy and each hub node is modeled as an $M/M/1$ queue system. Tavakkoli-Moghaddam et al. (2017) presented a new multi-objective model for a facility location problem with the pricing policy and congestion of immobile service facilities by a stochastic demand and an $M/M/m/K$ queue system. They proposed the multi-objective vibration damping optimization (MOVDO) and non-dominated ranking genetic algorithm (NRGA) for solving large-sized problems.

In spite of the significance of social responsibility, the relevant literature is very scares in the context of hub-and-spoke networks. However, some insights can be gained from the studies that addressed the supply chain network design (SCND) with social responsibility and corporate sustainability. Dehghanian et al. (2009) developed a three-objective mathematical programming model to maximize the economic and social benefits and minimize negative environmental impacts, simultaneously. Several measures of SR including employment, local development and damage to workers were considered in this model. Pishvaee et al. (2012) designed a socially responsible supply chain with the aim of minimizing the total cost and maximizing the SR of the supply chain. The created job opportunities, amount of produced waste and number of potentially hazardous products were considered as the social metrics. Devika et al. (2014) presented a mixed-integer programming model with a multi-objective closed-loop supply chain network problem. Workers safety and job opportunities are the two measures of SR that were quantified and modelled as separate objective functions. Pishvaee et al. (2014) proposed a multi-objective possibilistic programming model for the design of a sustainable medical supply chain under uncertainty considering economic, environmental and social objectives. Ahmadi-Javid and Seddighi (2012) considered an integrated problem with location, inventory and routing decisions in a multi-source distribution network design that minimizes the total related cost. They proposed a three-phase heuristic algorithm for solving large-sized problems. Mousavi et al. (2013) presented a fuzzy possibilistic two-phase mixed-integer programming model for a cross-docking system. They considered the multi-period location of multiple cross-docks and scheduling of vehicle routing problems under a fuzzy environment.

Designing a hub-and-spoke network that can fulfill multiple objectives at the same time has drawn a lot of attention recently. Da Graça Costa et al. (2008) presented a capacitated single-allocation model, in which the first objective is to minimize the total cost and the second one limits the number of flows that can be received by the hub nodes. They applied an interactive decision-aid method to solve the bi-objective model. Ghodratnama et al. (2015) developed a multi-objective single-allocation model with a supply chain overview. Three objective functions include the total transportation and installation costs, the total greenhouse gas emitted and the weighted sum of service times in the hub nodes. Masoumzadeh et al. (2016) proposed a multi-objective p-hub protection model with backup hub nodes. The first objective function in their model maximizes the potential flow between the O-D nodes with the minimum potential flow, and the second one minimizes the total installation costs. Mohammadi et al. (2016) and Parvaresh et al. (2014) introduced a bi-objective single-allocation p-hub center-median problem under uncertainty in flows, times, costs and hub operations. The first objective tends to minimize the total transportation cost and the fixed cost of locating the hub and the second one minimizes the maximum travel time between each pair of O-D in the network. Rahimi et al. (2016) suggested a new $M/M/c/K$ queue model for a location-allocation problem that minimizes concurrently two objectives: (i) the total transportation cost of the hub network and (ii) the maximum travel time between each O-D pair. Zhalechian et al. (2017a) presented a multi-objective mathematical model for a multi-model HLP with the aim of minimizing the total transportation and traffic noise pollution costs. In addition, they minimized the maximum transportation time between O-D nodes in order to ensure a high probability of guarantying the service deliveries. Furthermore, Zhalechian et al. (2017b) introduced a new multi-objective model for a hub location problem under uncertainty that considers economic, responsiveness and social responsibility at the same time. They also used an $M/M/c$ queuing system to calculate the waiting time at each node and increase the responsiveness.

According to the above discussion, the literature review shows that there is a gap in incorporating social responsibility in designing hub-and-spoke networks. Although there are a number of studies that considered total costs, responsiveness and SR in their literature separately, a few papers have employed all of them at once. There are also three main assumptions in routing problems that have never been simultaneously considered so far. These three assumptions are (i) the set of nodes where local tour can be established is the same as the set of demand nodes, (ii) there is no limitation on the number of local tours, and (iii) there is limitation for the flow capacity for each local tour. Moreover, there is no any study in the context of routing problems that incorporates the routing cost in the local tour as a function of both distance traversed and flow carried and there is a gap in proposing a network that considers location, allocation and establishment of local tours simultaneously.

To overcome these deficiencies, we propose a mathematical model for the HLP that addresses in an integrated way the trade-off between the total cost, responsiveness and social responsibility. Also, an iterative two-phase clustering-routing heuristic model is developed in order to obtain a near-optimal solution with a reasonable CPU time. Thus, the main contributions of this paper with respect to the previous related studies can be summarized as follows:

- Introducing a new multi-objective mixed-integer mathematical model with a quadratic cost function to design a hub-and-spoke network;
- Developing a generalized version of the hub location-routing problem that jointly considers the location, allocation and establishment of local tours;
- Introducing a new objective function in the HLP that maximizes the employment and economic development based on the unemployment rate and the level of regional development;
- Developing an $M/M/c/K$ queuing system to calculate the waiting times at hub nodes and enhance the responsiveness of the designed network.

3. Problem description and proposed model

3.1. Modeling framework

This paper introduces a multi-objective single-allocation hub location-routing problem. The first objective aims at reducing the transportation and installation costs. The cost of routing per unit of flow from node i to node j is defined as c_{ij} and g_{ij} is the cost of using arc (i, j). These cost parameters are dependent on the traversed distance between each pair of nodes. By establishing local tours and direct links with a hub, each non-hub node has two options: either being directly connected to a hub or be visited through a local tour. Local tours have the maximum capacity of Q units of flow. The parameters α and β are discount factors for the routing costs of the traffic and flow through local tours, respectively.

The model involves two additional new objective functions. The first one maximizes the social responsiveness by creating both fixed and variable job opportunities through establishing a hub node and also by promoting the economy in the region where the hub node is set up. The second one aims at minimizing the maximum travel time between each pair of O-D nodes. Due to the finite capacity of hub nodes, the arrival flow must wait in a queue for receiving service. The total service time is the sum of queue waiting time and the processing time. In order to calculate the waiting time within the stochastic flow setting, a queue model needs to be developed. Each queue has a limited capacity K to control and limit the arrival flow (see Fig. 1). Service and average entering rates are constant and follow a Poisson distribution during peak hours.

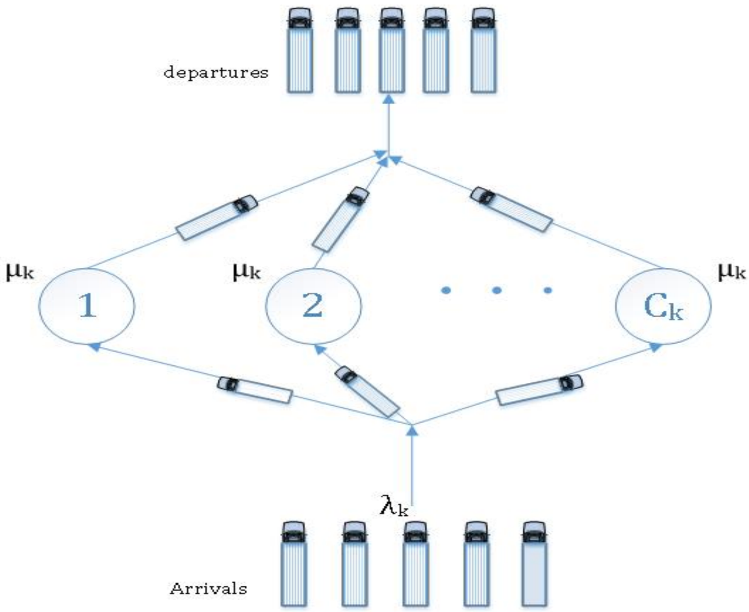

Fig. 1. $M/M/c/k$ queuing system at hub node.

Van Woensel et al. (2006) developed a queuing system by using both simulation and empirical data. Peterson et al. (1995) proposed queue system algorithms for transient congestion at airports and concluded that arrival rates and capacity levels follow a Poisson distribution when there is variation over the scheduled time. Aykin (1994), Ebery et al. (2000) and Sasaki et al. (2003) added a capacity level constraint at a network in order to control the congestion. Rahimi et al. (2016) studied an $M/M/c/K$ queue system with different capacity levels for the hub. In their study, hubs with more capacity need higher cost and equipment to be built and in return, they attract a higher volume of flow and control better the congestion. In this paper, a Poisson distribution will be used to calculate the waiting time from the arrival flow at hub nodes and an $M/M/c$ model for the queue management is proposed.

3.2. Mathematical Model

Before presenting our model, sets, parameters, and decision variables are defined below.

Indices

i, j, k, l	Indices representing the non-hub nodes (set *I*) and hub nodes (set *J*)

Decision variables

x_{ij}	1, if node *i* is assigned to hub *j*; 0 otherwise
y_{ijk}	1, if node *i* precedes node *j* at the route that completes on hub *k*; 0 otherwise
f_{jl}^{i}	Flow that originates at node *i* and travels from hub *j* to hub *l*
r_{ij}^{k}	Flow that travels from node *i* to node *j* in the route that completes on hub *k*
Φ	Maximum traveling time between each O-D nodes

Queuing system parameters

W_k	Total service time at hub node *k*
Wq_k	Total queue waiting time at hub node *k*
λ_k	Arrival rate of flow units to hub k
μ_k	Service rate of hub *k*
c_k	Number of service providers at hub *k*
K_k	Finite capacity of a queue at hub *k*
$P_{n_k,k}$	Probability of *n* flow units in the queue at hub node *k*
Lq_k	Length of the queue at node *k*
$\widetilde{W}_{ij}$	Flow units between nodes *i* and *j*
$\tilde{O}_i = \sum_j \widetilde{W}_{ij}$	Total amount of flow units originating from node *i*
$\widetilde{D}_i = \sum_j \widetilde{W}_{ji}$	Total amount of flow units delivering at node *i*

Based on the above notation, the arrival rate of flow unit to hub *k* is calculated by:

$$\lambda_k = \sum_i (\tilde{O}_i + \widetilde{D}_i) x_{ik} \tag{1}$$

Moreover, the total service time (W_k) is calculated as the sum of waiting time in the queue and the process time, as:

$$W_k = Wq_k + \frac{1}{\mu_k}, \tag{2}$$

and the waiting time of the arrival flow unit into hub *k* is defined as.

$$Wq_k = \frac{Lq_k}{\lambda_k (1 - P_{n_k,k})} \tag{3}$$

And according to the queuing theory laws, the involved entities can be calculated by means of the following equations:

$$P_{n_k,k} = \frac{(\lambda_k)^{K_k}}{K_k!\,(\mu_k)^{K_k}} P_{0k} \tag{4}$$

$$Lq_k = \frac{(a_k)^{c_k}(\rho_k)}{c_k!\,(1-\rho_k)^2} P_{0k}[1-(\rho_k)^{K_k-c_k+1}-(1-\rho_k)(K_k-c_k+1)(\rho_k)^{K_k-c_k}] \tag{5}$$

$$P_{0k} = [\frac{(a_k)^{c_k}(1-(a_k)^{K_k-c_k+1})}{c_k!\,(1-a_k)} + \sum_{v=0}^{c_k-1}\frac{(a_k)^v}{v!}]^{-1} \tag{6}$$

$$a_k = \frac{\lambda_k}{\mu_k} \tag{7}$$

$$\rho_k = \frac{\lambda_k}{c_k\mu_k} \tag{8}$$

Fuzzy parameters

$\widetilde{FJ}_k$	Number of fixed job opportunities created through establishing a hub at node *k*
$\widetilde{VJ}_k$	Number of variable job opportunities created through establishing a hub at node *k*
$\widetilde{Ev}_k$	Economic value of hub node *k*
$\tilde{g}_{ij}$	Cost of using arc (*i, j*)
$\tilde{c}_{ij}$	Cost of routing a unit of flow from node *i* to node *j*
$\tilde{T}_{ij}$	Transportation time between nodes *i* and *j*

Deterministic parameters

rd_k	Level of regional development at node *k*
w_{em}	Importance weight of the employment measure
w_{ed}	Importance weight of the economic development measure
ur_k	Unemployment rate at node *k*
p	Number of hubs that must be located in the network
α	Node-hub transportation discount factor
β	Local tour transportation discount factor
Q	Maximum units of flow that a vehicle can carry

On the basis of the above notation, our suggested model can be expressed as:

$$\textbf{Min } Z_1 = \sum_{i\in I}\sum_{j\in J}\sum_{l\in J\setminus\{j\}} \alpha\,\tilde{c}_{jl} f_{jl}^i + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} 2\beta\,\tilde{c}_{ij} r_{ij}^k + \sum_{i\in I}\sum_{j\in J} 2\tilde{O}_i\,\tilde{c}_{ij} x_{ij} + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} \tilde{g}_{ij} y_{ijk} \tag{9}$$

$$+ \sum_{i\in I}\sum_{j\in J} \tilde{g}_{ij} x_{ij} + \sum_{j\in J:j<k}\sum_{k\in J} \tilde{g}_{jk} x_{jj} x_{kk}$$

$$\textbf{Max } Z_2 = w_{em}\left(\sum_k \left(\widetilde{FJ}_k + \widetilde{VJ}_k\right) ur_k x_{kk}\right) + w_{ed}\left(\sum_k \widetilde{Ev}_k (1-rd_k) x_{kk}\right) \tag{10}$$

$$\textbf{Min } Z_3 = \phi \tag{11}$$

Subject to:

$$\sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}} \sum_{k\in J} y_{ijk} \geq 1 \qquad \forall i \in I \qquad (12)$$

$$\sum_{i\in I\setminus\{j\}} y_{ijk} - \sum_{i\in I\setminus\{j\}} y_{jik} = 0 \qquad \forall j \in I\,, k \in J \qquad (13)$$

$$y_{ikk} + y_{kik} \leq 1 \qquad \forall k \in J\,, i \in I : i \neq k \qquad (14)$$

$$y_{ijk} \leq x_{kk} \qquad \forall i \in I, j \in I, k \in J : i \neq j \qquad (15)$$

$$x_{ij} \leq x_{jj} \qquad \forall i \in I\,, j \in J \qquad (16)$$

$$\left(\tilde{T}_{ik} + W_k + \tilde{T}_{kl} + W_l + \tilde{T}_{lj}\right) y_{ijk} \leq \phi \qquad \forall i \in I\,, j \in J, k \in J \qquad (17)$$

$$\sum_{j\in J} x_{jj} = p \qquad (18)$$

$$\sum_{l\in J\setminus\{j\}} (f_{jl}^{i} - f_{lj}^{i}) = \sum_{m\in I} \widetilde{W}_{im} \Big(\sum_{k\in I\setminus\{i\}} y_{ikj}\Big) - \sum_{m\in I\setminus\{j\}} \widetilde{W}_{im} \Big(\sum_{k\in I\setminus\{m\}} y_{mkj}\Big) + \sum_{m\in I} \widetilde{W}_{im} (x_{ij} - x_{mj}) \qquad \forall i \in I, j \in J : i \neq j \qquad (19)$$

$$\sum_{l\in J\setminus\{j\}} (f_{jl}^{j} - f_{lj}^{j}) = \sum_{m\in I} \widetilde{W}_{jm} \Big(x_{jj} - x_{mj} - \sum_{k\in I\setminus\{m\}} y_{mkj}\Big) \qquad \forall j \in J \qquad (20)$$

$$\sum_{j\in J\setminus\{i\}} (r_{ij}^{k} - r_{ji}^{k}) = \tilde{O}_i \sum_{m\in I\setminus\{i\}} y_{imk} \qquad \forall i \in I, k \in J : i \neq k \qquad (21)$$

$$0 \leq r_{ij}^{k} \leq Q y_{ijk} \qquad \forall i \in I, j \in I, k \in J : i \neq j \qquad (22)$$

$$f_{jl}^{i} \geq 0 \qquad \forall i \in I, j \in J, l \in J \setminus\{j\} \qquad (23)$$

$$y_{ijk} \in \{0,1\} \qquad \forall i \in I, j \in I, k \in J : i \neq j \qquad (24)$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, j \in I \qquad (25)$$

The objective function (9) minimizes the total transportation cost that consists of six terms: (1) the routing cost of flows sent in the hub network that takes into account the discount factor α, (2) the routing cost of flows sent through the local tours, taking into account the discount factor β, (3) the routing cost of flows sent directly from single-assigned non-hub nodes to nodes, (4) the fixed cost of travelling the local tours expressed as a function of the distance traversed, (5) the fixed cost of travelling from single assigned non-hub nodes to hub nodes expressed as a function of distance traversed, and (6) the fixed cost of travelling through the hub network. It is to be noted that the second and third terms are multiplied by two in order to count for the delivery and pick-up costs. The second objective function (10) aims at maximizing the employment and economic development. The third objective (11) minimizes the maximum transportation time between each pair of O-D nodes.

Constraint (12) ensures that each node in the set *I* will be assigned directly to a hub or to a tour that completes its tour on a hub. Constraint (13) imposes that the number of incoming arcs to any node *i* is equal to the number of outgoing arcs from any node *i* that are assigned to a tour that completes its tour on hub *k*. Constraint (14) ensures that there is no local tour with just one node. Constraints (15) and (16) impose that if a node is not chosen to be a hub node, any demand node that is either part of a local tour or single cannot be assigned to this node. Constraint (17) calculates the maximum transportation time between O-D nodes. Constraint (18) indicates that *p* nodes should be chosen as hub locations.

Constraints (19) and (20) are flow balance constraints for the hub network. If node *j* is not a hub node, then the right sides of both constraints will be zero, which means there cannot be any flow sent through the hub network that visits node *j*. More specifically, constraint (19) apply when i node *i* is not a hub node, but node j is a hub node, then node i is either directly assigned to a hub node or to a local, tour as stated by constraint (12). If node i is assigned to hub node j, then $x_{ij} + \sum_{k\in I\setminus\{i\}} y_{ikj} = 1$ and the total flow emanating from node i will be $\sum_{m\in I} w_{im} (\sum_{k\in I\setminus\{i\}} y_{ikj}) + \sum_{m\in I} w_{im}\, x_{ij}$. Some flow will not go through the hub network but will be sent to nodes either individually or by a local tour to hub j, which is calculated by $\sum_{m\in I\setminus\{j\}} w_{im} (\sum_{k\in I\setminus\{m\}} y_{mkj}) + \sum_{m\in I} w_{im}\, x_{mj}$. Therefore, the flow emanating from node i and going through the hub network will be the total flow emanating from node i minus the flow sent to nodes either individually or by a local tour to hub j, $\sum_{m\in I} w_{im} (\sum_{k\in I\setminus\{i\}} y_{ikj}) - \sum_{m\in I\setminus\{j\}} w_{im} (\sum_{k\in I\setminus\{m\}} y_{mkj}) + \sum_{m\in I} w_{im} (x_{ij} - x_{mj})$. If node i is assigned to hub node j then $x_{ij} + \sum_{k\in I\setminus\{i\}} y_{ikj} = 0$ that ensures the flow originating node i cannot be sent from node j.

In the case of Constraint (20) in which node j is a hub node, the total flow emanating from node j will be $\sum_{m\in I} w_{jm}\, x_{jj}$; however, some flow will not go through the hub network, but will be sent to nodes either individually or by a local tour to hub j, which is calculated by $\sum_{m\in I} w_{jm}(x_{mj} + \sum_{k\in I\setminus\{m\}} y_{mkj})$. Hence, the flow emanating from node j and going through the hub network will be the total flow emanating from node j minus the flow sent to the nodes either individually or by a local tour to hub j (i.e., $\sum_{m\in I} w_{jm}(x_{jj} - x_{mj} - \sum_{k\in I\setminus\{m\}} y_{mkj})$).

Set of constraints (21) represents the flow balance for local tours. The total outgoing flow minus the total incoming flow from non-hub node i will be equal to its demand. Constraint (22) ensures that the capacity on the tours is not exceeded. Finally, constraints (23) to (25) are the variable restriction.

The objective function (9) of above problem is non-linear but can be easily linearized by defining a new variable $z_{jk} = x_{jj} x_{kk}$ and adding new constraints as follows:

$$z_{jk} \geq x_{jj} + x_{kk} - 1 \qquad \forall j \in J\,, k \in J{:}\, j < k \qquad (26)$$

$$z_{jk} \leq x_{jj} \text{ and } z_{jk} \leq x_{kk} \qquad \forall j \in J\,, k \in J{:}\, j < k \qquad (27)$$

4. Proposed solution approach

Solving large-sized instances of the proposed model is computationally challenging. Our preliminary computational experiments have shown that solving large-sized problems, i.e. having more than 15 nodes, required a huge computational time that is not compatible with the decisional process timing. To overcome this limitation, a two-phase approach is developed in this section:

Phase 1: Convert the fuzzy model (9)—(27) to its equivalent auxiliary crisp form

Phase 2: Develop a new multi-objective meta-heuristic algorithm to find optimal Pareto solutions

4.1 *Phase 1— Converting the model to its equivalent auxiliary crisp form*

The HLP model (9)—(27) is a fuzzy multi-objective linear program. Several approaches have been developed in the literature to transform a possibilistic model into an equivalent crisp one. The literature review illustrates that credibility-based possibilistic approaches, such as expected value (Liu et al. 2002) and chance-constrained programming (Lie et al. 1998), are the two most applied methods to handle the uncertainty in parameters of objective functions and constraints. There are several fuzzy measurements to transform a possibilistic chance constraint into its crisp form. Among them, Pos (possibility measures) and Nec (necessity measure) are the basic fuzzy measures to calculate the optimistic and pessimistic attitudes of the DM (Rabbani et al. 2018). The Cr (credibility) is another fuzzy measure, that can be defined as an average of the Pos and Nec, to measure and demonstrate the certainty degree of occurrence of an uncertain event.

Xu and Zhou (2013) presented a more flexible measure Me to avoid extreme attitudes. This approach is an extension of the Cr measure. It can also consider the combined attitude of the DM, which is between optimistic and pessimistic views. The concepts of possibility, necessity and credibility of a fuzzy event is defined below. The triple set (θ, P(θ), Pos), according to Dubois and Prade (2012), is called the possibility space, where θ is a non-empty set, P(θ) is the power of set θ, and Pos is a possibility measure. The fuzzy measure Me is defined by:

$$\text{Me}\{A\} = \text{Nec}\{A\} + \varepsilon(\text{Pos}\{A\} - \text{Nec}\{A\}), \tag{28}$$

where A is a set in P(θ) and ε is the pessimistic-optimistic parameter to be depend on the decision-maker preferences. The necessity and credibility measures of A are, respectively, defined as:

$$\text{Nec}\{A\} = 1 - \text{Pos}\{A^c\} \tag{29}$$

$$\text{Cr}\{A\} = \frac{1}{2}(\text{Pos}\{A\} + \text{Nec}\{A\}) \tag{30}$$

Several kinds of definitions for the expected value of the triangular fuzzy variable have been mentioned in the literature. Based on Xu and Zhou (2013), the expected value of triangular fuzzy variable $\xi = (\xi_1, \xi_2, \xi_2)$ when $\xi_1 \geq 0$ can be calculated as:

$$E[\xi] = \frac{(1-\varepsilon)}{2}\xi_1 + \frac{1}{2}\xi_2 + \frac{\varepsilon}{2}\xi_3 \tag{31}$$

In order to deal with such uncertainty in the parameters, the chance-constrained programming approach is used in this study, as briefly explained in the sequel.

$$\begin{aligned} &\textbf{Min } \tilde{c}x \\ &\text{Subject to:} \\ &\quad \text{Me}\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\ &\quad \text{Me}\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\ &\quad x \geq 0 \end{aligned} \tag{32}$$

where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_n)$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{N} = [\tilde{n}_{ij}]_{m\times n}$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2 \ldots, \tilde{b}_n)^t$, and $\tilde{d} = (\tilde{d}_1, \tilde{d}_2 \ldots, \tilde{d}_n)^t$ show the triangular fuzzy numbers in the objective functions and constraints. Also, α and β are the decision maker's minimum satisfaction levels of possibilistic constraints.

The foregoing model can be transformed into two approximation models, namely UAM (upper approximation model) and LAM (lower approximation model) defined as follows:

UAM:

$$\begin{aligned} &\textbf{Min } E[\tilde{c}]x \\ &\text{subject to:} \\ &\quad \text{Pos}\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\ &\quad \text{Pos}\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\ &\quad x \geq 0 \end{aligned} \tag{33}$$

LAM:

$$\begin{aligned} &\textbf{Min } E[\tilde{c}]x \\ &\text{subject to:} \\ &\quad \text{Nec}\{\tilde{A}x \geq \tilde{b}\} \geq \alpha. \\ &\quad \text{Nec}\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\ &\quad x \geq 0 \end{aligned} \tag{34}$$

The above possibilistic models can be transformed into two crisp equivalent linear models as:

UAM:

$$\begin{aligned} &\textbf{Min } \left(\frac{(1-\varepsilon)}{2}c_{(1)} + \frac{1}{2}c_{(2)} + \frac{\varepsilon}{2}c_{(3)}\right)x \\ &A_{(2)}x + (1-\alpha)\big(A_{(3)} - A_{(2)}\big)x \geq b_{(2)} - (1-\alpha)(b_{(2)} - b_{(1)}) \\ &N_{(2)}x - (1-\beta)\big(N_{(2)} - N_{(1)}\big)x \leq d_{(2)} + (1-\beta)(d_{(3)} - d_{(2)}) \\ &x \geq 0 \end{aligned} \tag{35}$$

and

LAM: (36)

$$\mathbf{Min}\ (\tfrac{(1-\varepsilon)}{2}\mathrm{c}_{(1)}+\tfrac{1}{2}\mathrm{c}_{(2)}+\tfrac{\varepsilon}{2}\mathrm{c}_{(3)})x$$
$$A_{(2)}x-\alpha\big(A_{(2)}-A_{(1)}\big)x\geq b_{(2)}+(1-\alpha)(b_{(3)}-b_{(2)})$$
$$N_{(2)}x+(1-\beta)\big(N_{(3)}-N_{(2)}\big)x\leq d_{(2)}-\beta(d_{(2)}-d_{(1)})$$
$$x\geq 0$$

By calculating the UAM and LAM, the decision maker has both the upper and lower bounds of the optimal solution. Therefore, more information is made available to select the final solution. Accordingly, the proposed model will be approximated by means of the following auxiliary crisp equivalent problems with triangular fuzzy parameters:

UAM:

$$\begin{aligned}\mathbf{Min}\ Z_1 = &\sum_{i\in I}\sum_{j\in J}\sum_{l\in J\setminus\{j\}}\alpha\,(\frac{1-\varepsilon}{2}c_{jl(1)}+\frac{1}{2}c_{jl(2)}+\frac{\varepsilon}{2}c_{jl(3)})f_{jl}^{i}+\sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J}2\beta\,(\frac{1-\varepsilon}{2}c_{ij(1)}\\ &+\frac{1}{2}c_{ij(2)}+\frac{\varepsilon}{2}c_{ij(3)})r_{ij}^{k}+\sum_{i\in I}\sum_{j\in J}2(\frac{1-\varepsilon}{2}O_{i(1)}+\frac{1}{2}O_{i(2)}+\frac{\varepsilon}{2}O_{i(3)})\,(\frac{1-\varepsilon}{2}c_{ij(1)}\\ &+\frac{1}{2}c_{ij(2)}+\frac{\varepsilon}{2}c_{ij(3)})x_{ij}+\sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J}(\frac{1-\varepsilon}{2}g_{ij(1)}+\frac{1}{2}g_{ij(2)}+\frac{\varepsilon}{2}g_{ij(3)})y_{ijk}\\ &+\sum_{i\in I}\sum_{j\in J}\left(\frac{1-\varepsilon}{2}g_{ij(1)}+\frac{1}{2}g_{ij(2)}+\frac{\varepsilon}{2}g_{ij(3)}\right)\,x_{ij}\\ &+\sum_{j\in J:j<k}\sum_{k\in J}\Big(\frac{1-\varepsilon}{2}g_{jk(1)}+\frac{1}{2}g_{jk(2)}\\ &+\frac{\varepsilon}{2}g_{jk(3)}\Big)x_{jj}x_{kk}\end{aligned}\tag{37}$$

$$\begin{aligned}\mathbf{Max}\ Z_2 = w_{em}&\left(\sum_{k}\left(\left(\frac{1-\varepsilon}{2}FJ_{k(1)}+\frac{1}{2}FJ_{k(2)}+\frac{\varepsilon}{2}FJ_{k(3)}\right)\right.\right.\\ &\left.\left.+\left(\frac{1-\varepsilon}{2}VJ_{k(1)}+\frac{1}{2}VJ_{k(2)}+\frac{\varepsilon}{2}VJ_{k(3)}\right)\right)ur_{k}x_{kk}\right)\\ &+w_{ed}\left(\sum_{k}\left(\frac{1-\varepsilon}{2}Ev_{k(1)}+\frac{1}{2}Ev_{k(2)}+\frac{\varepsilon}{2}Ev_{k(3)}\right)(1-rd_{k})x_{kk}\right)\end{aligned}\tag{38}$$

$$\mathbf{Min}\ Z_3=\phi\tag{39}$$

Subject to:

$$\lambda_k=\sum_{i}([O_{i(2)}+(1-\alpha)\big(O_{i(3)}-O_{i(2)}\big)]+[D_{i(2)}+(1-\alpha)\big(D_{i(3)}-D_{i(2)}\big))x_{ik}\tag{40}$$

$$\begin{aligned}\big([T_{ik(2)}-(1-\alpha)\big(T_{ik(2)}-T_{ik(1)}\big)]+W_k+[T_{kl(2)}-(1-\alpha)\big(T_{kl(2)}-T_{kl(1)}\big)]+W_l+[T_{lj(2)}\\ -(1-\alpha)\big(T_{lj(2)}-T_{lj(1)}\big)]\big)y_{ijk}\leq\phi\end{aligned}\tag{41}$$

$$\sum_{l\in J\setminus\{j\}}(f_{jl}^{i}-f_{lj}^{i})=\sum_{m\in I}[W_{im(2)}+(1-\alpha)\big(W_{im(3)}-W_{im(2)}\big)]\,(\sum_{k\in I\setminus\{i\}}y_{ikj}) - \sum_{m\in I\setminus\{j\}}[W_{im(2)}+(1-\alpha)\big(W_{im(3)}-W_{im(2)}\big)]\,(\sum_{k\in I\setminus\{m\}}y_{mkj}) + \sum_{m\in I}[W_{im(2)}+(1-\alpha)\big(W_{im(3)}-W_{im(2)}\big)]\,(x_{ij}-x_{mj}) \tag{42}$$

$$\sum_{l\in J\setminus\{j\}}(f_{jl}^{j}-f_{lj}^{j})=\sum_{m\in I}[W_{jm(2)}+(1-\alpha)\big(W_{jm(3)}-W_{jm(2)}\big)]\,(x_{jj}-x_{mj}-\sum_{k\in I\setminus\{m\}}y_{mkj}) \tag{43}$$

$$\sum_{j\in J\setminus\{i\}}(r_{ij}^{k}-r_{ji}^{k})=[O_{i(2)}+(1-\alpha)\big(O_{i(3)}-O_{i(2)}\big)]\sum_{m\in I\setminus\{i\}}y_{imk} \tag{44}$$

The other relevant constraints.

LAM:

$$\text{Min } E[Z_1] \tag{45}$$

$$\text{Max } E[Z_2] \tag{46}$$

$$\text{Min } Z_3=\phi \tag{47}$$

Subject to:

$$\lambda_k=\sum_{i}([O_{i(2)}-\alpha\big(O_{i(2)}-O_{i(1)}\big)]+[D_{i(2)}-\alpha\big(D_{i(2)}-D_{i(1)}\big))x_{ik} \tag{48}$$

$$\big([T_{ik(2)}+(1-\alpha)\big(T_{ik(3)}-T_{ik(2)}\big)]+W_k+[T_{kl(2)}+(1-\alpha)\big(T_{kl(3)}-T_{kl(2)}\big)]+W_l+[T_{lj(2)} + (1-\alpha)\big(T_{lj(3)}-T_{lj(2)}\big)]\big)y_{ijk}\le\phi \tag{49}$$

$$\sum_{l\in J\setminus\{j\}}(f_{jl}^{i}-f_{lj}^{i})=\sum_{m\in I}[W_{im(2)}-\alpha\big(W_{im(2)}-W_{im(1)}\big)]\,(\sum_{k\in I\setminus\{i\}}y_{ikj}) - \sum_{m\in I\setminus\{j\}}[W_{im(2)}-\alpha\big(W_{im(2)}-W_{im(1)}\big)]\,(\sum_{k\in I\setminus\{m\}}y_{mkj}) + \sum_{m\in I}[W_{im(2)}-\alpha\big(W_{im(2)}-W_{im(1)}\big)]\,(x_{ij}-x_{mj}) \tag{50}$$

$$\sum_{l\in J\setminus\{j\}}(f_{jl}^{j}-f_{lj}^{j})=\sum_{m\in I}[W_{jm(2)}-\alpha\big(W_{jm(2)}-W_{jm(1)}\big)]\,(x_{jj}-x_{mj}-\sum_{k\in I\setminus\{m\}}y_{mkj}) \tag{51}$$

$$\sum_{j\in J\setminus\{i\}}(r_{ij}^{k}-r_{ji}^{k})=[O_{i(2)}-\alpha\big(O_{i(2)}-O_{i(1)}\big)]\sum_{m\in I\setminus\{i\}}y_{imk} \tag{52}$$

The other relevant constraints.

4.2 Phase 2—*Developing the meta-heuristic algorithm*

In order to solve the resulting deterministic equivalent model we adapted the method GVIWO, first introduced by Mohammadi et al. (2016), to find optimal Pareto solutions. The algorithm GVIWO is a new multi-objective meta-heuristic approach that is based on a combination of three different components: the fuzzy invasive weed optimization (Mehrabian and Lucas, 2006), the variable neighborhood algorithm, and game theory.

5. Computational experiments

This section in dedicated to the numerical experiments that have carried out to validate the proposed model and to compare the performance of our developed method (that we will still call GVIWO) with respect to the very well-known multi-objective algorithms NSGA-II (Non-dominated Sorting Genetic Algorithm-II, by Deb et al., 2002) and MOPSO (multi-objective particle swarm optimization, by Coello and Lechunga, 2002).

5.1. *Parameters Setting*

It is particularly important to tune each parameter of a meta-heuristic algorithm given its great impact on the quality of the produced solution. In our case, statistical approaches such as response surface methodology (RSM) can be employed to identify the best set of factor levels (Mohammadi et al. 2013). Tables 1 and 2 show the values of NSGA-II and MOPSO as well as the values of GVIWO parameters used along all the experiments, respectively.

Table 1. NSGA-II and MOPSO parameters settings.

NSGA-II		MOPSO	
Parameter	Value	Parameter	Value
Population size	400	Max iterations	200
Crossover rate	0.78	No. of particles	30
NFC	30000	Date of damping	0.989
Mutation rate	0.1	C_1	1.3
		C_2	2.7
		α	6%

Table 2. GVIWO parameters settings.

Algorithm	Parameters and values			
GVIWO	Pop_{Size}	Pop_{max}	$iter_{max}$	S_{max}
	30	96	180	8
	S_{min}	sd_{min}	sd_{max}	pow
	2	0.7	0.017	1
	P_E	$ItVNS_{max}$	n_{Repeat}	Itr_{max}
	0.45	19	7	250

5.2 *Quality of the Pareto Solutions*

In order to validate the proposed model and to check the quality of the Pareto solution produced by our GVIWO method we start by considering two small-sized test problems : (i) Problem 1 has 8 total nodes and 3 hub nodes and (ii) Problem 2 with 10 nodes and 4 hub nodes. The distribution functions of the randomly generated parameters for both problems are shown in Table 3.

Table 3. Source of randomly generated parameters

Parameters	Generated values	
	Problem 1 (8#3)	Problem 2 (10#4)
α, β	~*U(0,1)*	~*U(0,1)*
w_{em}	~*U(5,10)*	~*U(6,11)*
w_{ed}	~*U(10,15)*	~*U(12,17)*
ur_k	~*U(0,1)*	~*U(0,1)*
g_{ij}	~*U(15,40)*	~*U(17,44)*
T_{ij}	~*U(0,10)*	~*U(0,12)*
c_{ij}	~*U(20,50)*	~*U(22,55)*
Fj_k	~*U(10,20)*	~*U(11,22)*
Vj_k	~*U(50,100)*	~*U(55,110)*
Ev_k	~*U(50,100)*	~*U(55,110)*
rd_k	~*U(0,1)*	~*U(0,1)*
w_{ij}	~*P(100)*	~*P(110)*

The exact Pareto frontier has been obtained by coding the developed models in GAMS software that used BARON as solver (Branch and Reduce Optimization Navigator) solver. The aim here is to assess the gap between the optimal solution attained by GAMS and the Pareto solutions generated by GVIWO, as well as the other two methods. Figs. 2 and 3 depict the Pareto frontiers of the GVIWO, NSGA-II and MOPSO algorithms compared to the exact solution identified by GAMS package for both the test problems, respectively. The figures show that the proposed GVIWO method can obtain Pareto solutions that are very close to the optimal Pareto frontier extracted by GAMS. It is clear that the different components constituting our proposed GVIWO algorithm have contributed in reaching such high quality solution. In particular, the competition between the different objectives (that have the role of players in the game theory) succeeded to search better the solution space and to, consequently, reach high-quality solutions with respect to the classical NSGA-II and MOPSO algorithms.

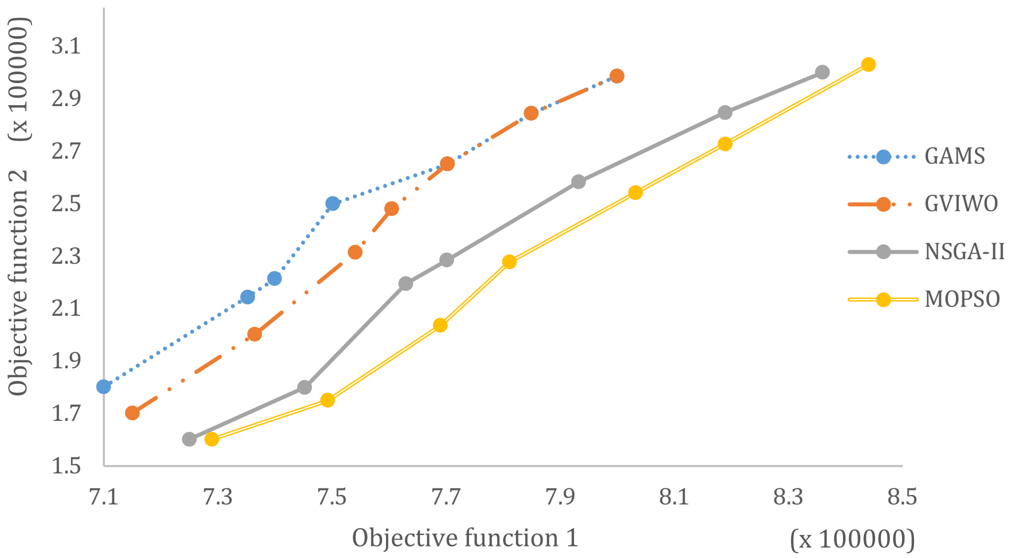

Fig. 2. Pareto frontier of GAMS, GVIWO, NSGA-II and MOPSO for Problem 1

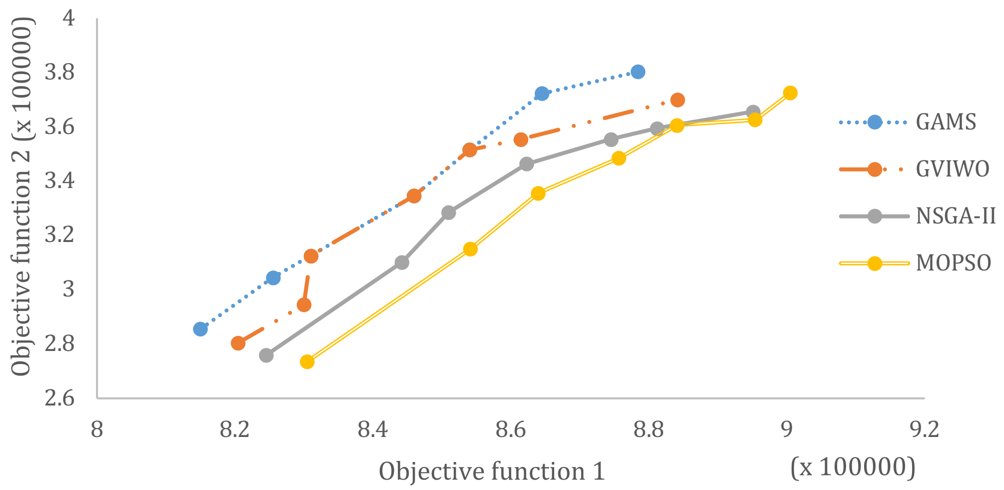

Fig. 3. Pareto frontier of GAMS, GVIWO, NSGA-II and MOPSO for Problem 2

5.3 *Comparative study*

The GVIWO algorithm is applied to a number of test problems and its performance is compared with MOPSO and NSGA-II with respect to four different comparison metrics, namely quality (QM), spacing (SM), diversity (DM) and mean ideal distance (MID) metrics (for the significance of each metric check Kaveh et al. 2020).

The parameters of the GVIWO, NSGA-II and MOPSO algorithms are first tabulated for small-sized problems consisting of 10 and 15 nodes. Problem instances are indicated as *"number of nodes # number of hubs"* (e.g., 10#3 means 10 nodes and 3 hubs to be located). Tables 4 and 5 show the comparison of the four QM, DM, SM and MID metrics considering these small–sized problems (best values are highlighted in bold font). The tables show that our GVIWO performs better than NSGA-II and MOPSO algorithms for all the metrics.

Table 4. Quality and spacing metrics for small-sized problems

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	MOPSO	NSGA-II	GVIWO	MOPSO	NSGA-II	GVIWO
10#3	0.2	0.1	**0.25**	0.434	0.479	**0.389**
10#4	0	0	**0.1**	0.476	0.511	**0.357**
15#3	0.3	0.1	**0.4**	0.328	0.586	**0.29**
15#4	0.1	0.05	**0.15**	0.572	0.611	**0.478**

Table 5. Diversity and mean ideal distance metrics for small-sized problems

Problem No.	Diversity metric (DM)			Mean ideal distance (MID)		
	MOPSO	NSGA-II	GVIWO	MOPSO	NSGA-II	GVIWO
10#3	0.933	0.812	**0.998**	0.308	0.469	**0.23**
10#4	0.946	0.874	**1**	0.579	0.654	**0.345**
15#3	0.961	0.552	**0.988**	0.519	0.458	**0.399**
15#4	0.725	0.728	**0.878**	0.447	0.468	**0.255**

The proposed algorithm is then applied to solve large-sized problems ranging from 30 to 70 nodes with different numbers of hubs to be located. The results reported in Table 6 (for QM and SM) and Table 7 (for DM and MID) show again that GVIWO outperforms the other methods for all the metrics.

Table 6. Quality and spacing metrics for large-sized problems

Problem No.	Quality metric (QM)			Spacing metric (SM)		
	MOPSO	NSGA-II	GVIWO	MOPSO	NSGA-II	GVIWO
30#7	0	0	**0.2**	0.638	0.759	**0.634**
30#8	0	0	**0.1**	0.968	1.011	**0.678**
40#8	0	0	**0.1**	0.601	0.780	**0.54**
40#10	0	0	**0.3**	0.457	0.746	**0.234**
50#8	0.1	0	**0.2**	0.511	0.484	**0.4**
50#10	0	0	**0.25**	0.638	0.632	**0.234**
50#12	0.05	0	**0.1**	0.695	0.558	**0.55**
70#10	0.1	0.05	**0.15**	0.547	0.637	**0.35**
70#12	0	0	**0.1**	0.757	1.063	**0.45**

Table 7. Diversity and mean ideal distance metrics for large-sized problems

Problem No.	Diversity metric (DM)			Mean ideal distance (MID)		
	MOPSO	NSGA-II	GVIWO	MOPSO	NSGA-II	GVIWO
30#7	0.974	0.818	**0.96**	0.464	0.677	**0.367**
30#8	0.754	0.624	**0.89**	0.456	0.529	**0.4**
40#8	0.917	0.879	**0.957**	0.609	0.669	**0.5**
40#10	0.857	0.583	**0.89**	0.575	0.575	**0.5**
50#8	**0.999**	0.784	**0.999**	0.645	0.734	**0.589**
50#10	0.765	0.594	**0.879**	0.664	0.708	**0.543**
50#12	0.731	0.847	**0.99**	0.597	0.667	**0.589**
70#10	0.733	0.643	**0.879**	0.400	0.745	**0.367**
70#12	0.819	0.780	**0.89**	0.660	0.796	**0.555**

The above results have been combined and depicted in graph form, as reported in the Appendix. Among the four metrics, QM is the most important since it affects directly the solution quality and, with a lower extend, spacing metric that measures the uniformity of the spread and spacing of the solutions. For both these metrics, the results reported in the above tables and those depicted in Fig. 10–13 in the Appendix show a clear superiority of GVIWO with respect to the other two algorithms.

Finally, we aim at testing further the performance of GVIWO compared to NSGA-II and MOPSO in terms of the DM, SM and MID metrics. For this purpose perform the "non-parametric Friedman test" (Scheff, 2016) on 56 test problems while using the SPSS software for analyzing the data. Table 8 shows how for 55 degrees of freedom the significances (2-tailed) are nearly 0.000. The table also shows that there are significant statistical differences between the solutions obtained by GVIWO and the other two algorithms.

Table 8. Detailed statistics of paired *t*-test

		Paired Differences					*t*	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
Metric	Pair	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	Mean	Std. Deviation	Std. Error Mean
SM	MOPSO - GVIWO	-.32184	.17026	.02080	-.36336	-.28031	-15.473	55	.001
DM		.30881	.19944	.02436	.26016	.35745	12.674	55	.002
MID		-.39251	.16623	.02031	-.43306	-.35196	-19.327	55	.000
SM	NSGA-II - GVIWO	-.22330	.14833	.01812	-.25948	-.18712	-12.322	55	.000
DM		.20752	.18412	.02249	.16261	.25243	9.226	55	.000
MID		-.22188	.18467	.02256	-.26692	-.17684	-9.835	55	.000

5.4 *Sensitivity analysis*

In order to assess the effect of the various parameters on the objective functions, some sensitivity analyses are performed and the results are depicted in this subsection. First, the sensitivity of all three objective functions upon increasing the number of hub nodes is investigated. More specifically, Fig. 4 (left) shows that increasing the number of hub nodes, will result in an increase in the total cost (i.e., the first objective function). Establishing a higher number of hub nodes will clearly involve a higher fixed cost and more significant routing cost.

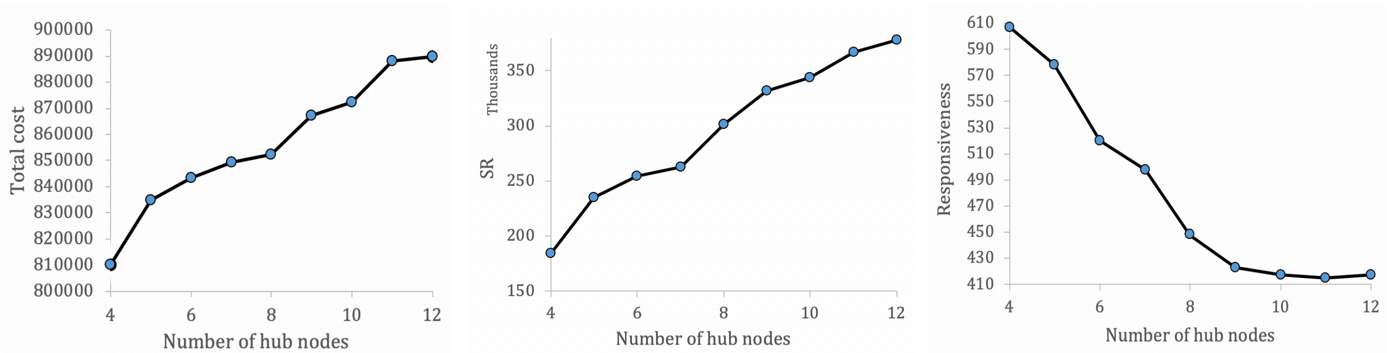

Fig. 4. Total cost, Social responsibility and Responsiveness vs. number of hub nodes

On the other hand, increasing the number of hubs has positive effect on the social responsibility. Fig. 4 (middle) shows that establishing more hubs will contribute in creating more job opportunities (thus, in decreasing the rate of unemployment) and impact positively on the regional economical development. Concerning the third objective function, a higher number of hub nodes will lead to a lower congestion at the hubs, which will reduce the waiting time and raise the responsiveness of the network. However, such an attractive effect can be observed till a certain extent. Indeed, Fig. 4 (right) shows that when the number of hub nodes exceeds 9, the systems cannot achieve further significant improvement in the third objective function value.

The second set of analyses the mean value of flow units increases, the first objective function will increase not only because of the augmented transportation cost but also because of it will be necessary to activate more hubs with larger capacity (see Fig. 5, left). In return to cope up with an increased flow, more hub nodes having larger capacity should be built which will engender more job vacancies and more development in the region. This can be seen in Fig. 5 (middle) where the value of SR increases, even though not monotonically, with the mean flow. Not surprisingly, when the mean of flow unit increases, the congestion at each hub node will also increase and that will lead to a higher flow waiting time which will deteriorate the network's responsiveness (as depicted in Fig. 5, right).

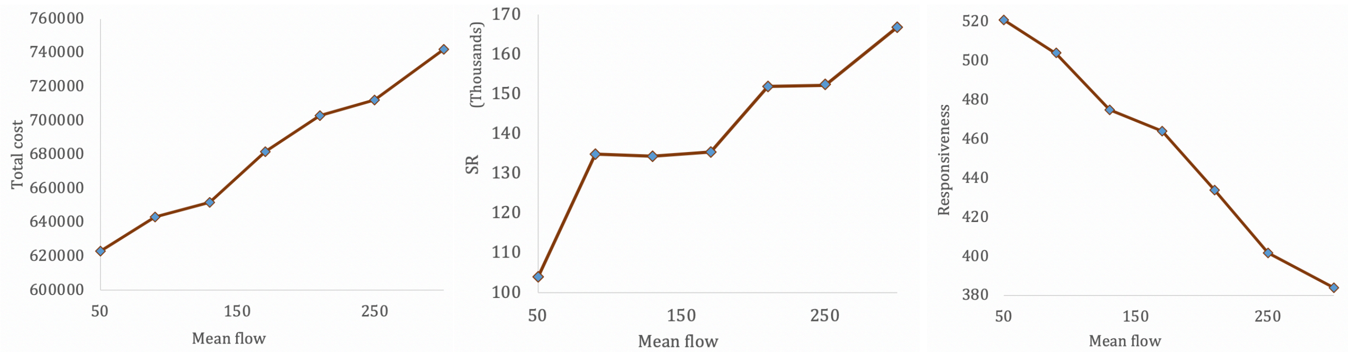

Fig. 5. Total cost, Social responsibility and Responsiveness vs. mean flow

As a final experiment within this section, we will check the effect of the queue capacity on the most important criteria related to the customers satisfaction, the responsiveness. Fig. 6 shows that a higher finite capacity of the queue will allow more flow to enter the hubs which will increase the congestion and the waiting time in the hubs and will, consequently, reduce the responsiveness ability of the network.

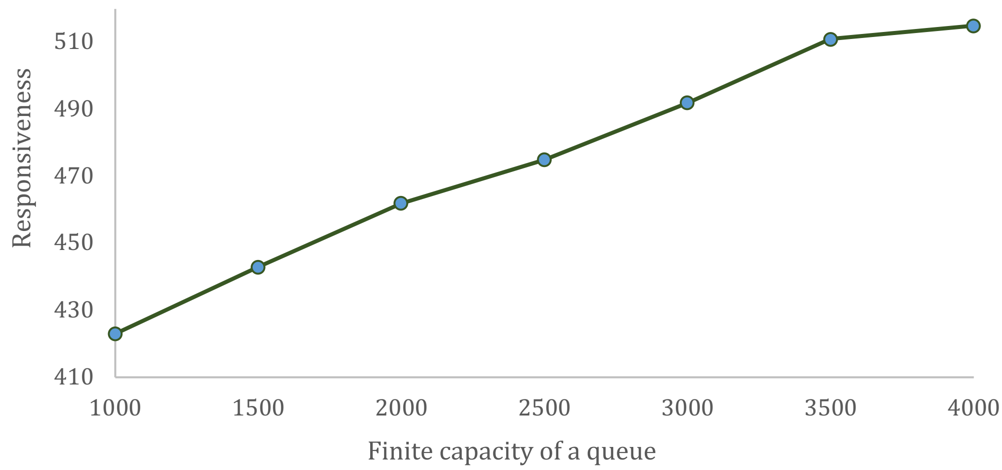

Fig. 6. Responsiveness vs. finite capacity of a queue

5.5 *Case study*

In this section, a real-case study of transportation in Iran is used to validate further the performance of the proposed model. The statistical data we employed are related to an instance of the Road Transportations of Iran (ROI) involving 37 cities, whose details (distance and fixed costs) are made available by the company. In this case, the costs of establishing hubs, hub-hub transportation and hubs-non-hubs transportation are minimized. The factors σ and β are set to 0.75 and 0. 8, respectively.

We solved the ROI instance by using our proposed GVIWO algorithm for two different values of number of hubs to be activated, i.e. p = 4 and p = 5. Fig. 7 depicts the solution of the transportation network that consists in activating the following 4 hub nodes: Mashhad, Tehran, Isfahan and Kerman, given their population and their importance in the country's economy. Each of the adjacent city is connected to one of the hubs either with a direct link or with a local tour. For instance, the figure shows that the city of *Rasht* is directly connected to *Tehran* given its high level of demand; On the other hand, the cities of *Qom* and *Arak* are assigned to *Tehran* by a local tour. Likewise, Fig. 8 shows the produced network involving 5 hub nodes. In this new topography, the city of *Tabriz* has been also selected as a hub and *Ardabil*, *Zanjan*, *Sanandaj* and *Uroomieh* are now connected to *Tabriz* instead of *Tehran*. The detailed results for the 4-hub and 5-hub scenarios are shown in Table 9. The last three columns demonstrate the first, second and third objective function values, respectively It is clear that by activating one more hub, the overall cost will decrease since the new topology succeed to reduce the transportation connections.

One may wonder if further savings can be achieved if more hubs are to be activated in the network. Our last experiment, whose results are summarized in Fig. 9, address this question and show that the total cost reaches its minimum when 5 hub nodes are established.

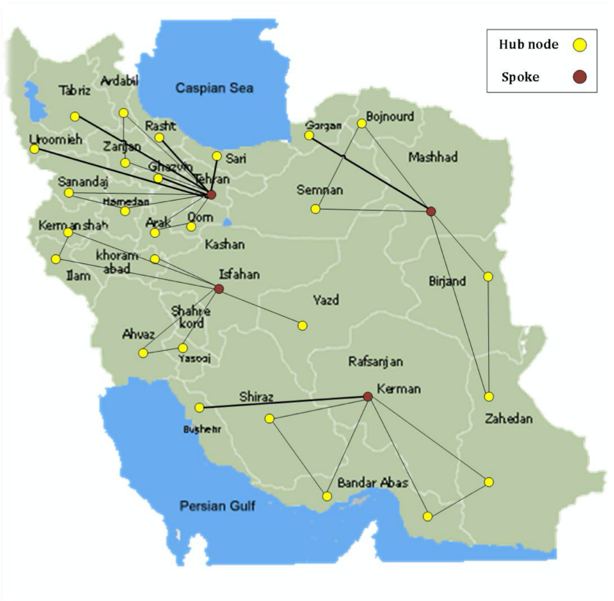

Fig. 7. Hub topography for 4 hub nodes

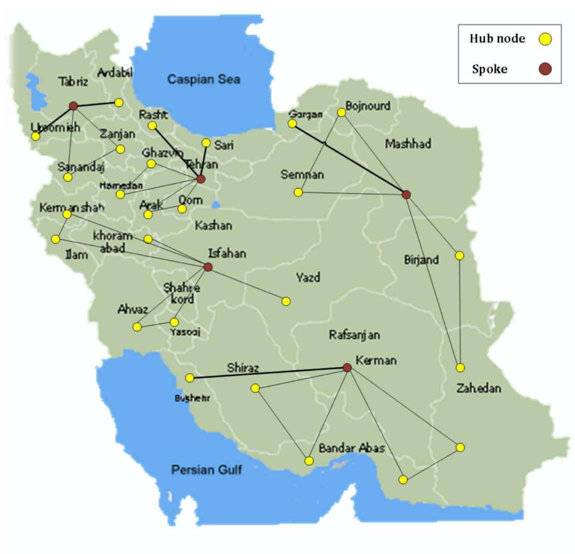

Fig. 8. Hub topography for 5 hub nodes

Table 9. Detailed cost of the real-case study

p	Hub cities	OF1 (Z_1)	OF2 (Z_2)	OF3 (Z_3)
4	Tehran, Isfahan, Mashhad, Kerman	62340000	171243	845
5	Tehran, Isfahan, Mashhad, Kerman, Tabriz	59340000	185456	794

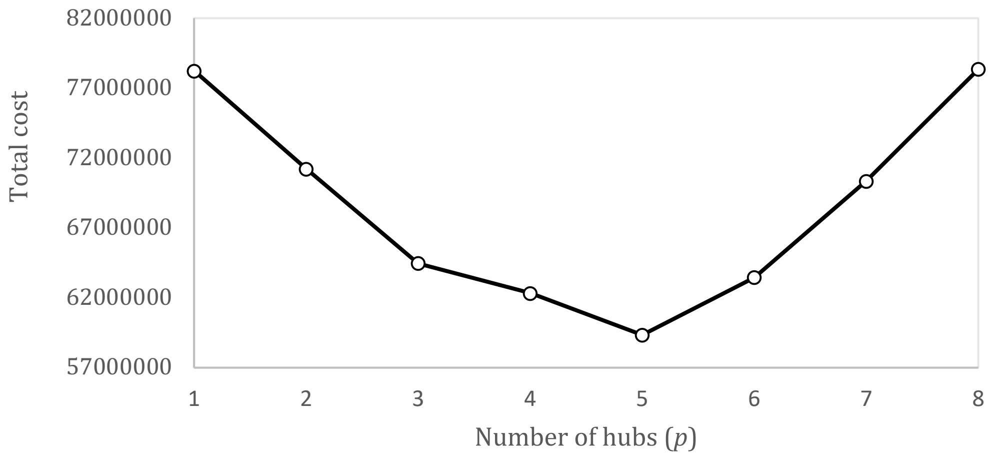

Fig. 9. Total cost vs. number of hub nodess

6 Conclusion

This paper proposes a multi-objective mixed-integer single-allocation hub location and routing problem considering the economic aspect, social responsibility and responsiveness of the network. Within this hub location-routing problem, every non-hub node has two options to be served: (i) either directly assigned to a hub node or (ii) visited through a local tour. Furthermore, to calculate the waiting time and minimize the maximum transportation cost between each pair O-D nodes, an $M/M/c/K$ queue system was applied. One of the major contribution of this study is to attempt to boost the employment and regional development through incorporating the social responsibility as objective function. Given the complexity of the resulting model, an efficient evolutionary approach (GVIWO) based on fuzzy invasive weed optimization, on the variable neighborhood algorithm and on game theory was developed to solve small- and large-sized problems and obtain the near-Pareto solutions. The intensive computational experiments we carried out showed that GVIWO overperformed well known approaches such as MOPSO and NSGA-II. Moreover, sensitivity analysis was carried out to show the influence of the mean flow, number of hubs, and queue capacity on the objective functions. Finally, a real transportation case in Iran was studied to validate the applicability of the proposed model and solution approach in the real world.

As possible future extensions on the presented study we can suggest incorporating the stochastic nature of the demand and also of the capacity on both hub and hub-to-hub flow into the model. Moreover, the transportation medium can be enriched by involving different shipping channels (modes) to enhance the capacity of the hub-to-hub links and reduce the disruption risk.

References

Ahmadi-Javid, A., & Seddighi, A.H. (2012). A location-routing-inventory model for designing multisource distribution networks, Engineering Optimization, 44 (6), 637-656.

Aykin, T. (1994). Lagrangian relaxation based approaches to capacitated hub-and-spoke network design problem. European Journal of Operational Research, 79(3), 501-523.

Campbell, J. F. (1994). Integer programming formulations of discrete hub location problems. European Journal of Operational Research, 72(2), 387-405.

Coello, C., & Lechunga, M. (2002). MOPSO: A Proposal for Multiple Objective Particle Swarm Optimization. In: Proceedings of the 2002 Congress on Evolutionary Computation, IEEE Press, 1051–1056.

Da Graça Costa, M., Captivo, M. E., & Clímaco, J. (2008). Capacitated single allocation hub location problem—A bi-criteria approach. Computers & Operations Research, 35(11), 3671-3695.

de Camargo, R. S., Miranda, G. D., & Luna, H. P. (2008). Benders decomposition for the uncapacitated multiple allocation hub location problem. Computers & Operations Research, 35(4), 1047-1064.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multi-objective genetic algorithm: NSGA-II. Evol. Comput. IEEE Trans. 6(2),182–197.

Dehghanian, F., & Mansour, S. (2009). Designing sustainable recovery network of end-of-life products using genetic algorithm. Resources, Conservation and Recycling, 53(10), 559-570.

Devika, K., Jafarian, A., & Nourbakhsh, V. (2014). Designing a sustainable closed-loop supply chain network based on triple bottom line approach: A comparison of metaheuristics hybridization techniques. European Journal of Operational Research, 235(3), 594-615.

Dubois, D., Prade, H., (2012). Possibility Theory: An Approach to Computerized Processing of Uncertainty. Springer Science & Business Media

Ebery, J., Krishnamoorthy, M., Ernst, A., & Boland, N. (2000). The capacitated multiple allocation hub location problem: Formulations and algorithms. European Journal of Operational Research, 120(3), 614-631.

Eiselt, H. A., & Marianov, V. (2009). A conditional p-hub location problem with attraction functions. Computers & Operations Research, 36(12), 3128-3135.

Elhedhli, S., & Hu, F. X. (2005). Hub-and-spoke network design with congestion. Computers & Operations Research, 32(6), 1615-1632.

Ernst, A. T., & Krishnamoorthy, M. (1998). Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem. European Journal of Operational Research, 104(1), 100-112.

Ghodratnama, A., Tavakkoli-Moghaddam, R., & Azaron, A. (2015). Robust and fuzzy goal programming optimization approaches for a novel multi-objective hub location-allocation problem: A supply chain overview. Applied Soft Computing, 37, 255-276.

Grove, Peter G., and Morton E. O'Kelly. 1986. "Hub networks and simulated schedule delay." Papers in Regional Science.

Ishfaq, R., & Sox, C. R. (2012). Design of intermodal logistics networks with hub delays. European Journal of Operational Research, 220(3), 629-641.

Kaveh, F., Tavakkoli-Moghaddam, R., Triki, C., Rahimi, Y., & Jamili A. (2020). A New Bi-objective Model of the Urban Public Transportation Hub Network Design Under Uncertainty. To appear on Annals of Operations Research.

Liu, B., & Iwamura, K. (1998). Chance constrained programming with fuzzy parameters. Fuzzy sets and systems, 94(2), 227-237.

Liu, B., & Liu, Y. K. (2002). Expected value of fuzzy variable and fuzzy expected value models. IEEE transactions on Fuzzy Systems, 10(4), 445-450.

Marianov, V., & Serra, D. (2003). Location models for airline hubs behaving as M/D/c queues. Computers & Operations Research, 30(7), 983-1003.

Masoumzadeh, S., Solimanpur, M., & Kamran, M. A. (2016). A multi-objective fuzzy goal programming P-hub location and protection model with back-up hubs considering hubs establishment fixed costs. Scientia Iranica. Transaction E, Industrial Engineering, 23(4), 1941.

Mehrabian, A. R., & Lucas, C. (2006). A novel numerical optimization algorithm inspired from weed colonization. Ecological informatics, 1(4), 355-366.

Meier, J. F. (2017). An improved mixed integer program for single allocation hub location problems with stepwise cost function. *International Transactions in Operational Research*, *24*(5), 983-991.

Mohammadi, M., Jolai, F., & Tavakkoli-Moghaddam, R. (2013). Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm. Applied Mathematical Modelling, 37(24), 10053-10073.

Mohammadi, M., Tavakkoli-Moghaddam, R., Siadat, A., & Rahimi, Y. (2016). A game-based meta-heuristic for a fuzzy bi-objective reliable hub location problem. Engineering Applications of Artificial Intelligence, 50, 1-19.

Mota, B., Gomes, M. I., Carvalho, A., & Barbosa-Povoa, A. P. (2015). Towards supply chain sustainability:

economic, environmental and social design and planning. Journal of Cleaner Production, 105, 14-27.
Mousavi, S.M., Tavakkoli-Moghaddam, R., & Jolai, F. (2013). A possibilistic programming approach for the location problem of multiple cross-docks and vehicle routing scheduling under uncertainty, Engineering Optimization, 45(10), 1223-1249.
O'kelly, M. E. (1987). A quadratic integer program for the location of interacting hub facilities. European Journal of Operational Research, 32(3), 393-404.
Parvaresh, F., Husseini, S. M., Golpayegany, S. H., & Karimi, B. (2014). Hub network design problem in the presence of disruptions. Journal of Intelligent Manufacturing, 25(4), 755-774.
Peterson, M. D., Bertsimas, D. J., & Odoni, A. R. (1995). Models and algorithms for transient queueing congestion at airports. Management Science, 41(8), 1279-1295.
Pishvaee, M. S., Razmi, J., & Torabi, S. A. (2012). Robust possibilistic programming for socially responsible supply chain network design: A new approach. Fuzzy sets and systems, 206, 1-20.
Pishvaee, M. S., Razmi, J., & Torabi, S. A. (2014). An accelerated Benders decomposition algorithm for sustainable supply chain network design under uncertainty: A case study of medical needle and syringe supply chain. Transportation Research Part E: Logistics and Transportation Review, 67, 14-38.
Rabbani, M., Zhalechian, M., & Farshbaf-Geranmayeh, A. (2018). A robust possibilistic programming approach to multiperiod hospital evacuation planning problem under uncertainty. International Transactions in Operational Research, 25(1), 157-189.
Rahimi, Y., Tavakkoli-Moghaddam, R., Mohammadi, M., & Sadeghi, M. (2016). Multi-objective hub network design under uncertainty considering congestion: An M/M/c/K queue system. Applied Mathematical Modelling, 40(5), 4179-4198.
Rahmati, S. H. A., Ahmadi, A., Sharifi, M., & Chambari, A. (2014). A multi-objective model for facility location–allocation problem with immobile servers within queuing framework. Computers & Industrial Engineering, 74, 1-10.
Rodriguez, V., Alvarez, M. J., & Barcos, L. (2007). Hub location under capacity constraints. Transportation Research Part E: Logistics and Transportation Review, 43(5), 495-505.
Sasaki, M., & Fukushima, M. (2003). On the hub-and-spoke model with arc capacity conatraints. Journal of the Operations Research Society of Japan, 46(4), 409-428.
Scheff, S. W. (2016). Fundamental Statistical Principles for the Neurobiologist. Academic Press, Alsevier Inc.
Skorin-Kapov, D., Skorin-Kapov, J., & O'Kelly, M. (1996). Tight linear programming relaxations of uncapacitated p-hub median problems. European Journal of Operational Research, 94(3), 582-593.
Tavakkoli-Moghaddam, R., Noshafagh, S.V., Taleizadeh, A.A., Hajipour, V., & Mahmoudi, A. (2017). Pricing and location decisions in multi-objective facility location problem with M/M/m/k queuing systems, Engineering Optimization, 49(1), 136-160.
Van Woensel, T., Wuyts, B., & Vandaele, N. (2006). Validating state-dependent queueing models for uninterrupted traffic flows using simulation. 4OR: A Quarterly Journal of Operations Research, 4(2), 159-174.
Xu, J., & Zhou, X. (2013). Approximation based fuzzy multi-objective models with expected objectives and chance constraints: Application to earth-rock work allocation. Information Sciences, 238, 75-95.
Zhalechian, M., Tavakkoli-Moghaddam, R., Rahimi, Y., & Jolai, F. (2017a). An interactive possibilistic programming approach for a multi-objective hub location problem: Economic and environmental design. Applied Soft Computing, 52, 699-713.
Zhalechian, M., Tavakkoli-Moghaddam, R., & Rahimi, Y. (2017b). A self-adaptive evolutionary algorithm for a fuzzy multi-objective hub location problem: An integration of responsiveness and social responsibility. Engineering Applications of Artificial Intelligence, 62, 1-16.

Appendix:

Fig. 10, 11, 12 and 13 summarize the behavior of the three meta-heuristic algorithms with respect to each of the four QM, DM, SM and MID metrics. All the four figures show clearly the good performance of our GVIWO compared to both NSGA-II and MOPSO algorithms.

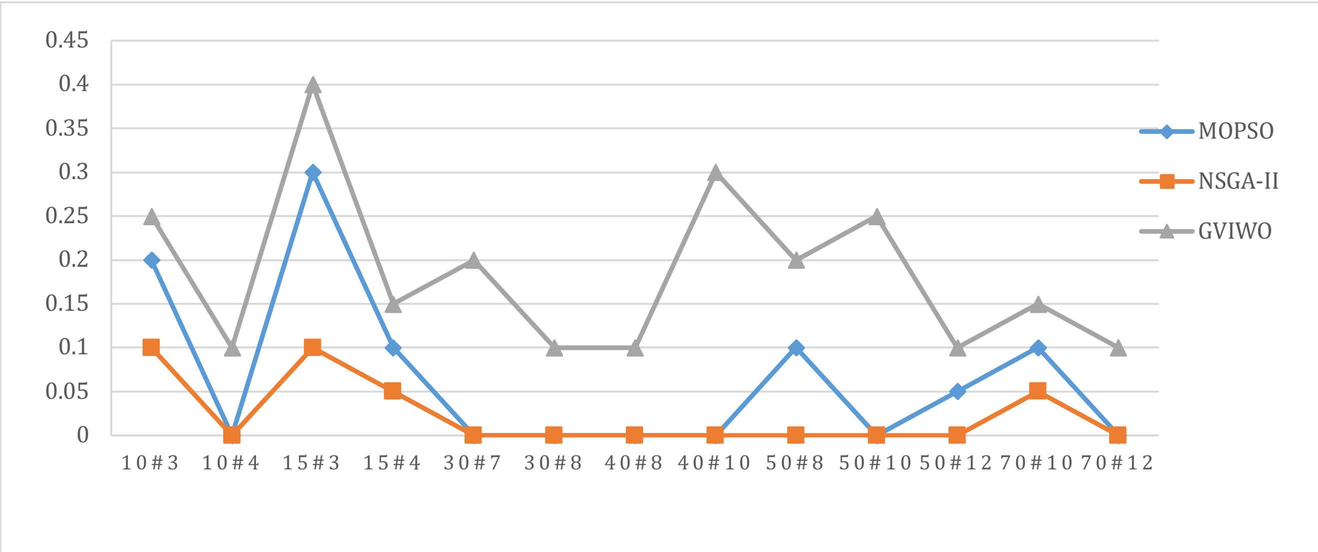

Fig. 10. Comparing the three multi-objective meta-heuristic algorithms with respect to Quality Metric

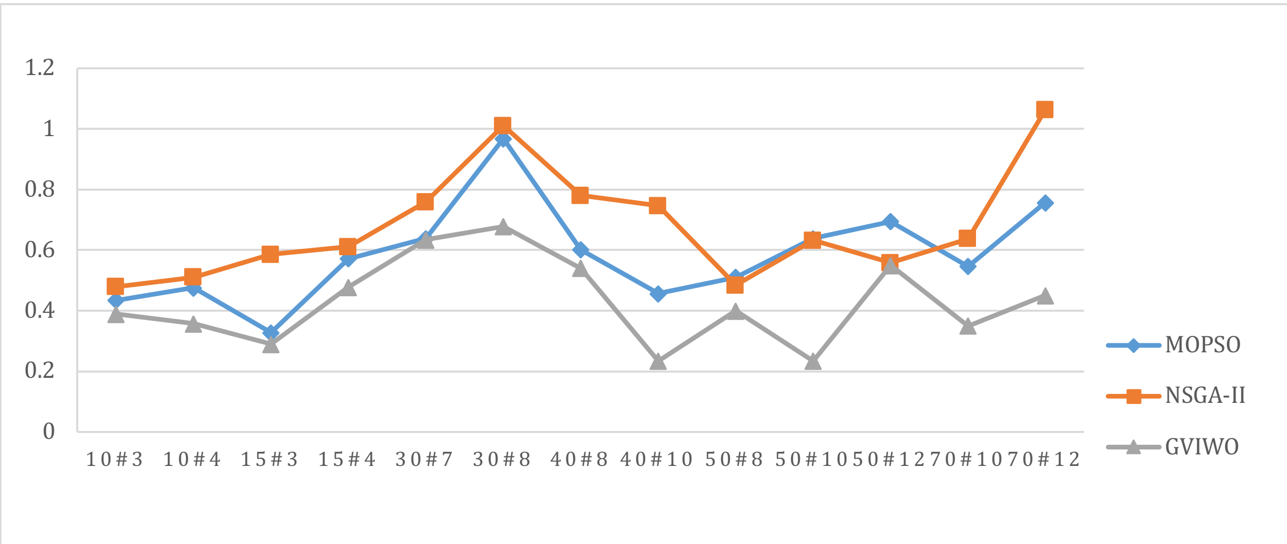

Fig. 11. Comparing the three meta-heuristic algorithms with respect to Spacing metric

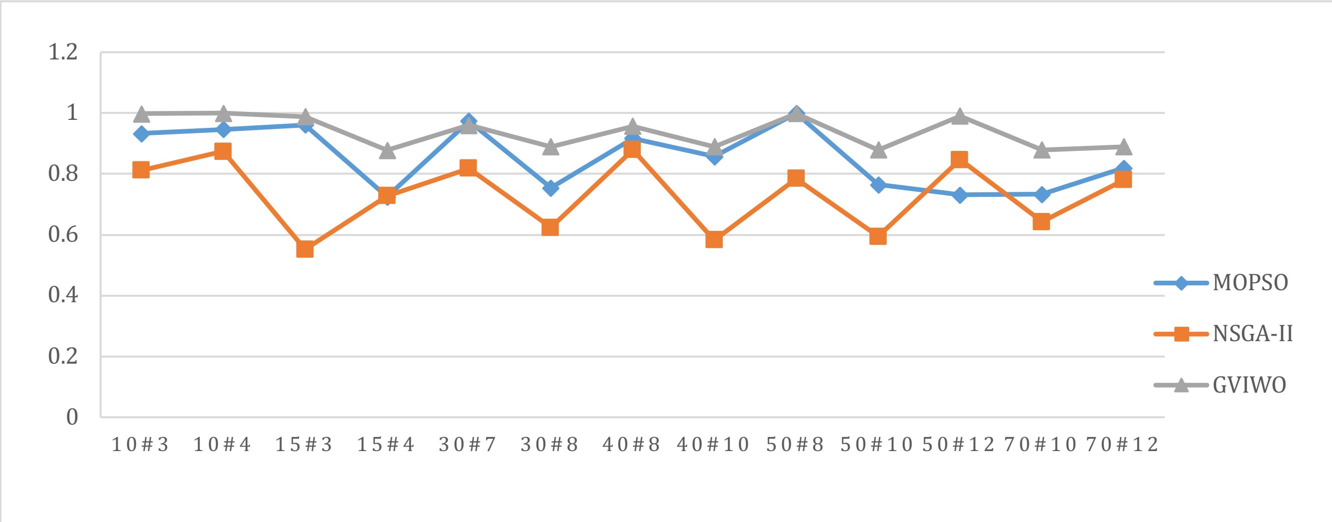

Fig. 12. Comparing the three meta-heuristic algorithms with respect to Diversity metric

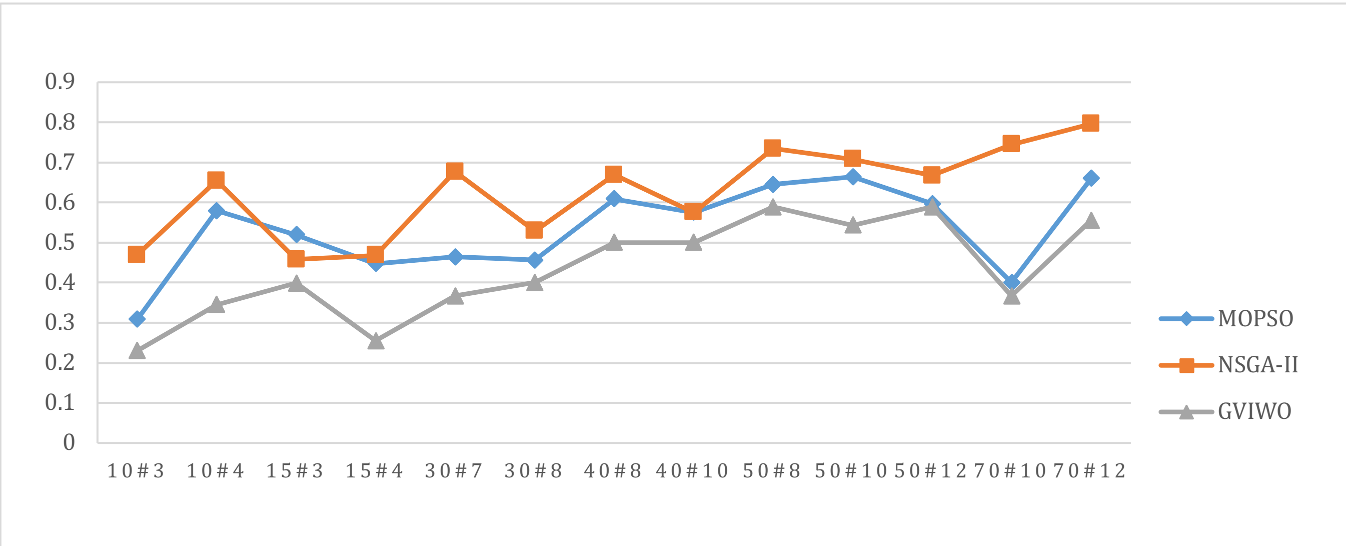

Fig. 13. Comparing the three meta-heuristic algorithms with respect to Mean ideal distance