**Reasoning without Contradiction**

Adding or subtracting a tautology to its premises will have no effect on the validity of an argument, so it is reasonable to believe that tautologies are not required for reasoning. But contradictions, it seems, feature in tried and trusted proof procedures, so one might suppose that, were contradiction somehow banned from proof procedures, incompleteness would result. But I think that this is untrue and that a case can be made that reasoning without contradiction has some very beneficial consequences. Limitations of space prevent me giving more than a sketch, but I set out the argument in such a way that the reader can easily identify where, if anywhere, s/he thinks it goes wrong:

1. The utterance ‘It is raining or it is not’ literally gives no information at all about the weather (Wittgenstein, *Tractatus* 4.461) and provides the hearer no guide to action, such as to take or not take an umbrella.
2. The utterance says nothing about anything else. (If it is not about the weather, what is it about?)
3. Therefore it says nothing at all; *aliter*, is completely uninformative.
4. *A fortiori*, it says nothing true and nothing false, and a similar conclusion will hold for any tautology.
5. Therefore its negation says nothing true and nothing false (think of prefixing a meaningless word-salad with ‘Not’).
6. The negation of a tautology is a contradiction.
7. So equally a contradiction says nothing. In particular,
8. A contradiction says nothing false.

This is Wittgenstein’s thesis, enunciated in his dictation to Moore in Norway, 1914 (*Notebooks 1914-16*: 108), in his 1939 Cambridge *Lectures on the Foundations of Mathematics* (p.222) and in his late manuscripts, where he insists that, although the sentence ‘It is raining and it is not’ can be used to mean that there is a very light drizzle, there can be no contradictions where one deliberately speaks against (contra-dicere) oneself, uttering a sentence of the form ┌A & ~A┐ and meaning it literally. Such contradictions have no sense; we have no clear-cut use for them (*Remarks on the Philosophy of Psychology, Vol.2*: §290).

But surely, it might be said, we make frequent use of contradictions in reasoning. Well, in classical logic, we derive conclusions (in fact, any conclusion) from contradictions. But if a contradiction says nothing, and nothing follows from nothing, then we need to abandon the classical principle *ex contradictione quodlibet* as being not characteristic of good reasoning*.* Several logics do just that and, as Graham Priest has shown (‘Negation as Cancellation, and Connexive Logic’, *Topoi* 18 (1999): 141-148), consistent connexivist logics do, and are tied to a view of negation as cancellation that is, in my view, attractive and natural. The rule &-E is valid except where the conjunction contains a wff and its negation.

*Reductio ad Absurdum* is a healthy rule of inference unaffected by the claim that contradictions say nothing. Informally, the rule is*:* if you want to get something useful (non-vacuous) from an inconsistent set of premises, eliminate at least one of those premises. The contradiction reached in a *reductio* proof is not *used*; it simply serves as a sign warning us to turn back and revise the argument in the way just suggested. Sextus Empiricus (c. 160-210 AD) expressed this beautifully: ‘Suppose there were a road leading up to a chasm, we do not push ourselves into the chasm just because there is a road leading to it but we avoid the road because of the chasm; so, in the same way, if there should be an argument which leads us to a confessedly absurd conclusion, we shall not assent to the absurdity just because of the argument but avoid the argument because of the absurdity’ (Sextus Empiricus, *Outlines of Pyrrhonism,* 1933, Vol. 1, p. 252).

Reasoning does not require contradiction.

A corollary:

1. Nothing follows from nothing, in that you can get no information from no information.
2. ┌ ((x)(x ϵ R & ~(x ϵ x)) ┐ is a contradiction — instantiate x with R.
3. Therefore nothing follows from it.
4. Therefore in any logic (e.g. classical) in which ┌ A & ~A ┐ is truth-functionally, or informationally, equivalent to ┌ A ↔ ~A ┐, nothing follows from ┌ ((x)(x ϵ R ↔ ~(x ϵ x)) ┐.
5. Russell proved that Frege’s *Grundgesetze* system is inconsistent by invoking the impredicative function ┌ ~(x ϵ x) ┐. Russell’s proof did not establish the existence of a Russell class R, because *nothing*, including the existence of R, follows from ┌ ((x)(x ϵ R ↔ ~(x ϵ x)) ┐.
6. In general, in any definition of a set by the abstraction principle ┌ (∃y)(x)(x ϵ y ↔ Fx) ┐, we must forbid substituends for x, y and F in the scope ┌ (x ϵ y ↔ Fx)  ┐ such that the result of any such substitution entails a tautology or a contradiction; for tautologies and contradictions say nothing and, in particular, fail to state conditions or give a recipe for determining whether any given object belongs in the set. The Russellian ‘definition’ ┌ (∃y)(x)(x ϵ R & ~(x ϵ x)┐, as we have seen, falls foul of this prohibition, but there is, for example, nothing to prohibit the definition of the empty set as ┌(∃y)(x)(x ϵ y ↔ ⊥)┐.
7. Naïve set theory (which surely captures rather faithfully our intuitions about sets), plus some principled, intuitive restriction, is superior to a philosophically ill motivated axiomatic system. The Wittgenstein-inspired proposal here is motivated not by the need to avoid contradiction, but by the requirement to avoid illicit definition.

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